Using single-objective heuristics to solve bi-objective problems:

Short Introduction & Insights

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Outline

Motivation and Basics	• Side-objectives in travel and routing contexts • Pareto-dominance, solution sets, trade-offs
The epsilon-constraint method	• Principles • Use with heuristics (?!)
Bi-objective CVRP example	• Implementation • Results compared to specialized methods

Best departing flights

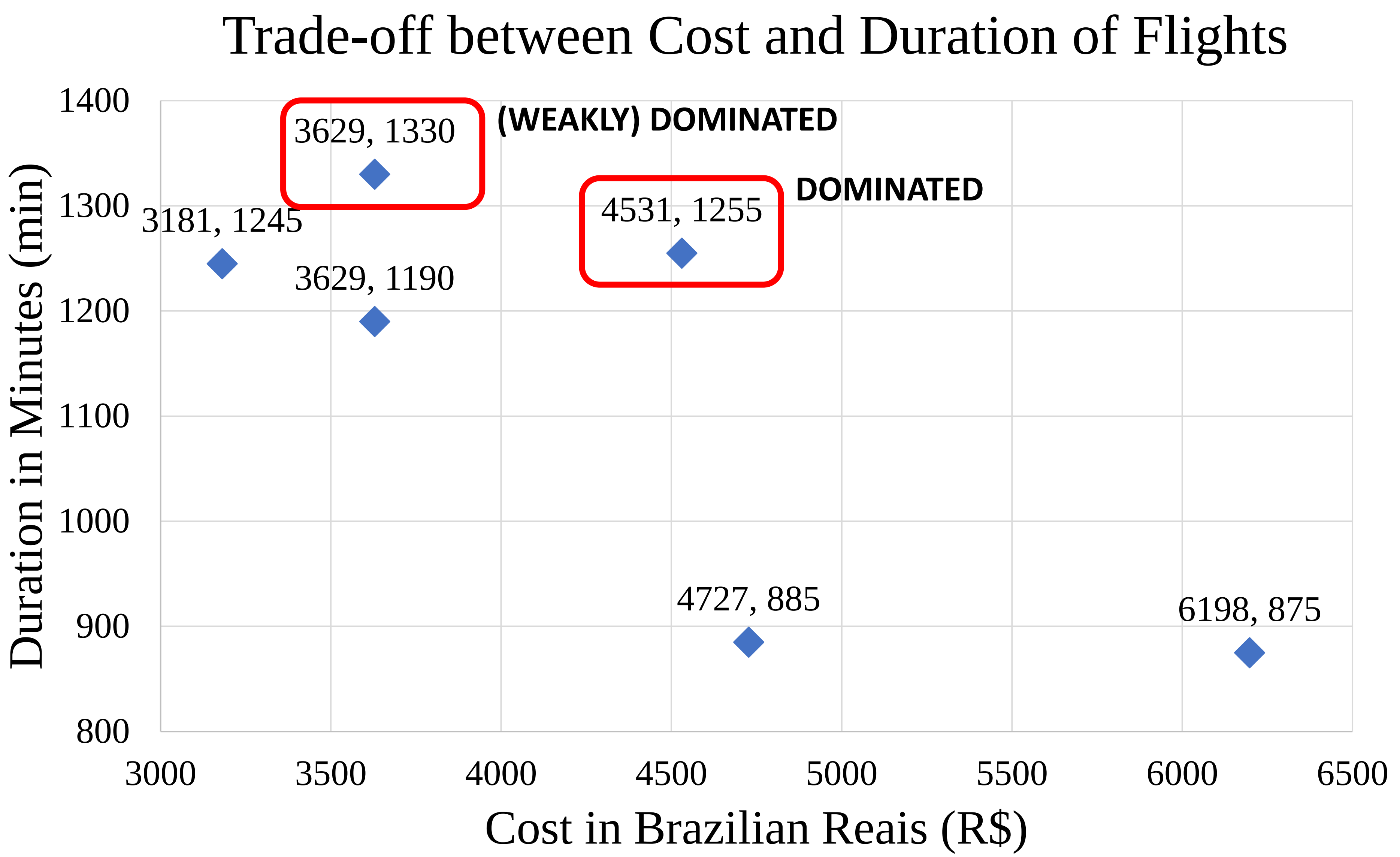
We chose options that give you the best trade-off between price and convenience, based on factors such as duration, number of stops, and airport changes during layovers.

8:05 PM – 10:5 Austrian, SWISS, L			
7:10 PM – 4:55 AM+1 Lufthansa	14h 45m VIE–GIG	1 stop 1h 40m FRA	R$4,727 round trip
7:45 PM – 5:20 AM+1 Tap Air Portugal	14h 35m VIE–GIG	1 stop 1h 5m LIS	R$6,198 round trip

Other departing flights

7:00 PM – 10:45 AM+1 SWISS, LATAM · Operated by Latam Airlines Brasil	20h 45m VIE–SDU	2 stops Change of airport	R$3,181 round trip
8:05 PM – 1:15 PM+1 Austrian, SWISS, Avianca Brazil	22h 10m VIE–GIG	2 stops ZRH, GRU	R$3,629 round trip
7:00 PM – 10:55 AM+1 SWISS, LATAM · Operated by Latam Airlines Brasil	20h 55m VIE–GIG	2 stops ZRH, GRU	R$4,531 round trip

85 longer or more expensive flights


Trade-off between Cost and Duration of Flights
(WEAKLY) DOMINATED
DOMINATED
3629, 1330
3181, 1245
3629, 1190
4531, 1255
4727, 885
6198, 875
Duration in Minutes (min)
1400
1300
1200
1100
1000
900
800
3000
3500
4000
4500
5000
5500
6000
6500
Cost in Brazilian Reais (R$)

Why not use a constraint?

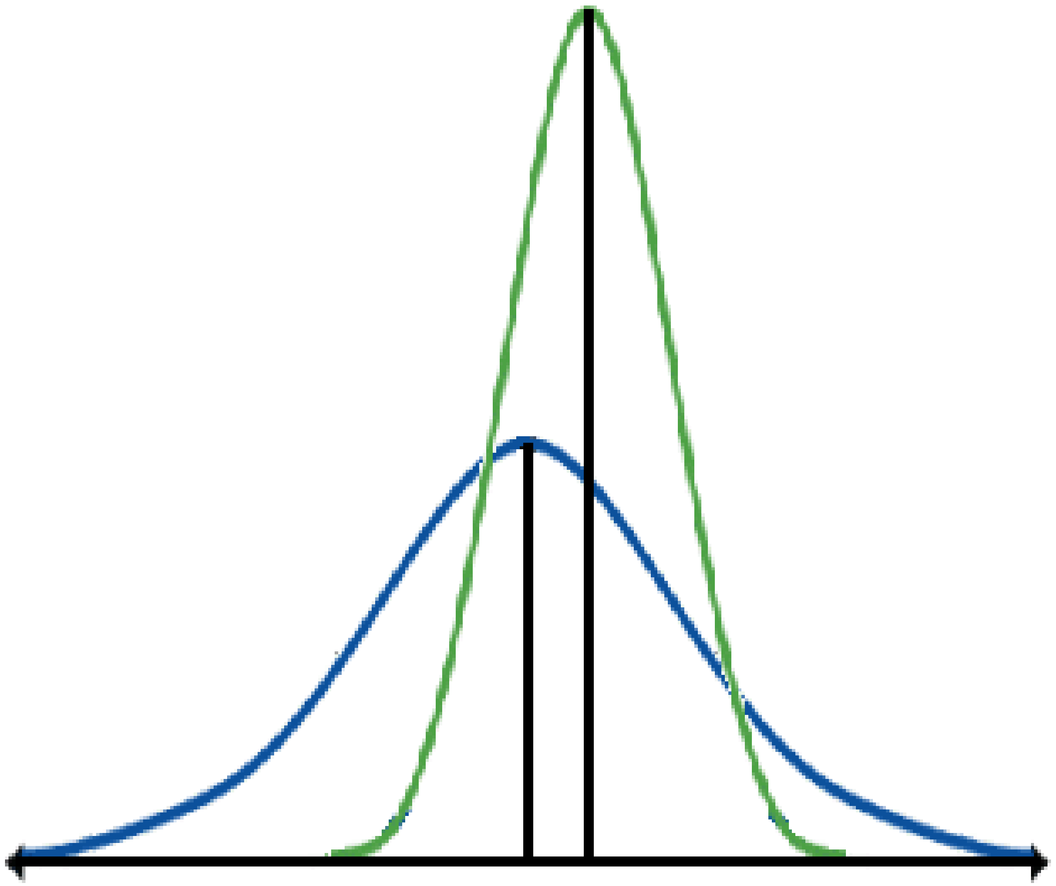
Maximum duration too strict…

- If we allowed only *a little* longer duration, perhaps we could reduce cost *a lot*.

Maximum duration not strict enough…

- If we pay only *a little* more, perhaps we could reduce the duration *a lot*.

Why consider side-objectives in VRPs?



An “optimal” solution should be…

… **robust** to noise in the input.

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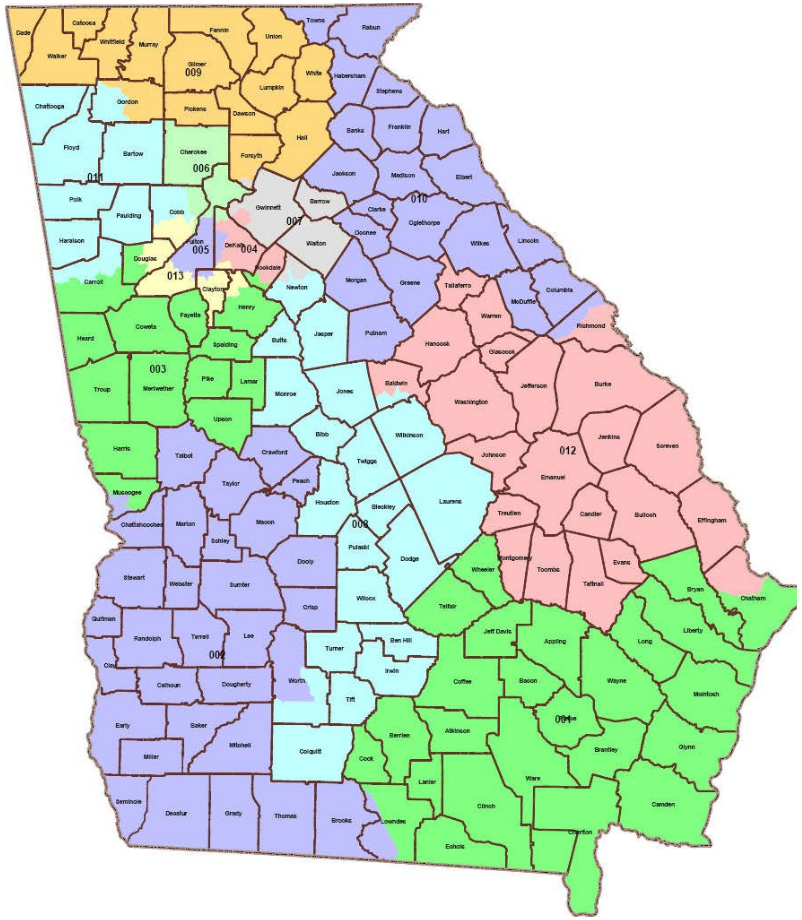


An "optimal" solution should be…

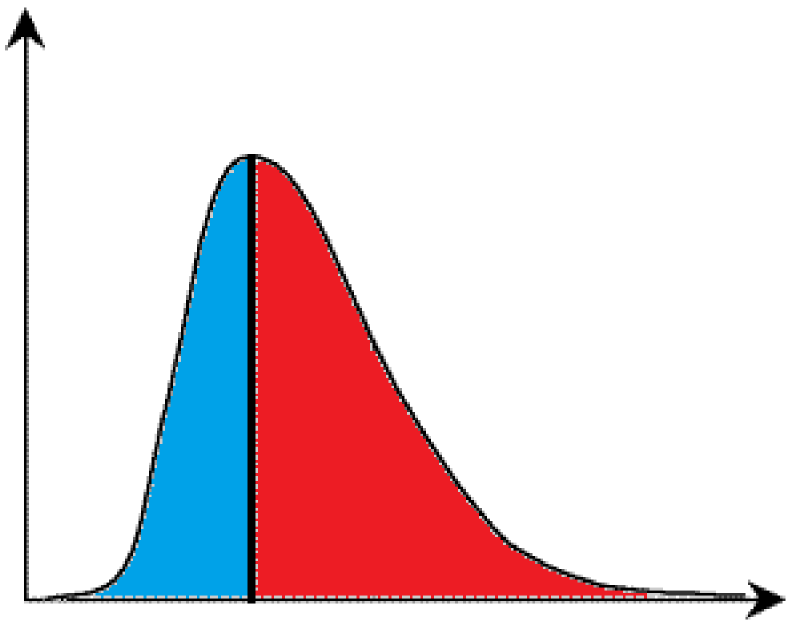
… **robust** to noise in the input.

… **balanced** in terms of workload.

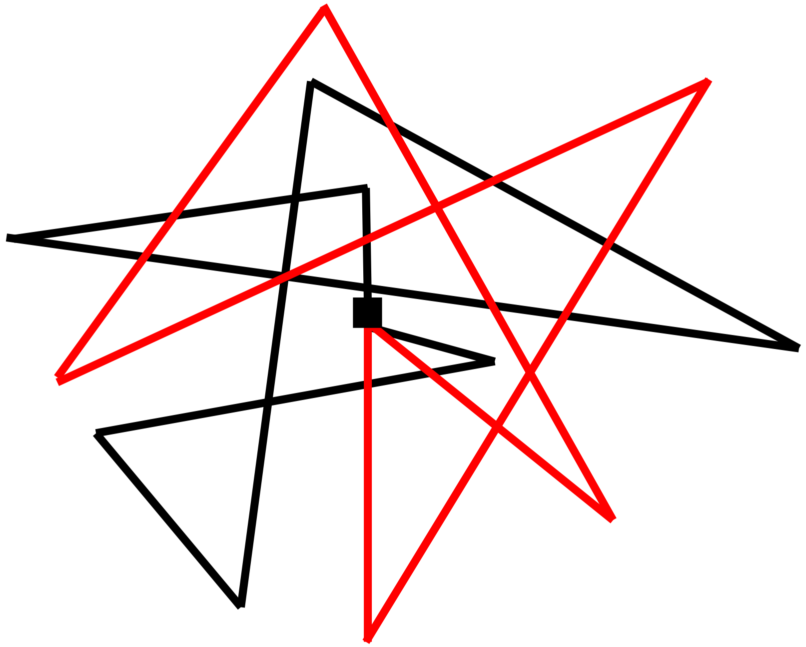
… **consistent** regarding service.

Take care with modeling!

Example: Robustness not “symmetric”



Example: “Artificial” balance



Danger: ‘optimal’ solutions for poorly defined objectives!

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multi-objective

multi objective **optimization**
multi objective **genetic algorithm**
multi objective **decision making**
multi objective **programming**
multi objective **particle swarm optimization**
multi objective **pso**
multi objective **evolutionary algorithm**
multi objective **problems**
multi objective **function**
multi objective **evolutionary**

NSGA-II
SPEA
IBEA
MOGLS
Path Relinking
Scatter Search
...

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Bi-objective CVRP example

- Implementation
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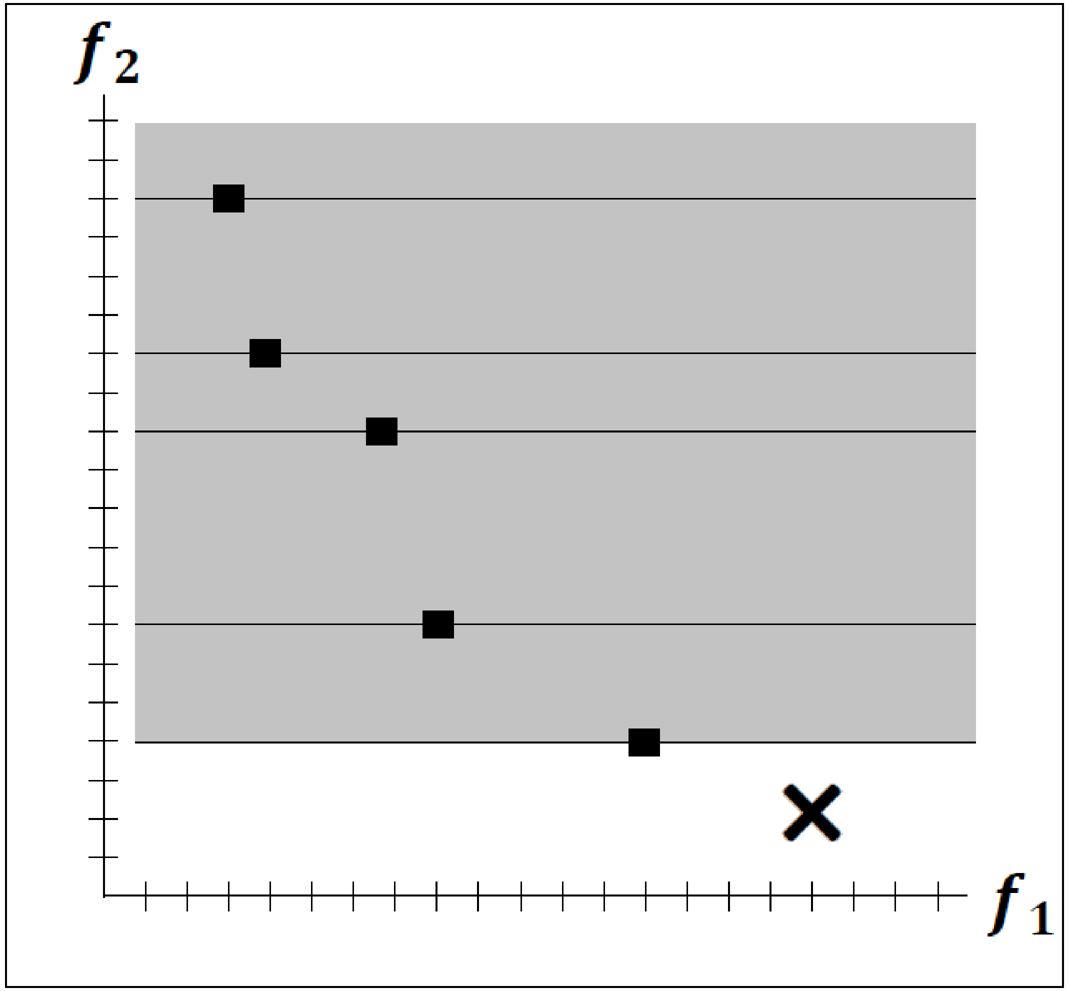
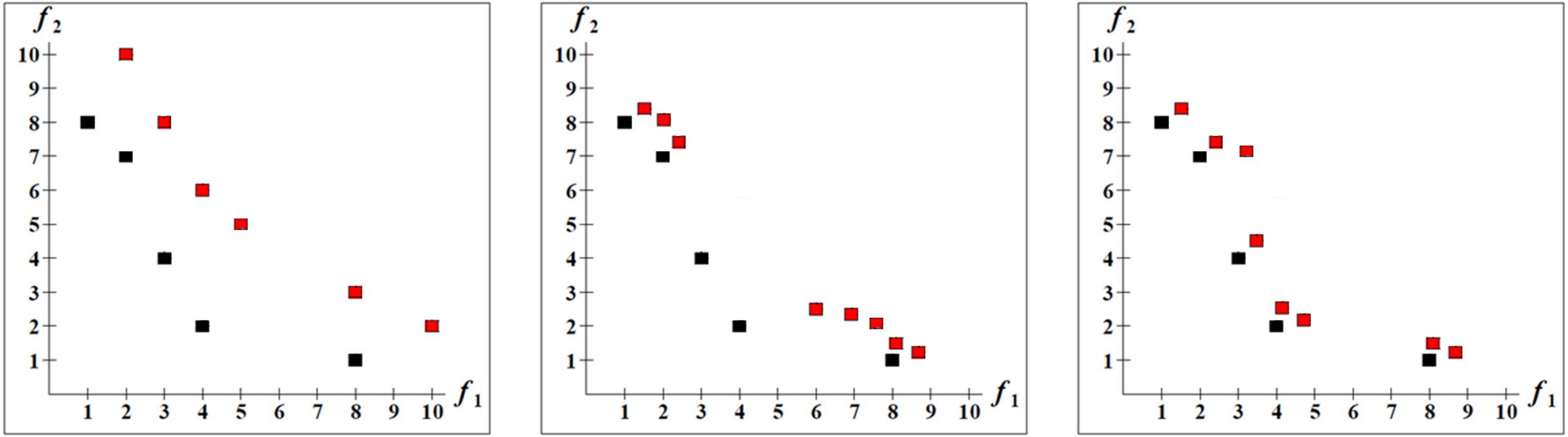
The classical ε-constraint method



Advantages

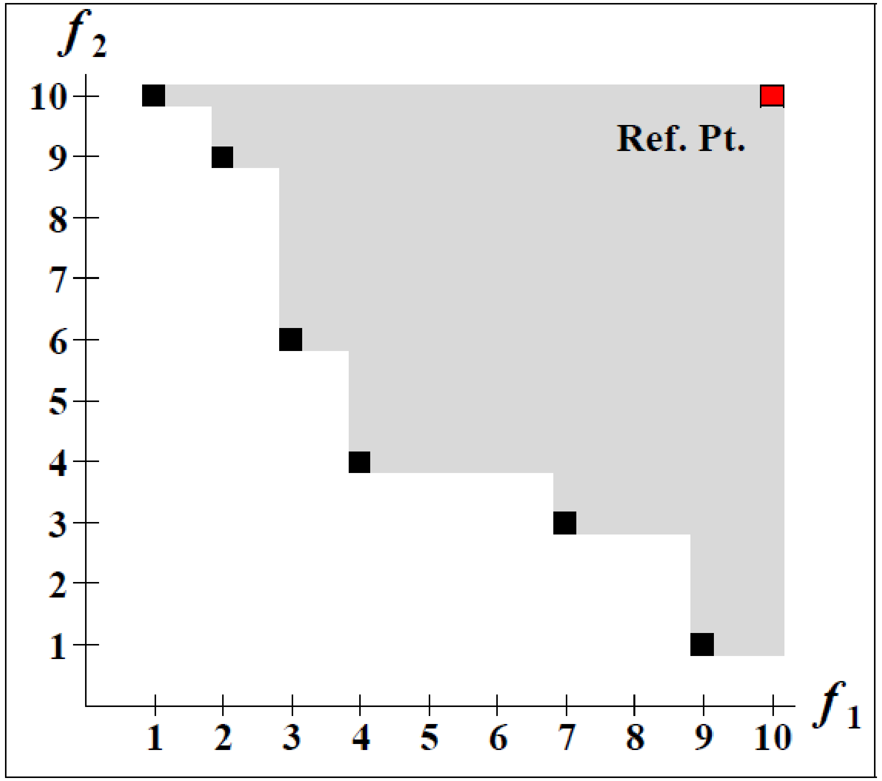
- ✓ Every iteration except the last yields a non-dominated solution, by definition
- ✓ Each sub-problem is completely independent of the others
- ✓ Exploit similarities between consecutive Pareto-efficient solutions
- ✓ General, easy to understand and implement, 1 simple parameter
- ✓ All problem-specific aspects in 1 sub-routine, no dependencies with other levels

Heuristic Approximation Sets

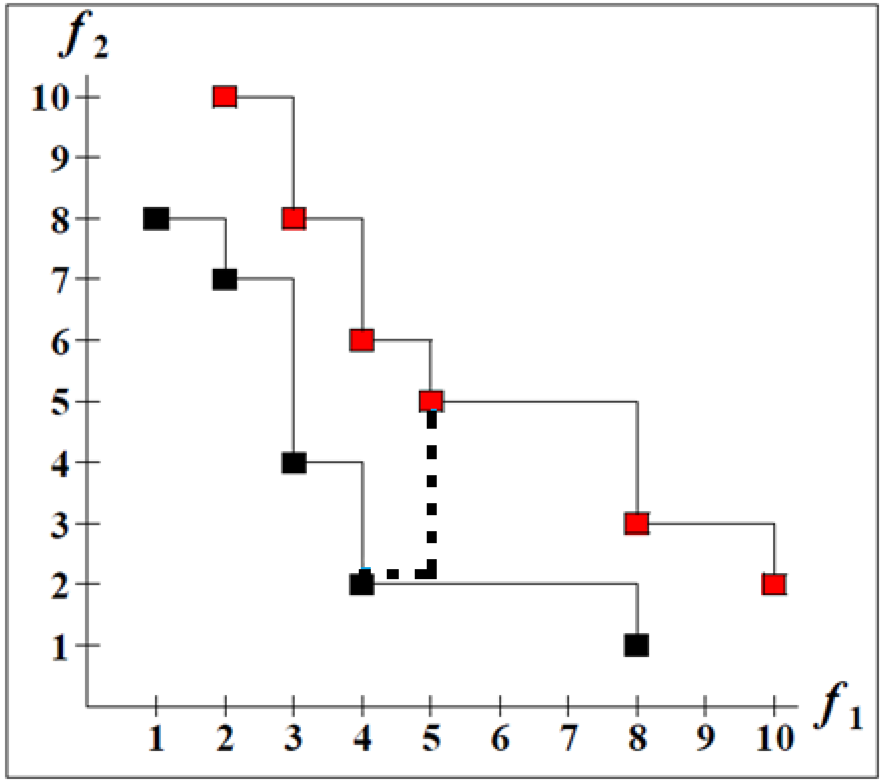


Bad Better Good

Quality Metrics



(a) Hypervolume



(b) Unary Epsilon

But how can we identify these solution sets?

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VRP with Route Balancing

Workload Resource/Metric

What is being balanced?

A. Tour length / distance

B. Tour demand / service time

C. Number of stops / customers

Balance/Equity Function

How is the balance quantified?

1. Min-Max
2. Lexicographic
3. Range
4. Mean Absolute Deviation
5. Standard Deviation
6. Gini Coefficient

How to consider all this variety in a simple but flexible way?

Implementation

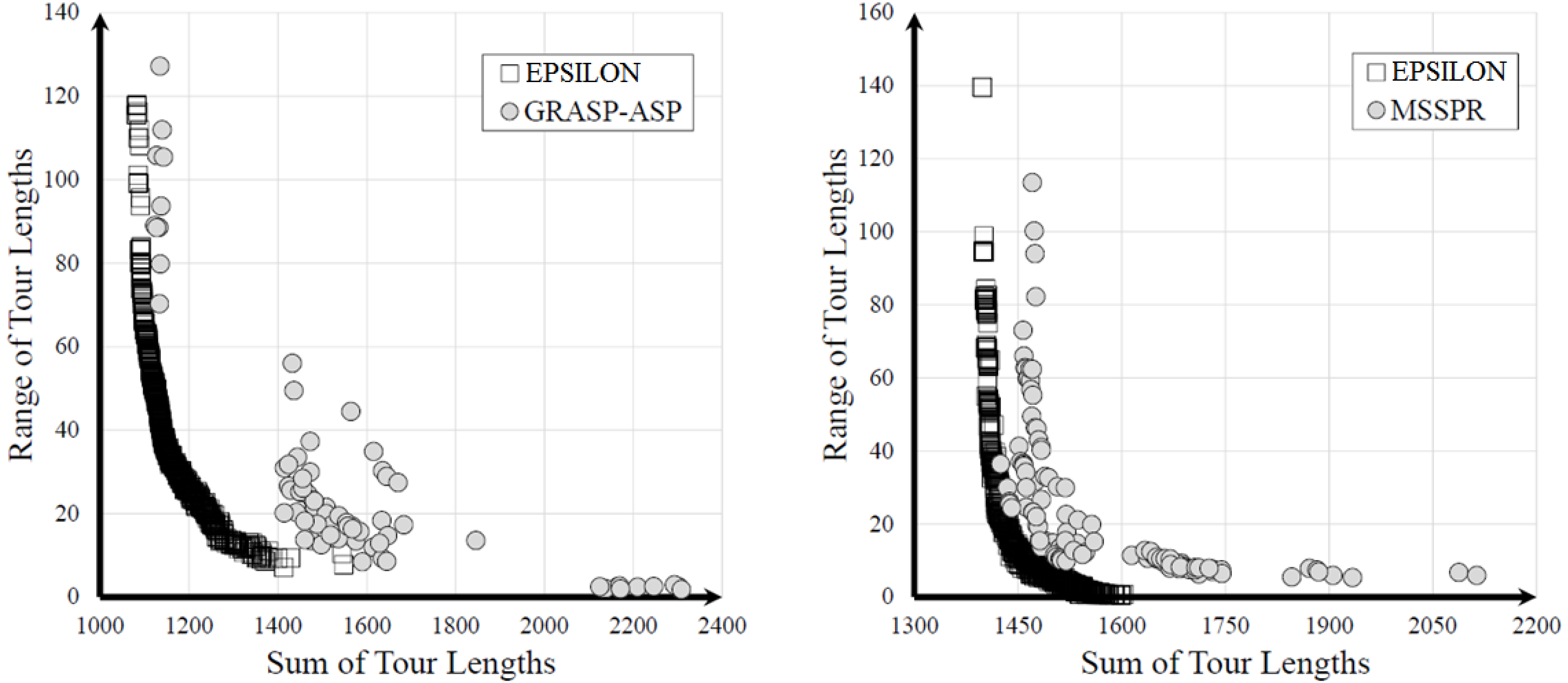
- We have to handle the epsilon-constraint on the balance objective.
- The most general way is to add a penalty term to the cost objective.

- Let c be the constraint value for the maximum allowed imbalance.
- Let $b(x)$ be the imbalance of solution x acc. to the chosen function.
- Then the penalty for imbalance p_{bal} is calculated as:

$$p_{bal} = max\{ c - b, 0\} * w_{bal}$$

Implementation

- Critical for efficient local search:
  - efficient delta evaluations for each balance function
  - sorted list of route workloads for quick re-evaluation
  - examine a wider set of moves than for cost-minimization only
- Critical for efficient search along the Pareto front:
  - enable warm-starts from search states of previous sub-problems
  - i.e. start each search from a solution “closest” to the new epsilon-constraint
- Save all local optima (they could be non-dominated)
- Re-evaluate and repair solutions after the epsilon-constraint is tightened



EPSILON
GRASP-ASP
Range of Tour Lengths
Sum of Tour Lengths
0
20
40
60
80
100
120
140
1000
1200
1400
1600
1800
2000
2200
2400
EPSILON
MSSPR
Range of Tour Lengths
Sum of Tour Lengths
0
20
40
60
80
100
120
140
160
1300
1450
1600
1750
1900
2050
2200

Summary

- In practice, optimization problems can often be multi-objective.
- A multi-objective approach can identify attractive compromises.

- Single-objective heuristics can be used for bi-objective problems.
- A simple ε-constraint framework can outperform specialized multi-objective metaheuristics when used with a state-of-the-art SO solver.

- General, flexible, and modular algorithm design is the key.

References

- Matl, P., Hartl, R. F., & Vidal, T. (2018). *Heuristic Rectangle Splitting: Leveraging Single-Objective Heuristics to Efficiently Solve Multi-Objective Problems*. https://arxiv.org/abs/1705.10174
- Matl, P., Hartl, R. F., & Vidal, T. (2018). Workload Equity in Vehicle Routing Problems: A Survey and Analysis. *Transportation Science, 52*(2), 239–260. https://arxiv.org/abs/1605.08565
- Matl, P., Hartl, R., & Vidal, T. (2018). Workload Equity in Vehicle Routing : The Impact of Alternative Workload Resources. https://arxiv.org/pdf/1803.01795