Multi-Attribute Vehicle Routing Problems (Part I)

→ Analysis of heuristics and winning strategies

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Mini-Course, ANPET

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Join Work With

- **A large part of this content derives from my Ph.D thesis with**
  - Teodor Gabriel Crainic – UQAM, Montréal
  - Michel Gendreau – Polytechnique, Montréal
  - Christian Prins – UTT, Troyes, France

Join Work With

- Plus recent and current works on specific vehicle routing problems:
  - **Prize-collecting VRP**
    - Nelson Maculan – UFRJ
    - Puca Huachi Vaz Penna – UFF
    - Luis Satoru Ochi – UFF
  - **Heterogeneous VRP**
    - Puca Huachi Penna – UFF
    - Luis Satoru Ochi – UFF
  - **Clustered VRP**
    - Maria Battarra – U. Southampton.
    - Gunes Erdogan – U. Southampton.
    - Anand Subramanian – UFPB
  - **Online/Stochastic VRP**
    - Patrick Jaillet -- MIT
    - Richard Hartl – U. Vienna
  - **Vehicle Routing and Truck Driver Scheduling Problem**:
    - Asvin Goel – Jacobs University, Bremen
  - **Workover Rig Routing Problem**:
    - G. Ribeiro, B. Vieira – UFRJ
    - G. Desaulniers, J. Desrosiers – U. Montréal
  - **Pollution Routing Problem**:
    - A. Subramanian – UFPB
    - R. Kramer – UFPB

Presentation outline

PART 1)

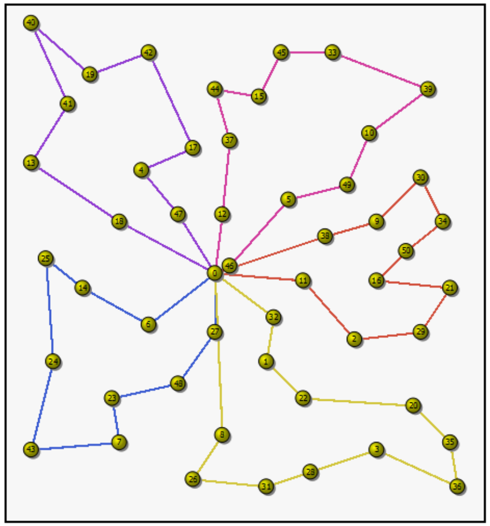
- I) Vehicle Routing Problem, and attributes.
- II) Classic Heuristics and metaheuristics for vehicle routing
- III) An analysis of some winning strategies

PART 2)

- IV) A new general-purpose solution approach
  - Attribute-based modular design
  - Unified Local Search
  - Unified Hybrid Genetic Search
  - Computational Experiments
- V) Some application cases

Vehicle Routing Problem, and attributes

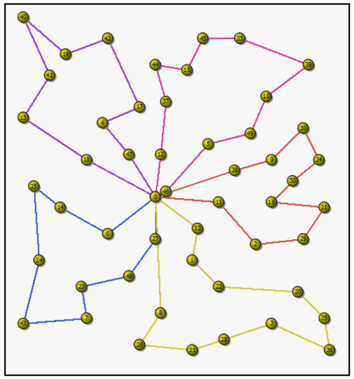
- **Capacitated vehicle routing problem:**
  - **INPUT :** *n* customers, with locations & demands. All-pair distances. Homogeneous fleet of *m* capacitated vehicles located at a central depot.
  - **OUTPUT** : Least-cost delivery routes (at most one route per vehicle) to service all customers.



- NP-Hard problem
- Exact resolution impracticable for most problem instances of interest (≥ 200 customers).
- "Scopus" facts : 2007-2011 = 1258 articles with the key *vehicle routing*.
- Massive research effort on heuristics.

Vehicle Routing Problem, and attributes

- **Capacitated vehicle routing problem:**
  - Combinatorial optimization problem, for a problem with n=100 customers and a single vehicle, the number of possible solutions is:



$$n! = 933262154439441526816992388562667004907159682643816\,2146859296389521759999322991560894146397615651828625369\,792082722375825118521091686400000000000000000000000000 \approx 10^{158}$$

  - Even with a grid of computers which…

    Contains as many CPU as the estimated nb atoms in the Universe : $n_{CPU} = 10^{80}$

    Does one operation per Planck time : $t_P = 5.39 \times 10^{-44}$ s

    We need $T = 10^{158} \times 5.39 \times 10^{-44} / 10^{80} = 5.39 \times 10^{34}$ s to enumerate all solutions.

    Compare this to the estimated age of Universe : 4.33×10^{17} s …

Vehicle Routing Problem, and attributes

- **Vehicle routing "attributes"** : *Supplementary decisions, constraints and objectives which complement the problem formulations*
  - Modeling the specificities of application cases, customers requirements, network and vehicle specificities, operators abilities…
  - E.g. Time windows, Multiple periods, multiple depots, heterogeneous fleet, 2D-3D loading, time-dependent travel times…

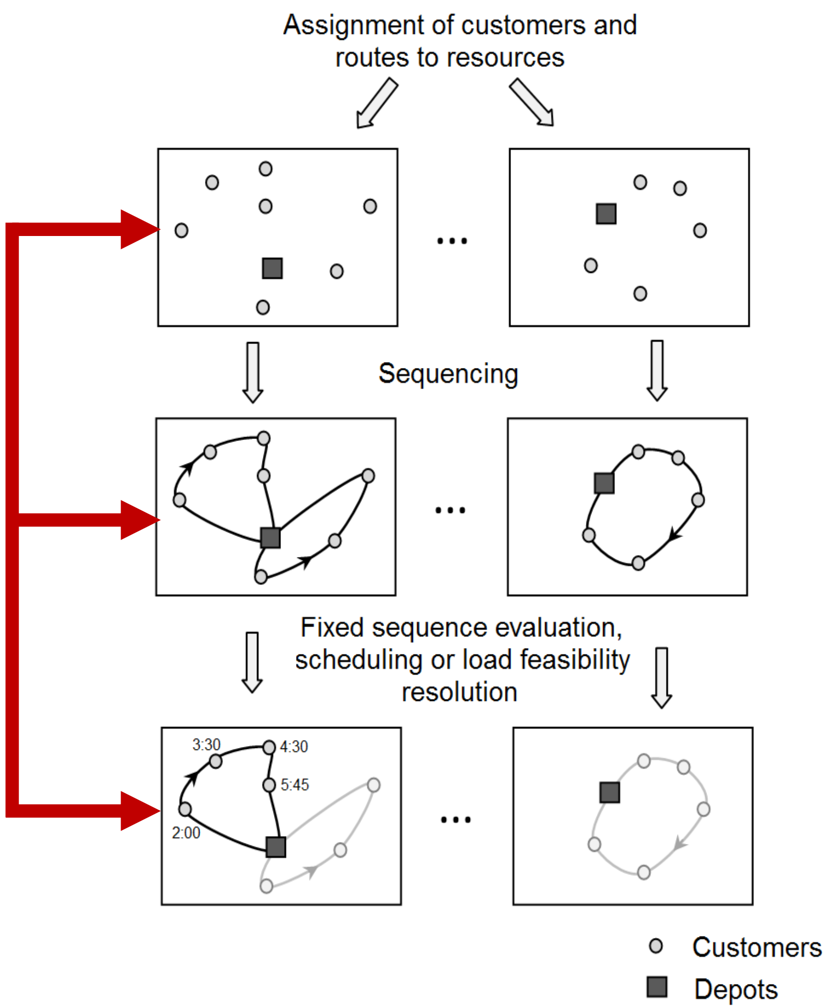
- Multi-Attribute Vehicle Routing Problems (MAVRP)
  - Challenges : **VARIETY** of attributes
  - Challenges : **COMBINATION** of attributes
  - Plethora of attribute-specific methods in the literature, but no unified approach.

Vehicle Routing Problem, and attributes

- **ASSIGNMENT**: assignment customers and routes to days and depots
  - **Take into account Periodic, Multi-Depot, Heterogeneous Fleet problems**

- **SEQUENCING:** create the sequence of visits to customers

- **ROUTE EVALUATION:** Evaluate each route generated during the search
  - **Time windows, Time-dep. travel time, Loading constraints, HOS regulations Lunch breaks, Load-Dependent costs…**



Vehicle Routing Problem, and attributes

- **A need for more flexible and general purpose solvers**
  - Solvers that can address a wide range of problems without need for extensive adaptation or user expertise.
  - **Necessary tools** for the timely application of current optimization methods to industrial settings.
  - Few such methods in the literature

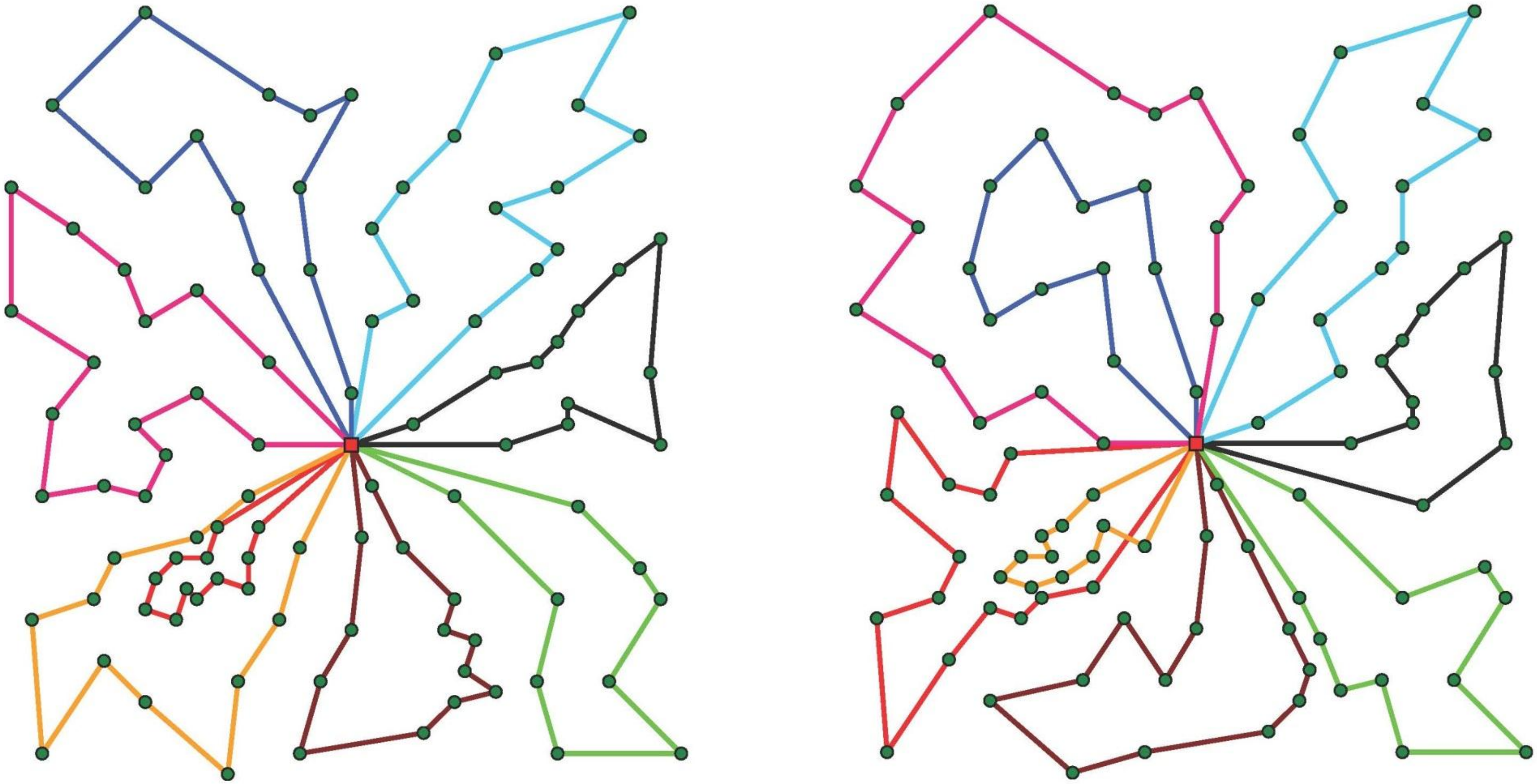
Vehicle Routing Problem, and attributes

- Quick look on several solutions for different vehicle routing problems…

Vehicle Routing Problem, and attributes

- Capacitated VRP (CVRP)

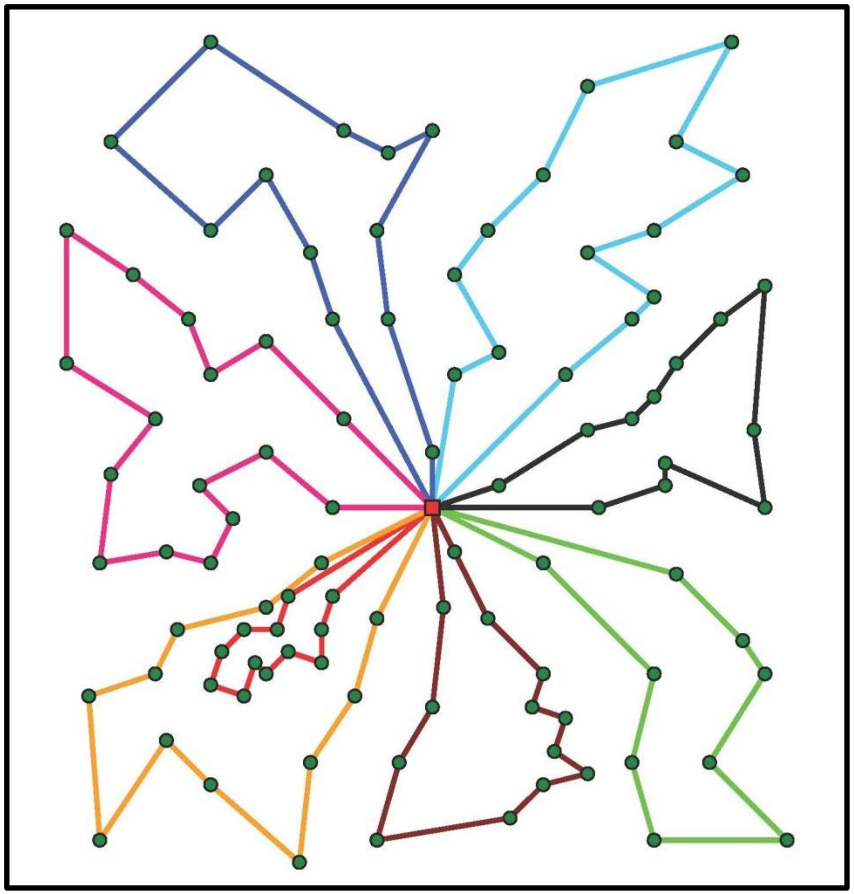
A) B)



Vehicle Routing Problem, and attributes

- **Capacitated VRP (CVRP)**

A) 827



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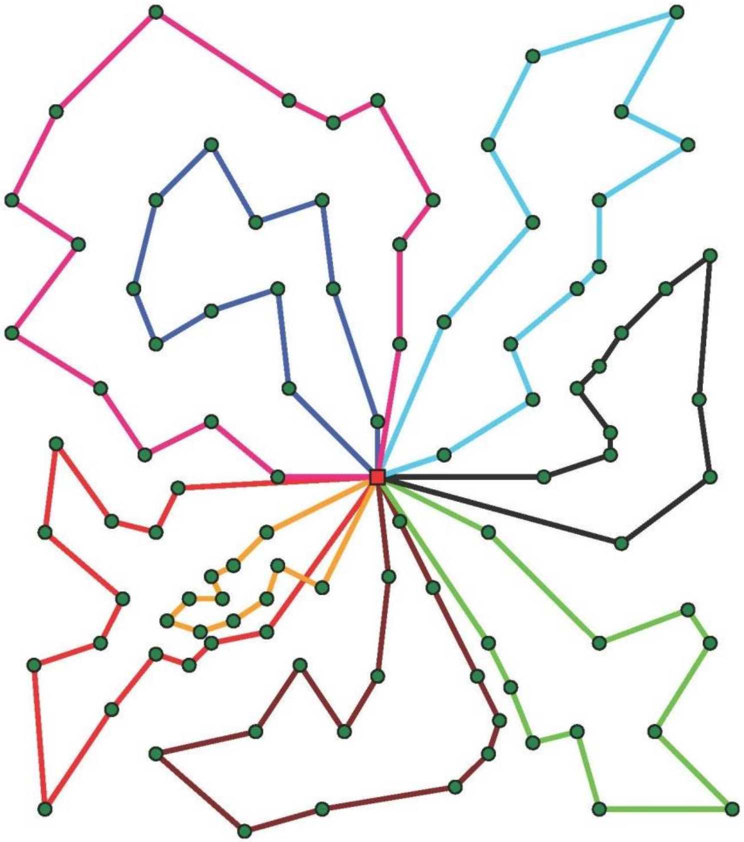
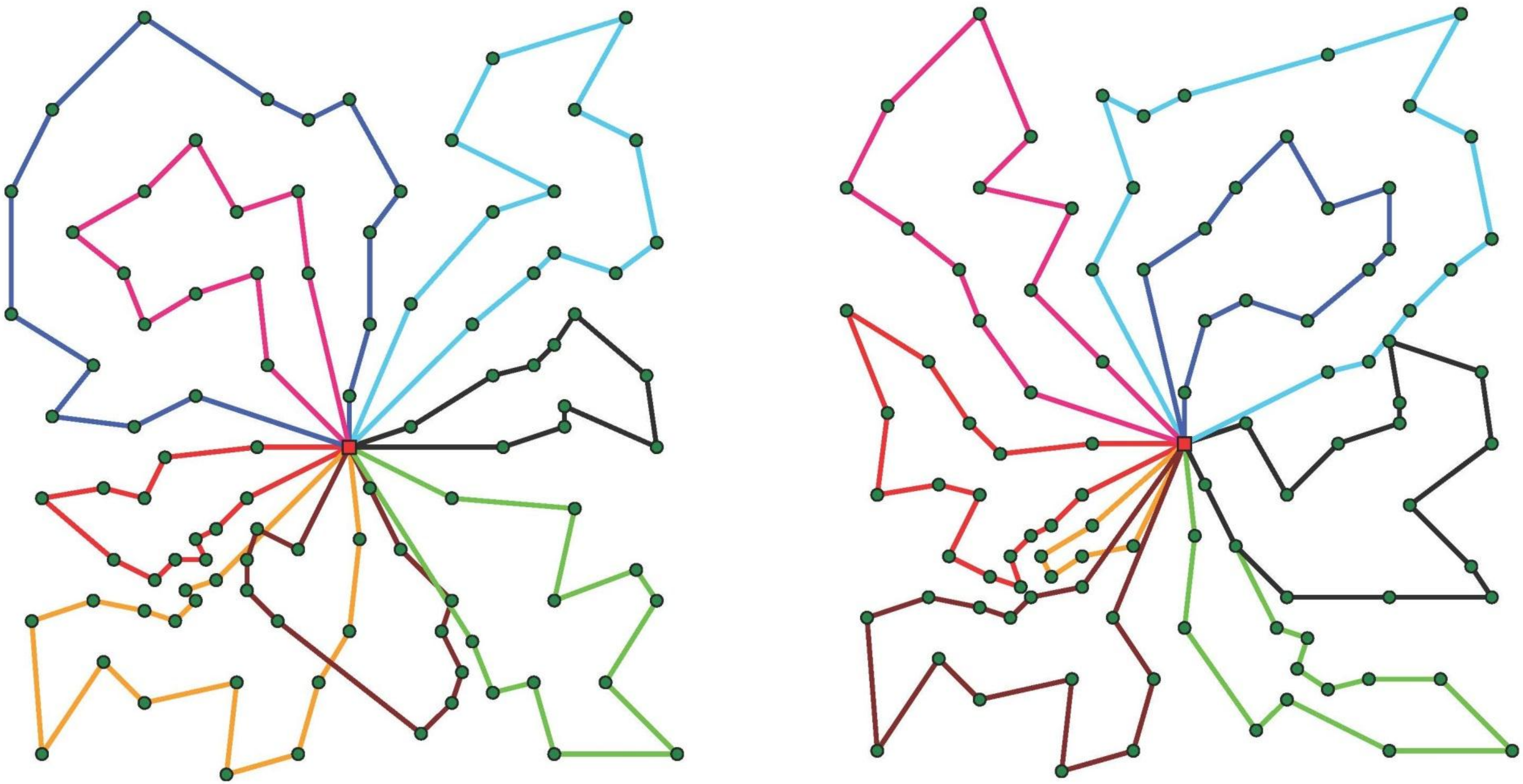


- **Capacitated VRP (CVRP)**

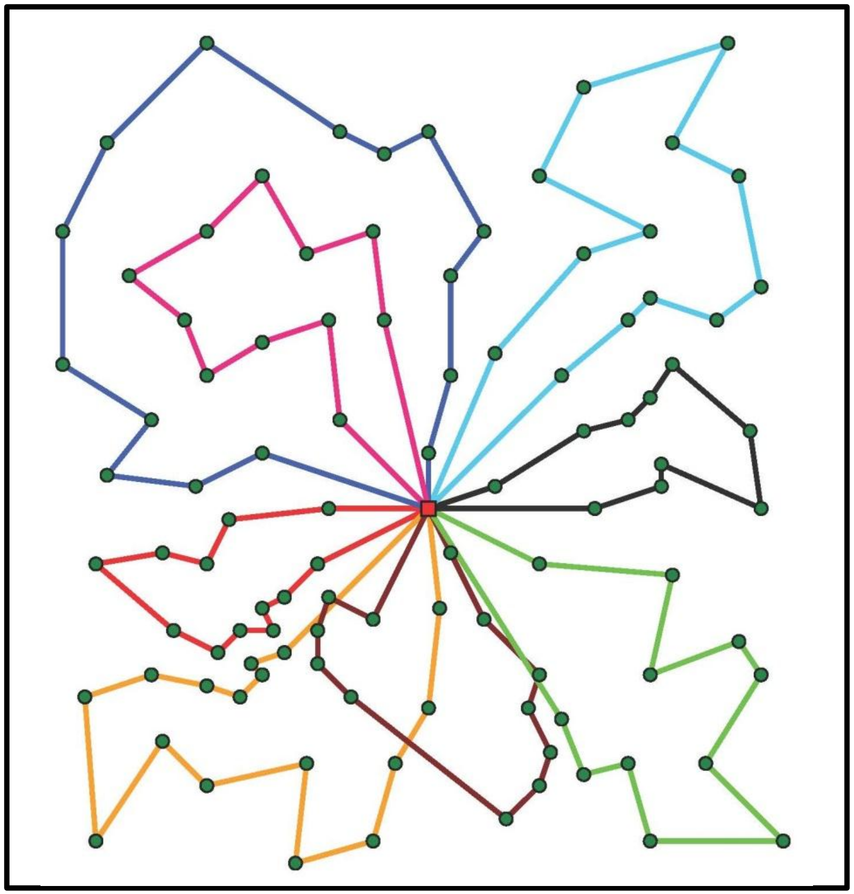
A) B)



Vehicle Routing Problem, and attributes

- **Capacitated VRP (CVRP)**

A) 840



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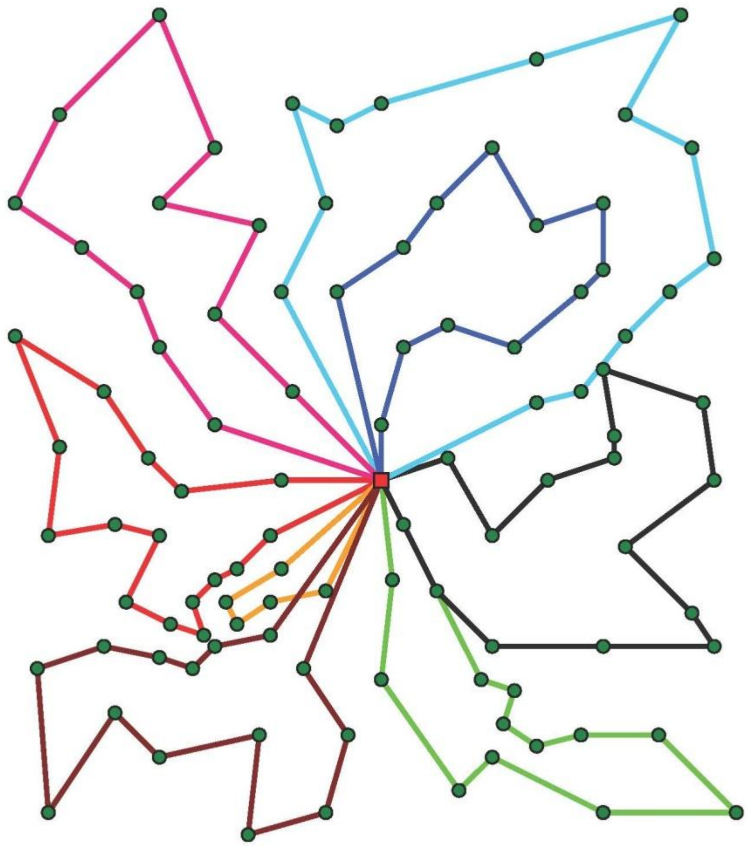
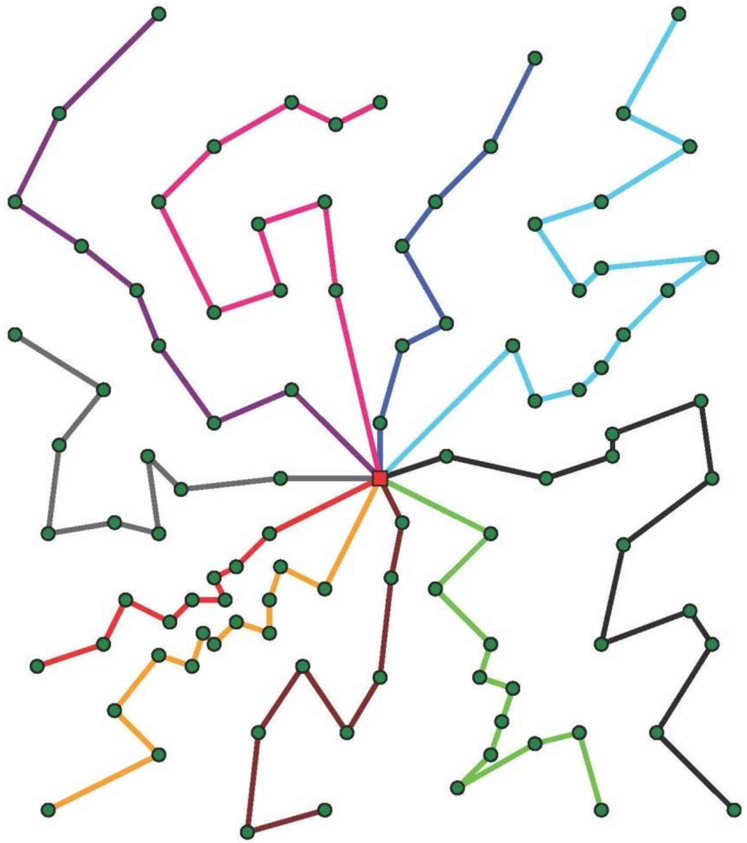


Vehicle Routing Problem, and attributes

- Open VRP (OVRP)

A)



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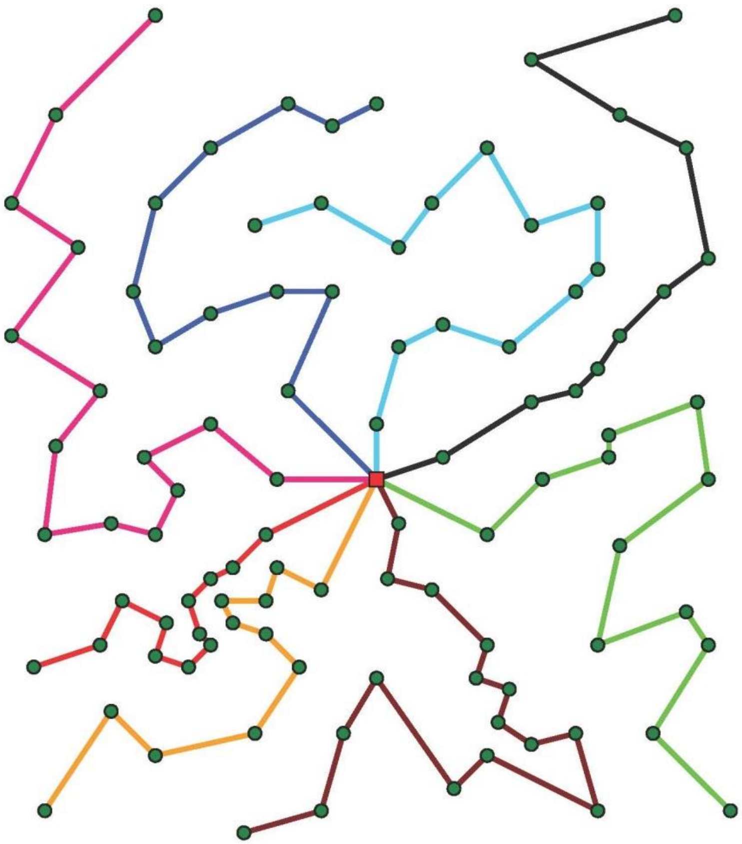
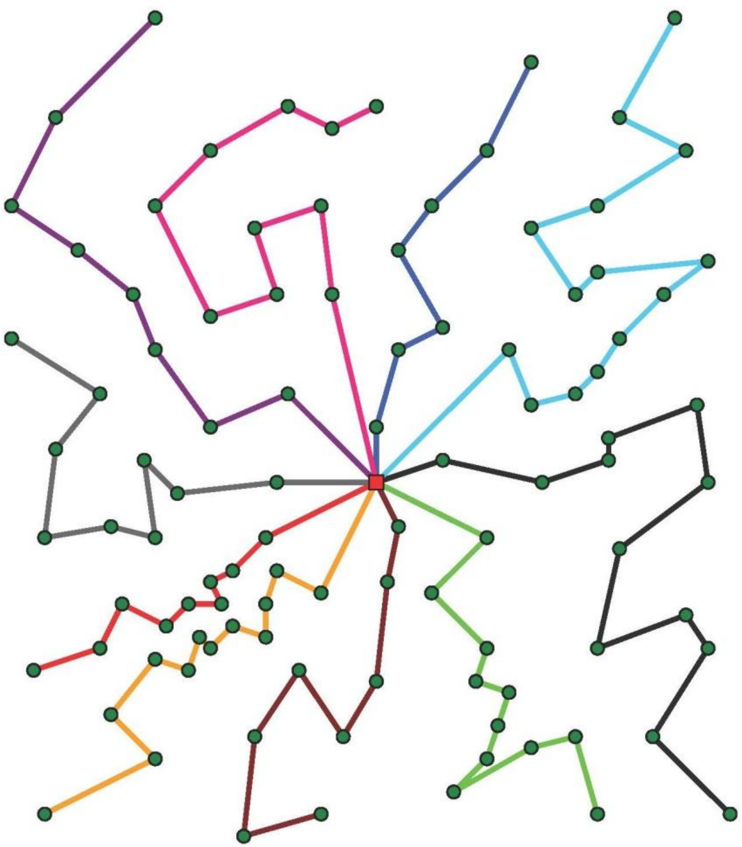
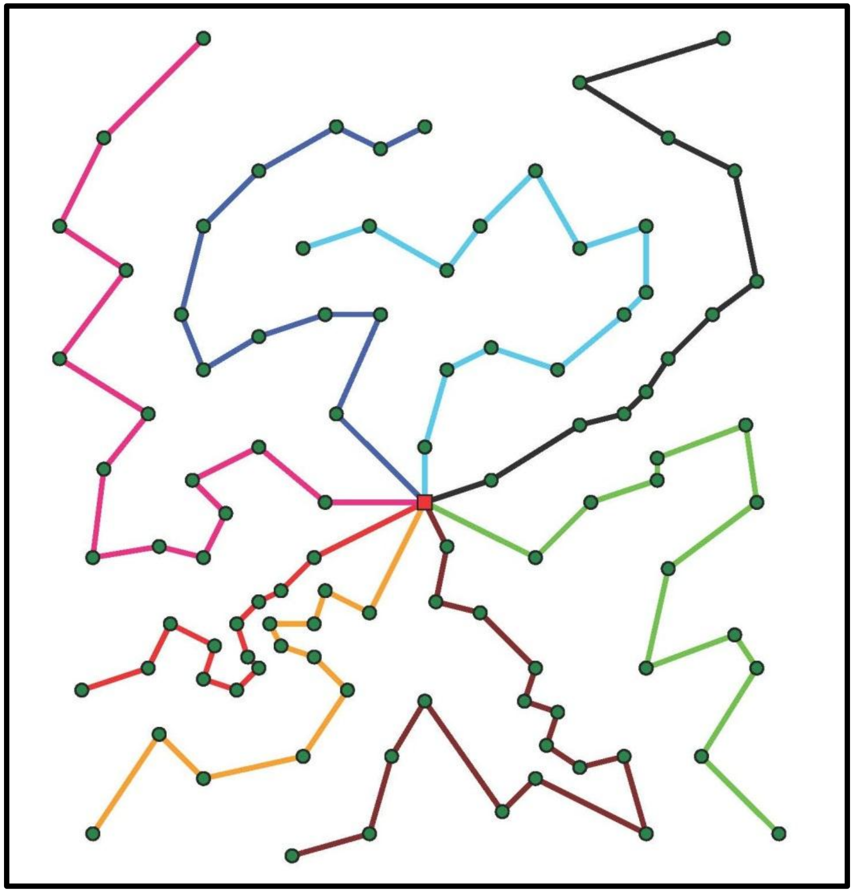


- **Open VRP (OVRP)**

A) 661

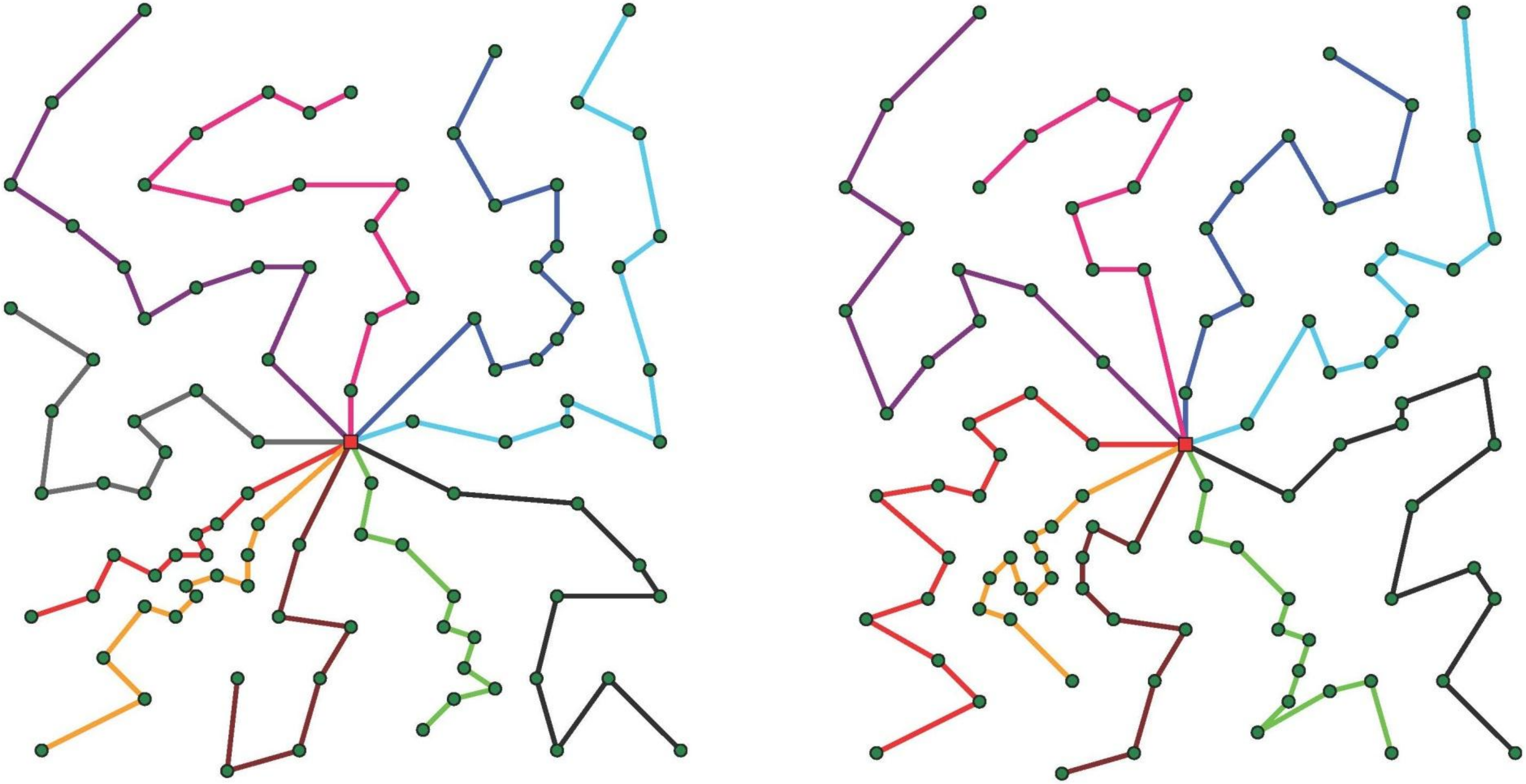


B) 640



- **Open VRP (OVRP)**

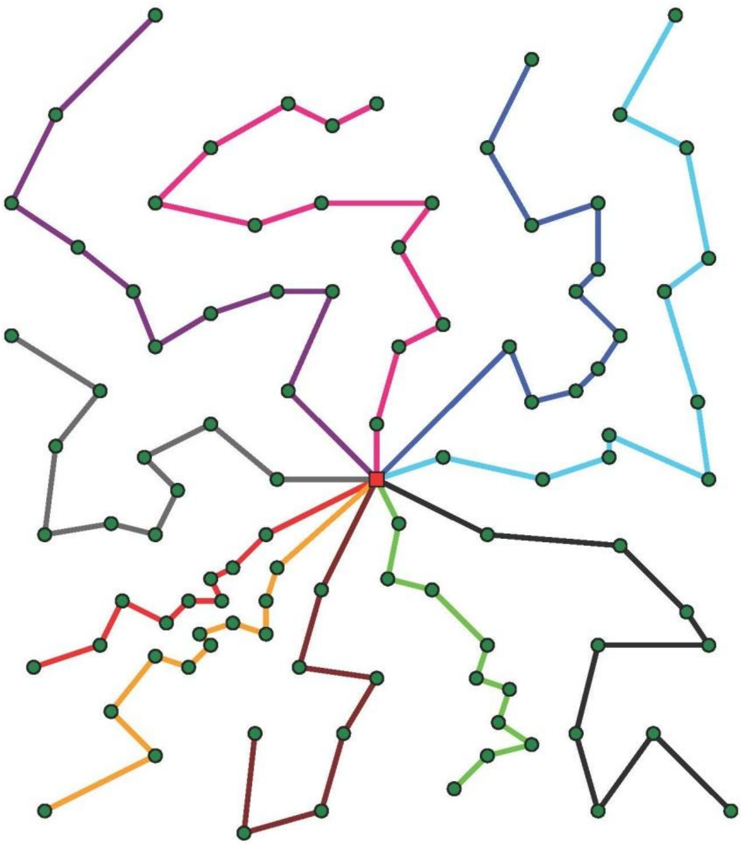
A) B)



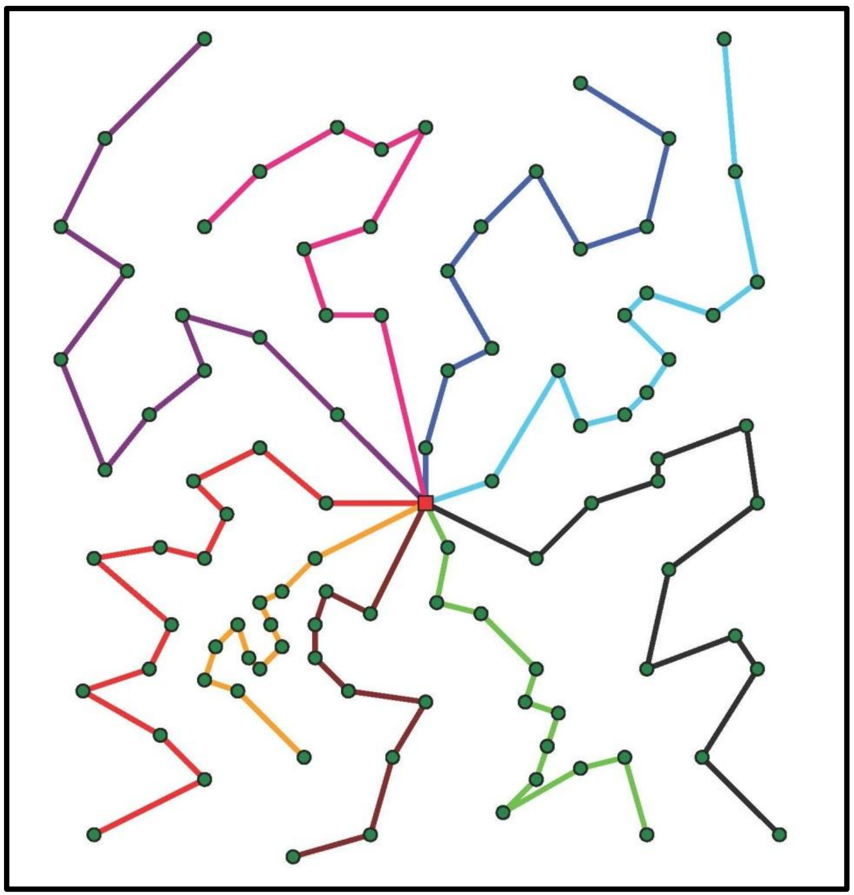
Vehicle Routing Problem, and attributes

- Open VRP (OVRP)

A) 652



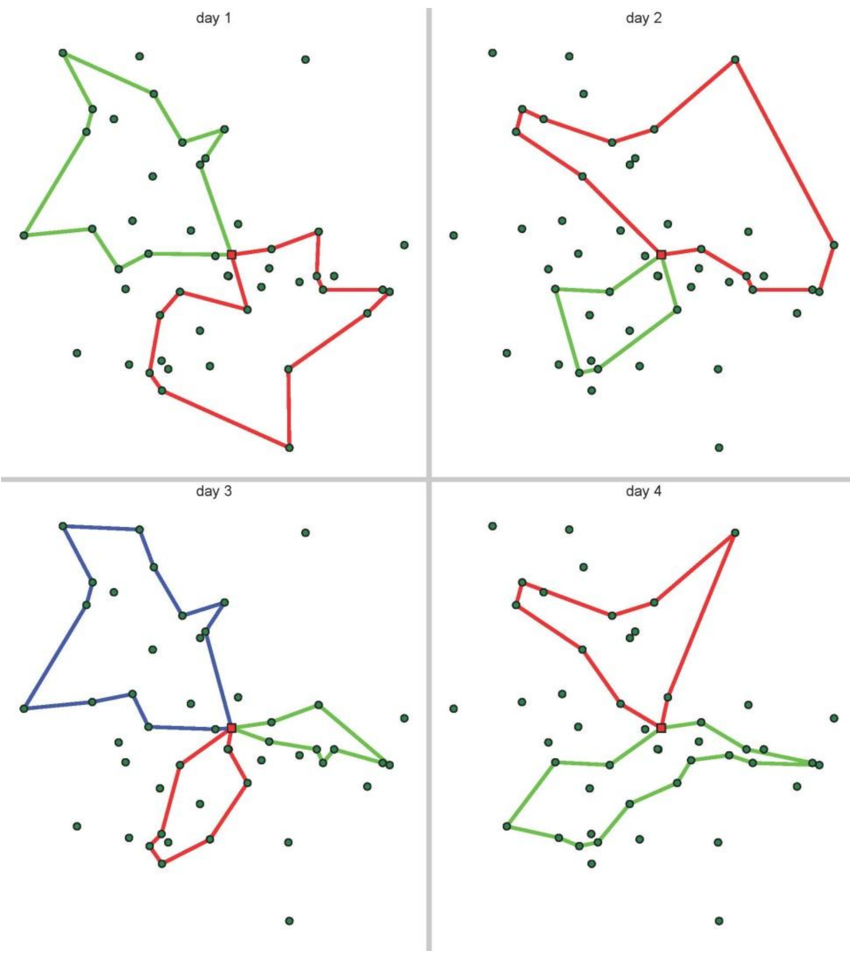
B) 649



Vehicle Routing Problem, and attributes

- **Multi-Period VRP (PVRP)**

A)



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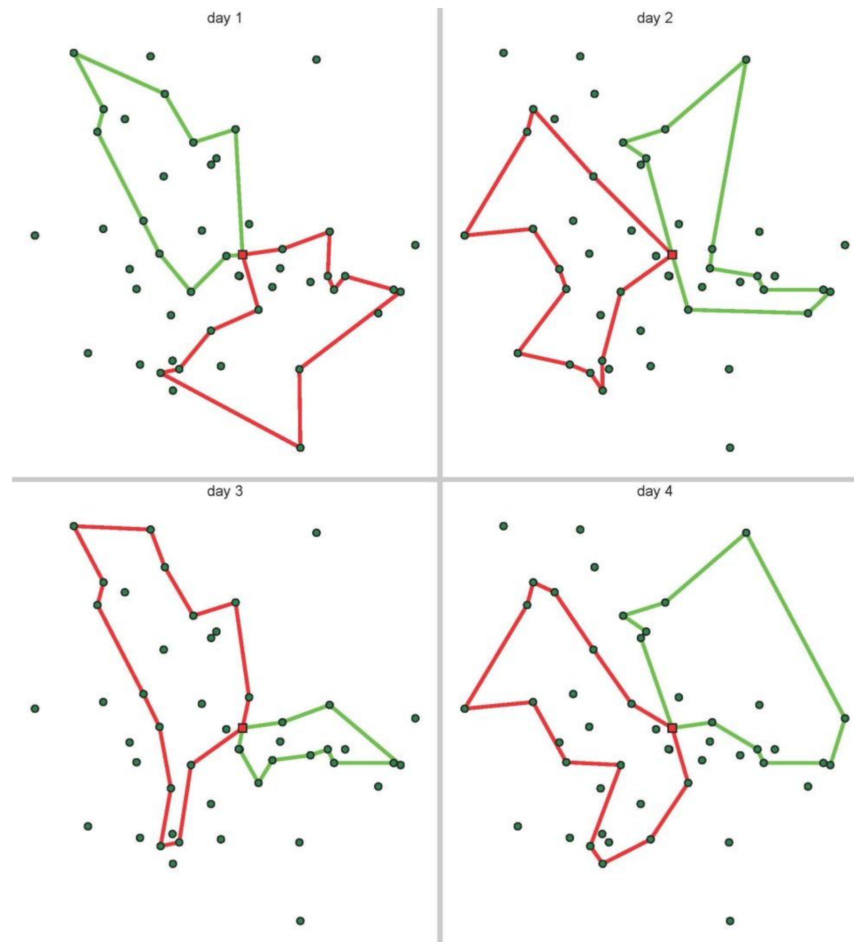
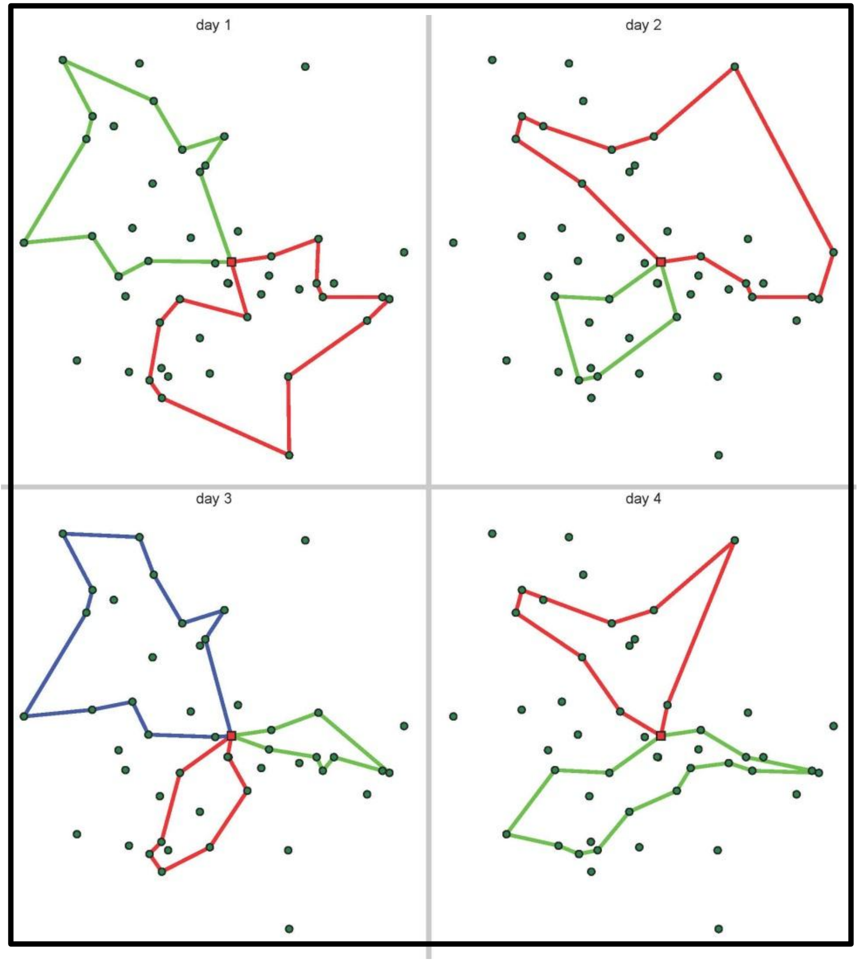


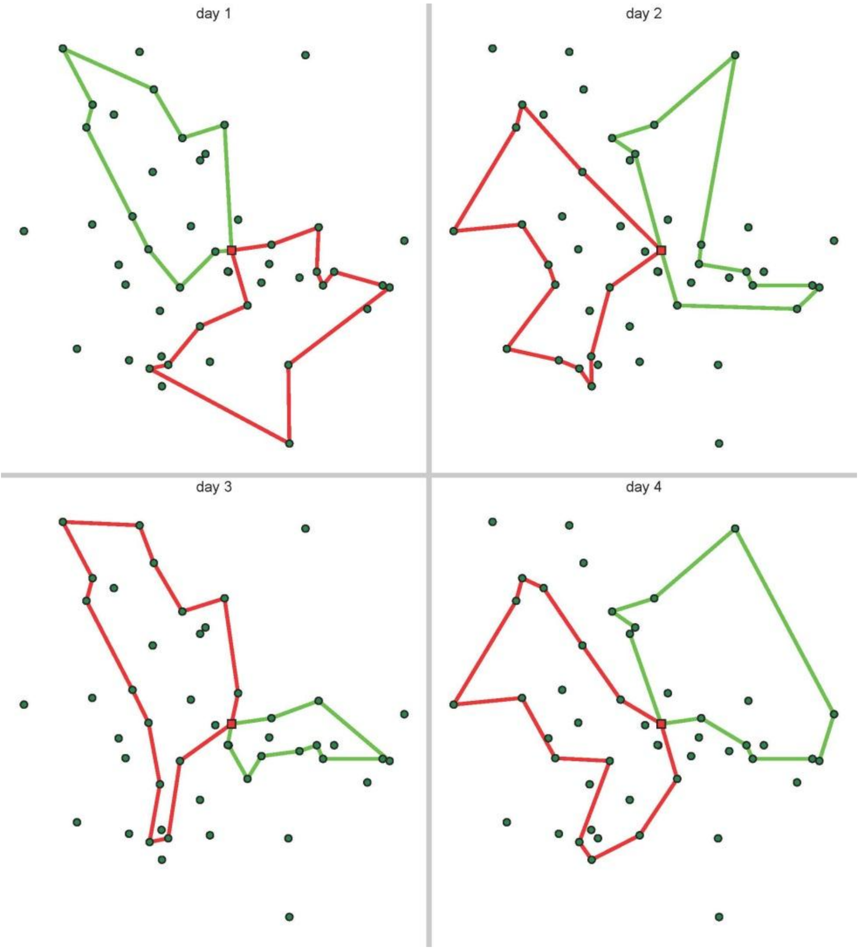
Vehicle Routing Problem, and attributes

- Multi-Period VRP (PVRP)

A) 2261

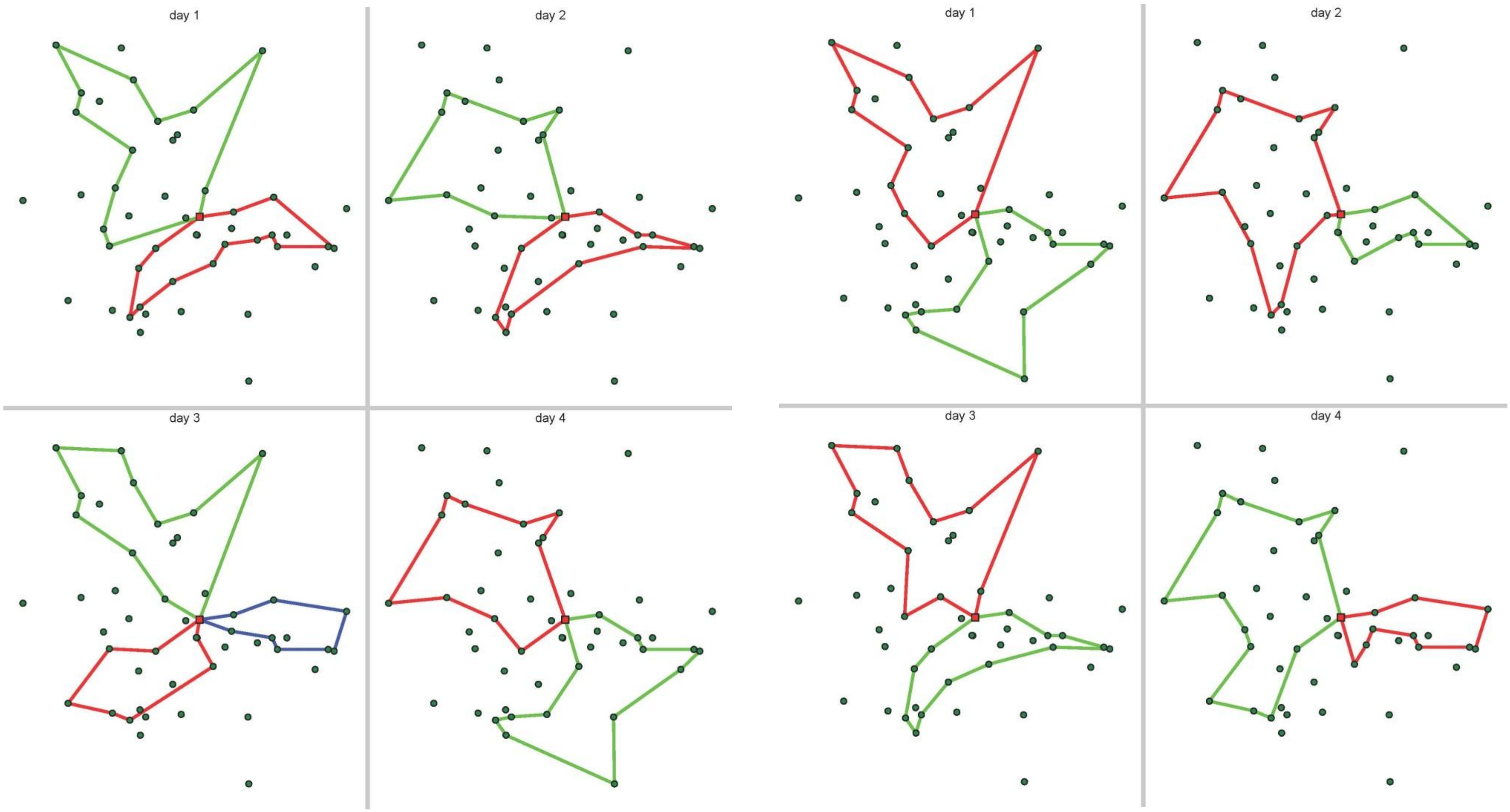


B) 2316



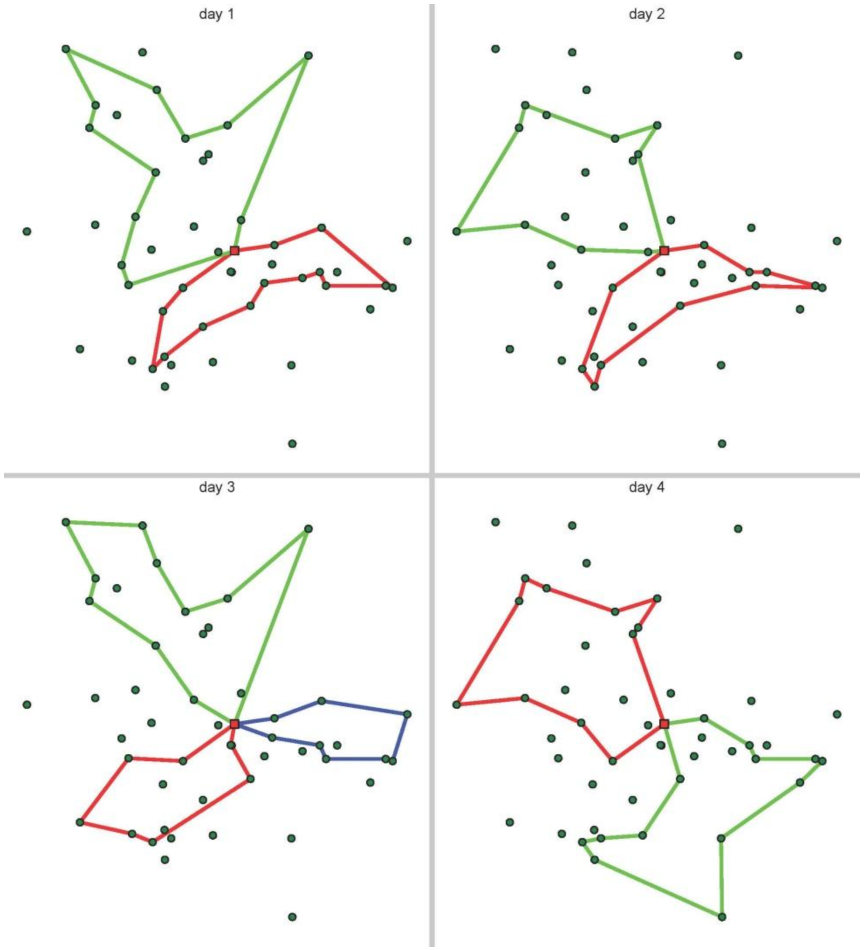
- **Multi-Period VRP (PVRP)**

A) B)

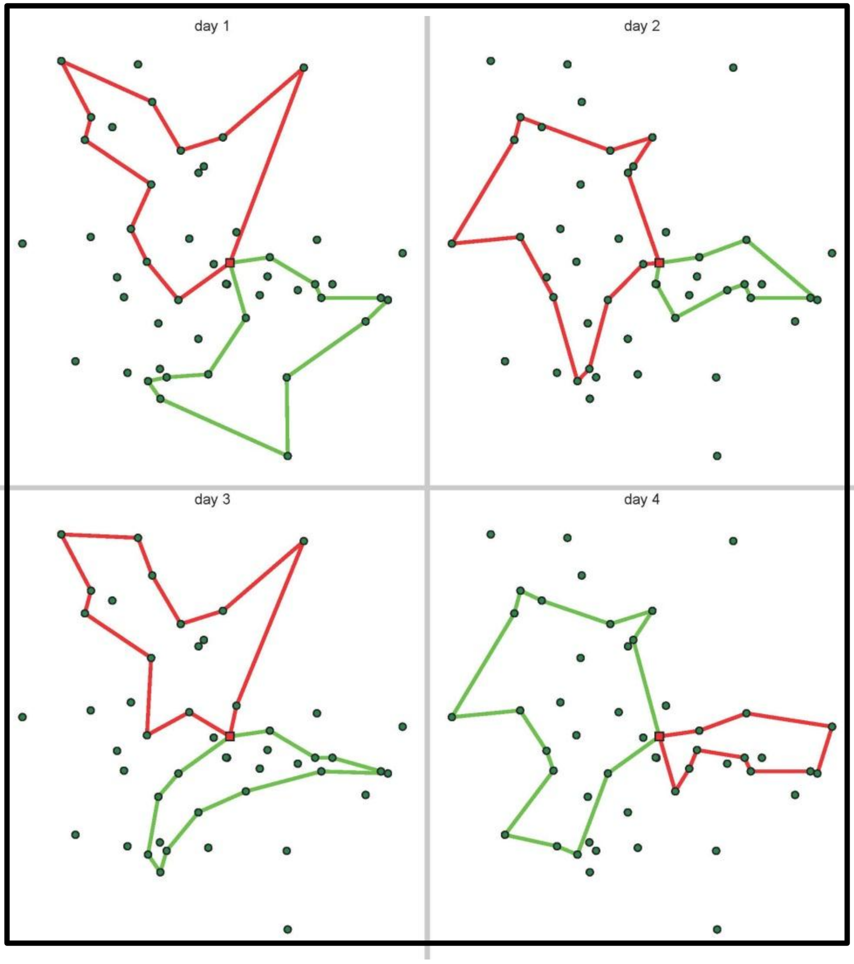


- **Multi-Period VRP (PVRP)**

A) 2289



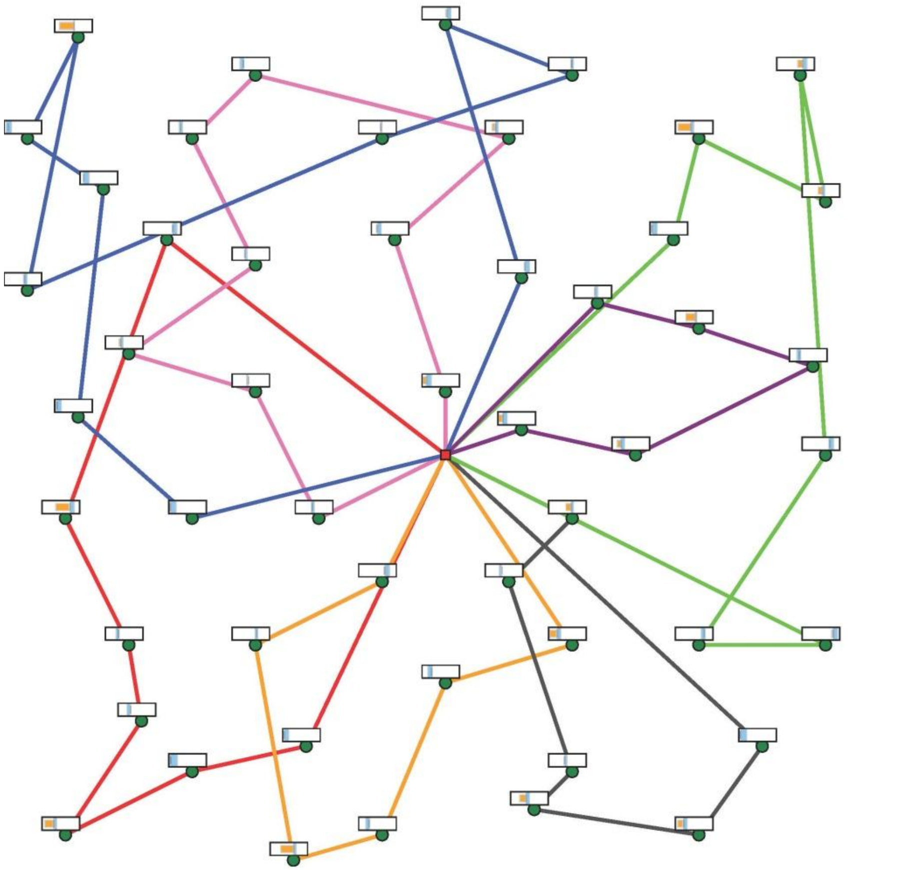
B) 2209



Vehicle Routing Problem, and attributes

- VRP with time windows (VRPTW)

A)



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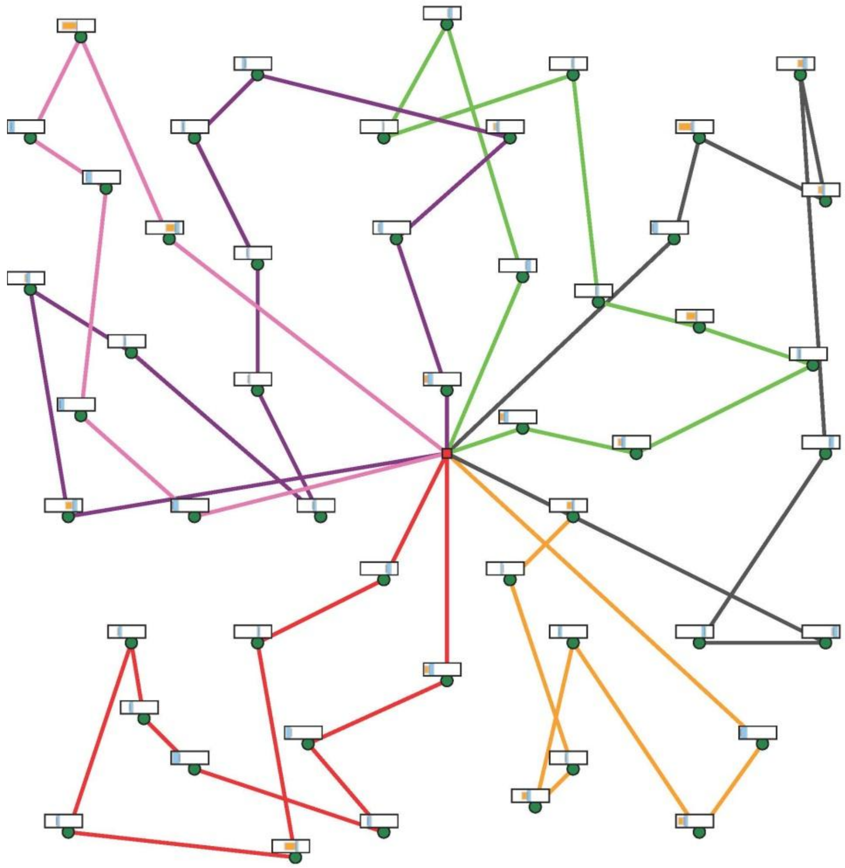
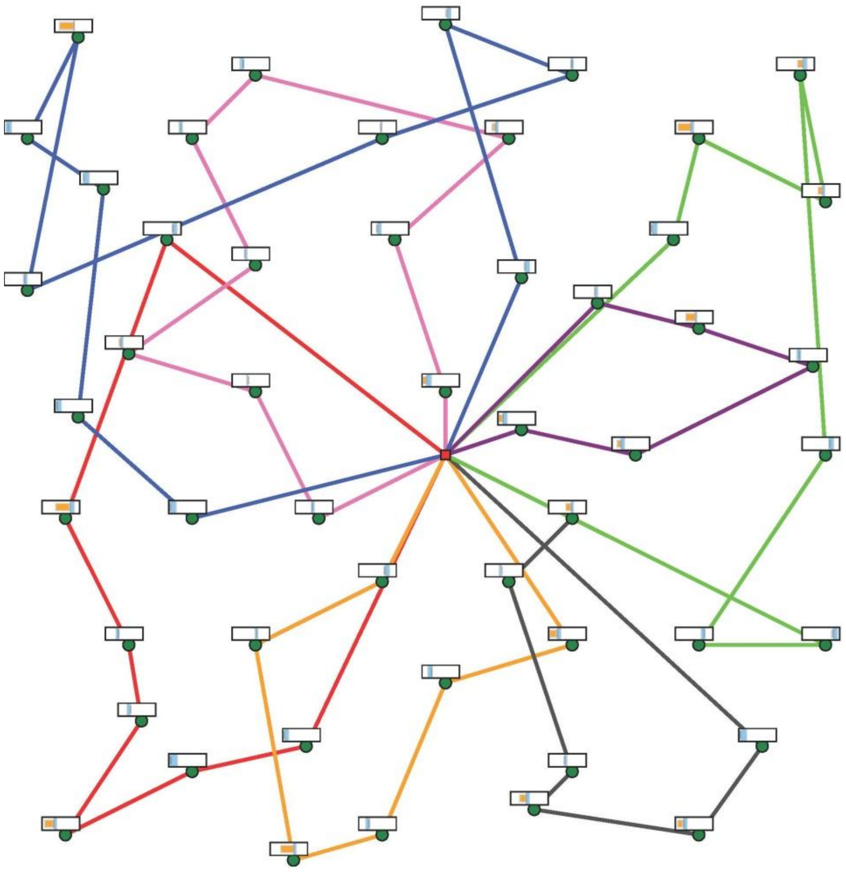
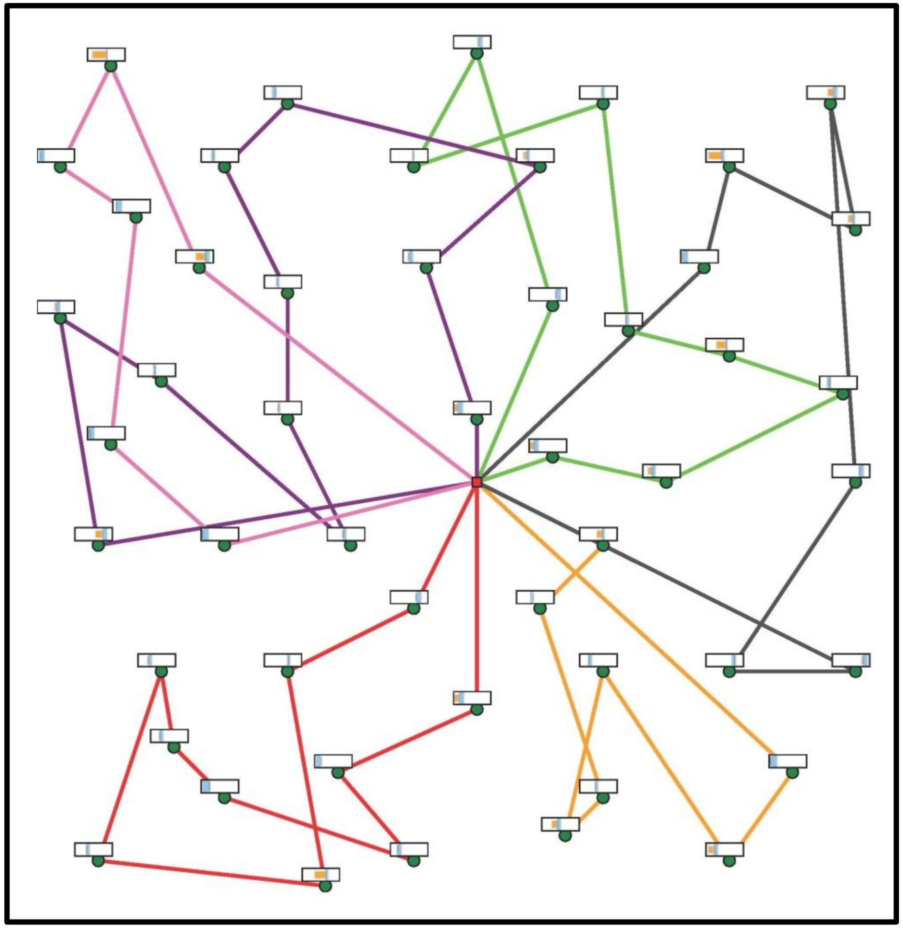


- **VRP with time windows (VRPTW)**

A) 813



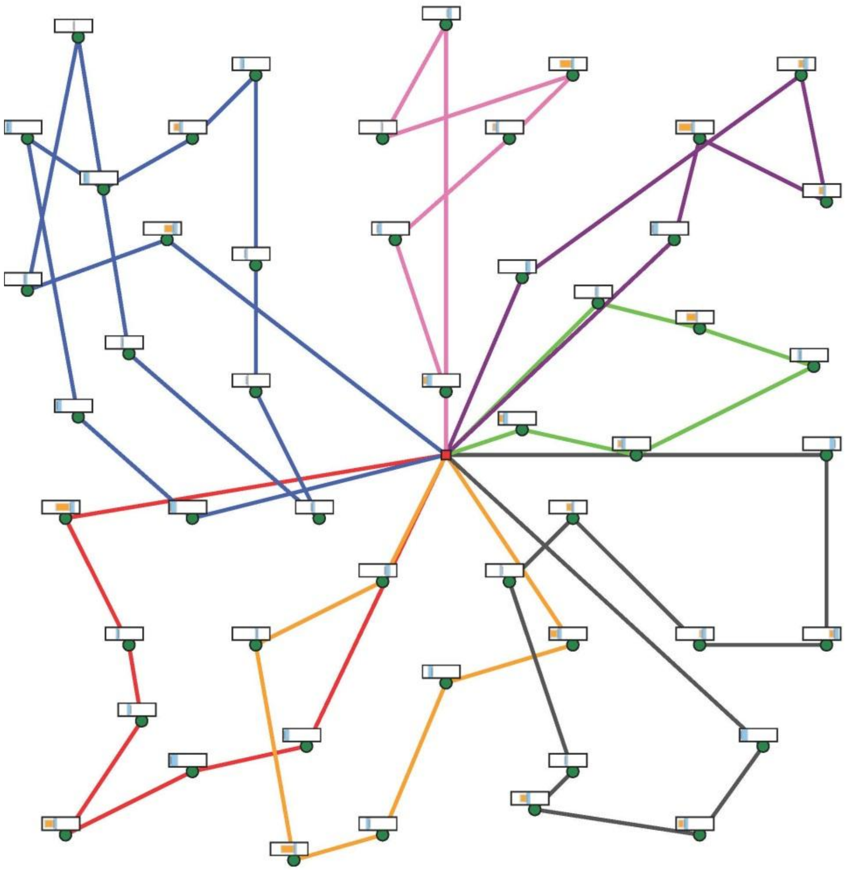
B) 808



Vehicle Routing Problem, and attributes

- VRP with time windows (VRPTW)

A)



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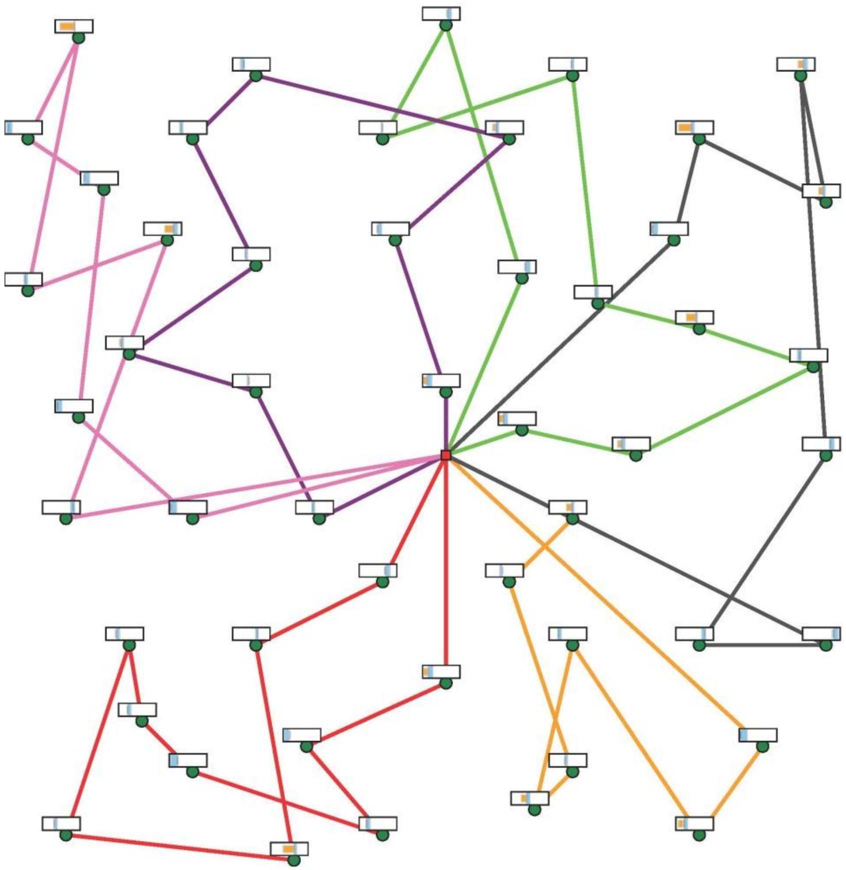
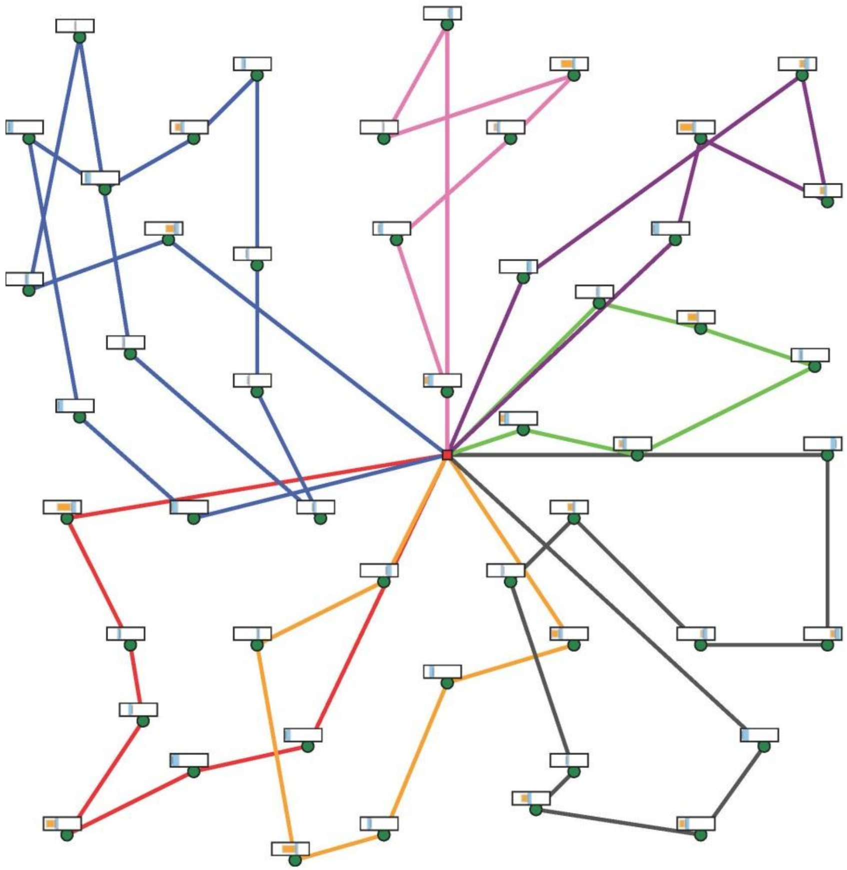


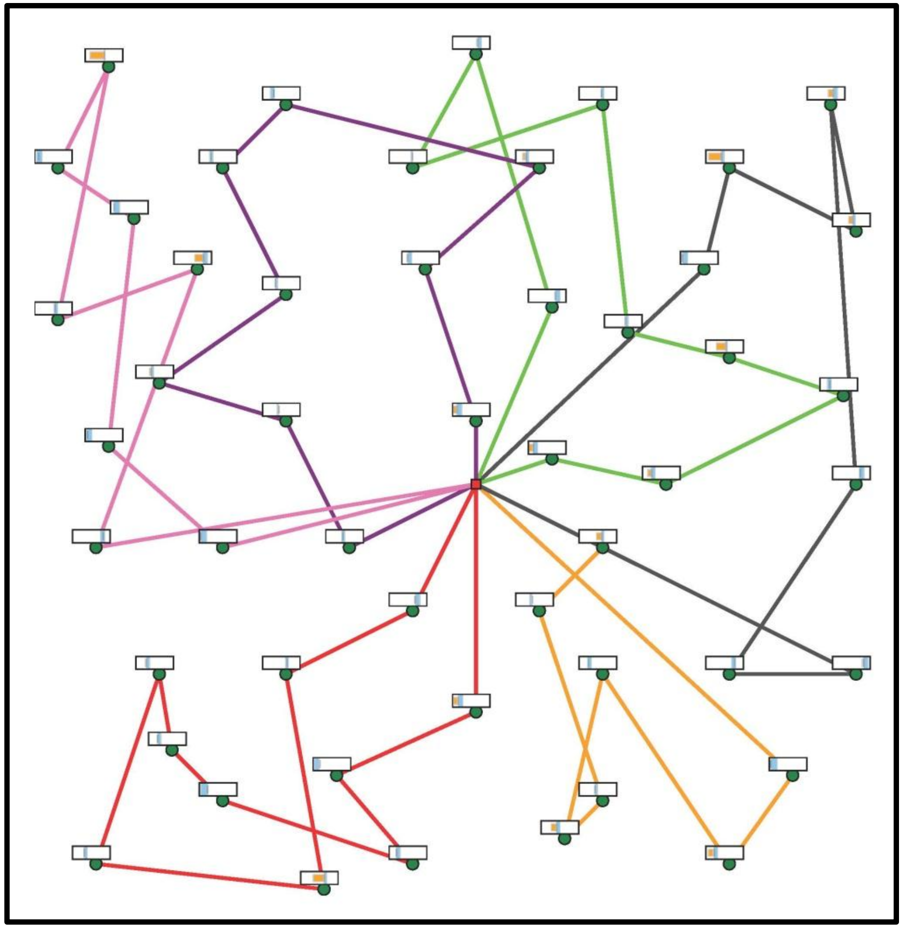
Vehicle Routing Problem, and attributes

- VRP with time windows (VRPTW)

A) 822



B) 794



Presentation outline

PART 1)

- I) Vehicle Routing Problem, and attributes.
- II) Classic Heuristics and metaheuristics for vehicle routing
- III) An analysis of some winning strategies

PART 2)

- IV) A new general-purpose solution approach
  - Attribute-based modular design
  - Unified Local Search
  - Unified Hybrid Genetic Search
  - Computational Experiments
- V) Some application cases

Classic Heuristics and Metaheuristics

- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

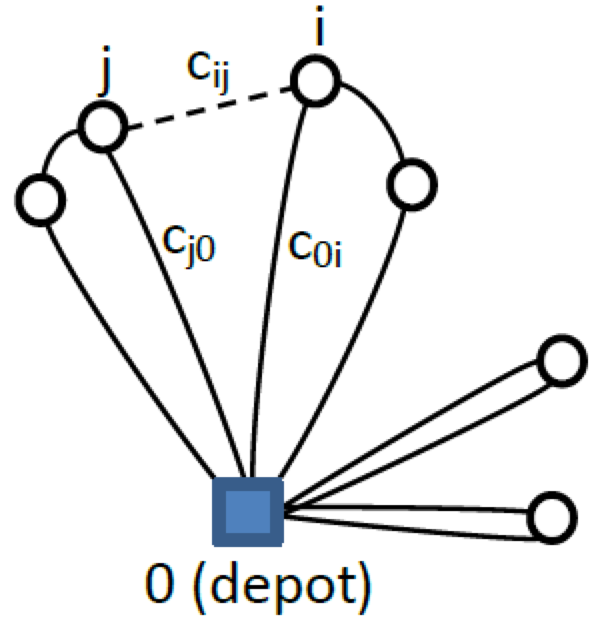
- Savings method (Clarke and Wright 1964)
  - Merge routes step by step based on a *savings* measure s_{ij}

$$s_{ij} = c_{i0} + c_{0j} - c_{ij}$$

  - Some refinements by Gaskell (1967) and Yellow (1970) :

$$s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$$

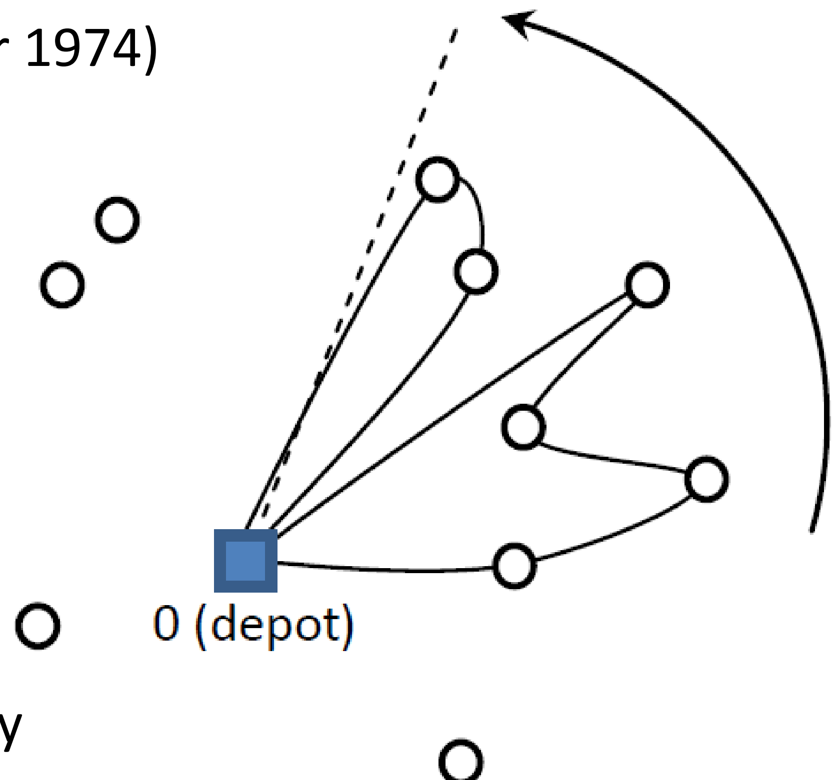
  - Mole and Jameson (1976) and Solomon (1987) generalize the concepts and also consider insertions inside the routes.



Classic Heuristics and Metaheuristics

- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

- Sweep algorithm (Gillett and Miller 1974)
  - Sweep the deliveries in circular order to create routes, a new route is initiated each time the capacity is exceeded.



- “Petal” methods : generate several alternative routes, called petals, and select a subset by solving a set-covering linear program.

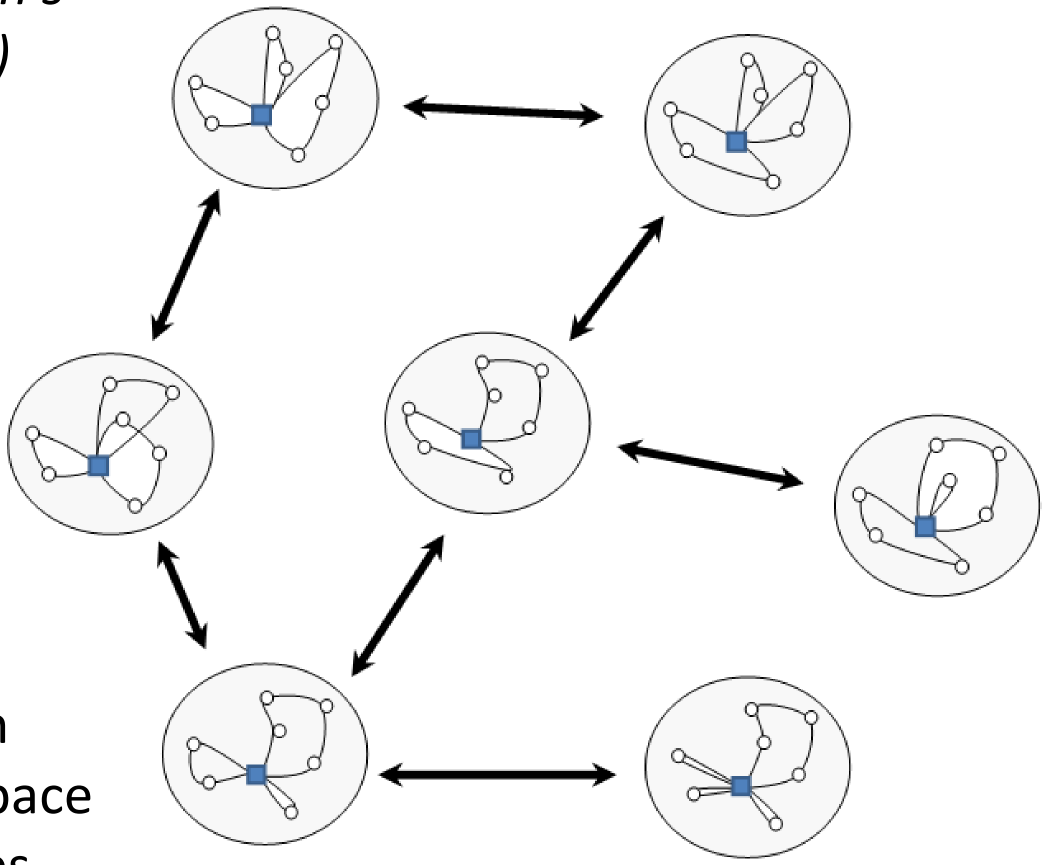
Classic Heuristics and Metaheuristics

- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

- Route first cluster second (Newton and Thomas 1974, Bodin and Berman 1979, Beasley 1979)
  - construct a giant circuit (TSP tour) that visits all customers.
  - Segmenting this tour into several routes. Optimal segmentation = assimilated to a shortest path problem in a auxiliary directed acyclic graph.

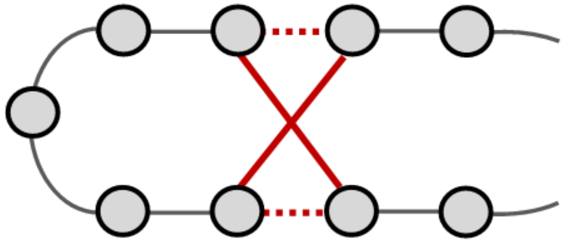
- **Local-improvement procedures** :

  - From an *incumbent solution s* define a *neighborhood N(s)* of solutions obtained by applying some changes
  - The set of solutions, linked by neighborhood relationships = *search space.*
  - LS-improvement method progress from one solution to another in this search space as long at the cost improves.

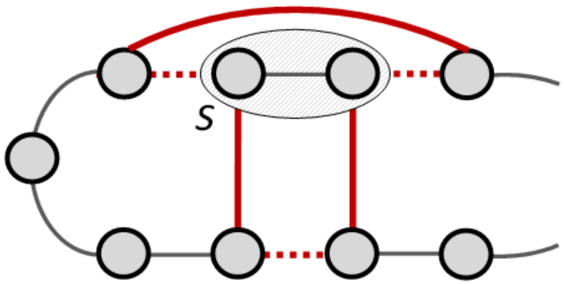


Classic Heuristics and Metaheuristics

- For optimizing a single route (TSP tour);
  - in the terminology of Lin (1965), *λ-opt* neighborhood = subset of moves obtained by deleting and reinserting *λ* arcs.
  - 2-opt and 3-opt are commonly used,
  - Or-opt which comes to relocate sequences of bounded size, and is a subset of 3-opt.



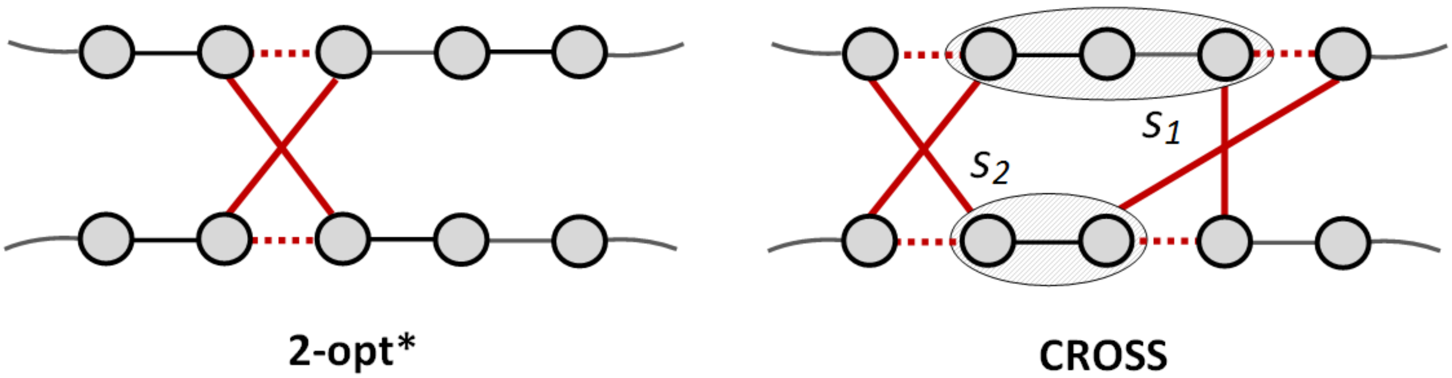
2-opt



Or-exchange

Classic Heuristics and Metaheuristics

- For optimizing multiple routes together,
  - Insert neighborhood (relocate a delivery)
  - Swap neighborhoods (swap two deliveries from different routes)
  - CROSS-exchange (exchange two sequences of visits)
  - I-CROSS (exchange and reverse two sequences)
  - 2-opt* exchange two route tails (special case of CROSS)



Classic Heuristics and Metaheuristics

- These neighborhoods contain a polynomial number of moves.
  - For all moves except CROSS and I-CROSS, the number of neighbors is proportional to n^2.
  - CROSS and I-CROSS are often limited of sequences of bounded size with less than k customers, in that case the number of neighbors is proportional to k^2n^2.

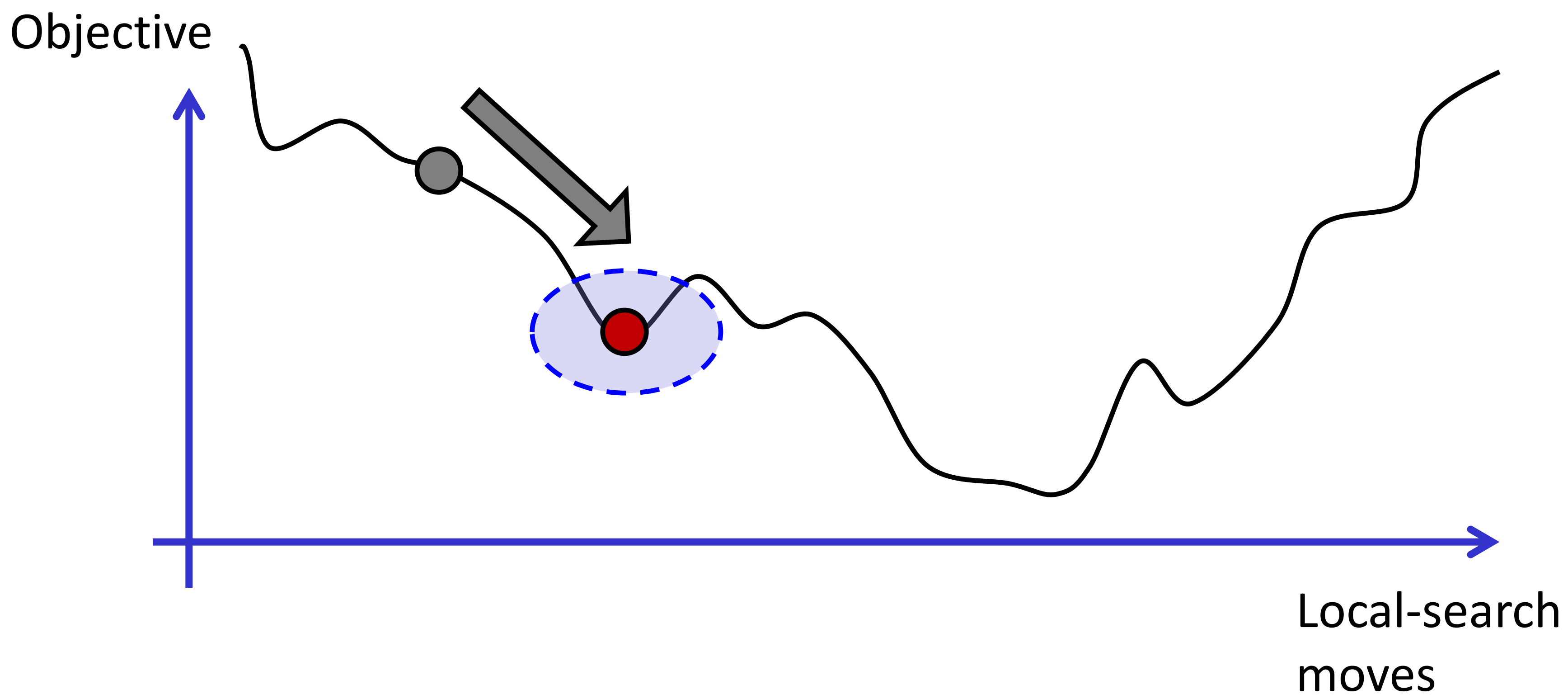
- Other non-enumerative large-scale neighborhoods:
  - Lin and Kernighan 1973
  - Ruin-and-recreate (Schrimpf 2000, Shaw 1998)
  - Ejection chains (Glover 1992,1996)

- For enumerative neighborhoods, efficient move evaluations and pruning procedures are critical to address large-scale problem instances.

  - *Granular search* (Johnson and McGeoch 1997, Toth and Vigo 2003) : restrain the subset of moves to spatially related customers.

  - *Sequential search* (Christofides and Eilon 1972 for the TSP, Irnich et al 2006 for the CVRP) : any profitable move can be broken down into a list of arc exchanges (a_1, …, a_λ) with gains (g_1, …, g_λ) such that for any k, $g_1+\ldots+g_k \geq 0$. This condition enables to dynamically prune many non-promising moves.

- Applying these neighborhoods with just improvement leads to a « local optimum » of the problem, which can be very different from the best solution (global optimum).



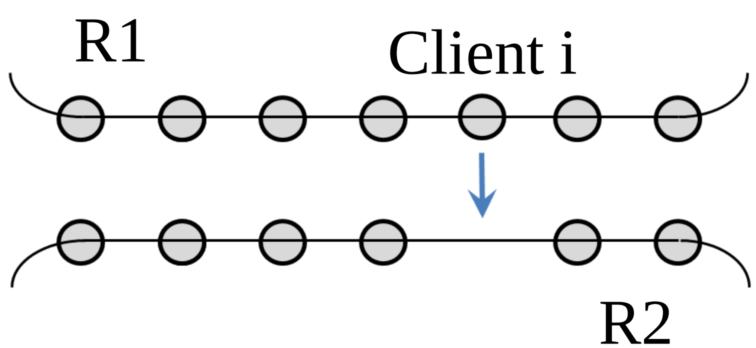
Classic Heuristics and Metaheuristics

- To escape from local optimums, efficient global strategies called "metaheuristics" have been developed in the past years.
- We discern two main classes of methods:
  - **Neighborhood-centered** search, concerned with the iterative improvement of one single solution
    - Tabu, Simulated Annealing, Iterated LS, VNS…
  - **Population-based** search, managing and improving a population of solutions.
    - Genetic or Evolutionary Algorithm, ACO, Scatter Search, PR…
- Most successful early approaches (before 2000) were neighborhood-centered, but nowadays population-based methods are the most successful.

1. UTS – Cordeau et al. (1997)

- **Tabu search** with choice of best neighbor at each step.
- Local-search neighborhoods based on single customer relocations
- Always perform the best move (possibly non-improving)
- To avoid cycling, registering some solution features as "tabu" for the next X iterations

  - For example, moving Client i from route R1 to R2. For X iterations it is not possible anymore to insert i back in route R1.



1. UTS – Cordeau et al. (1997)

- Furthermore:
  - Penalized infeasible solutions w.r.t. route constraints.
  - Continuous diversification strategy : penalizing recurrent attributes in the objective function.

2. Tabu Search with adaptive memory -- Rochat and Taillard (1995)

Diversification

- **Another Tabu Search** with several successful enhancements

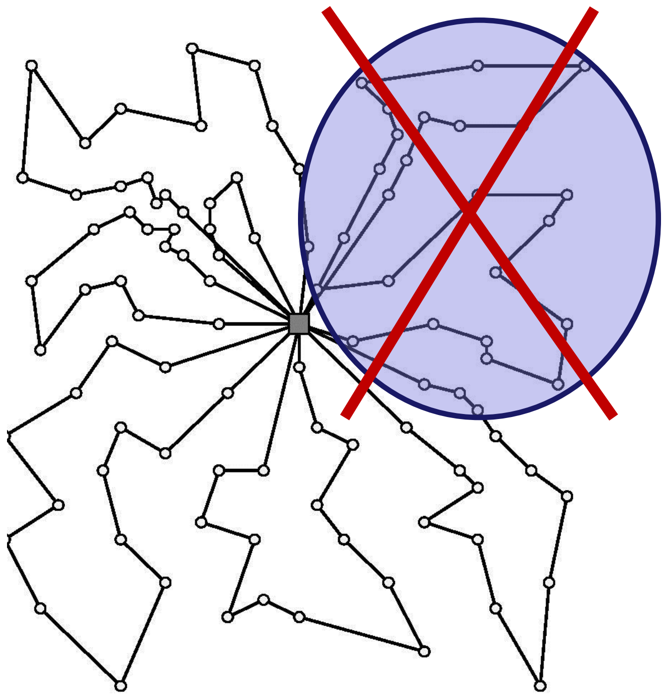
- Tabu memories based on route relocations
- Intelligent randomization of some decision variables, driven by measures of attractiveness.

- Detection of good components that consistently appear in elite solutions → and creating new solutions from them to generate new search starting points
- Decomposition phases based on spatial proximity

Intensification

3. ALNS – Pisinger and Ropke (2007)

- Large neighborhoods based on the **Ruin-and-recreate** principle.
- Variety of operators for destroying the solution
  - Using randomness, quality measures, relatedness, proximity, or history
  - Adaptive probabilities of operator selection
- Deteriorating solutions are accepted with some probability, as in Simulated Annealing.



4. ILS-RVND-SP – Subramanian et al. (2012)

- **Iterated local search :** at each iteration local search until a local optimum is encountered, *shaking* and local search again…

- A large diversity of neighborhoods is used
  - Relocate and Swap of one to three customers in different routes, 2Opt, 2Opt*, empty-route, swap depot…
  - Multiple shaking operators : multi-swap, Multi-shift, Double-bridge …

4. ILS-RVND-SP – Subramanian et al. (2012)

- Set covering model to create new solutions out of a set of high-quality routes. Adaptation of the pool size.

ELITE ROUTES found during the search

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k \in S} d_k x_k \\
\text{subject to} \quad & \sum_{k \in S} a_{ik} x_k = 1 && i = 1, \ldots, n \\
& x_k = 0 \text{ or } 1 && k \in S.
\end{aligned}
$$

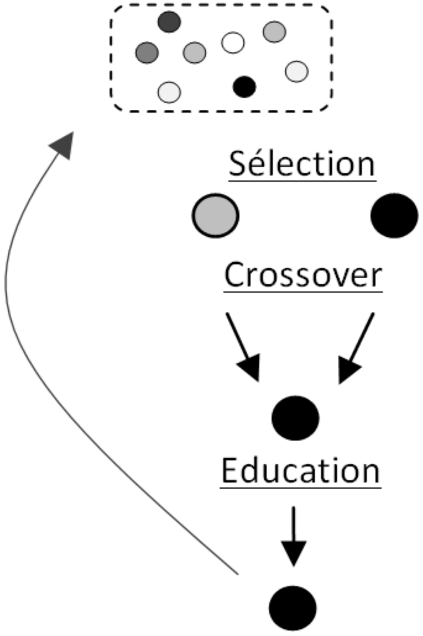
Solver for integer linear programming (Cplex)

BEST SOLUTION CREATED FROM THESE ROUTES

5. HGA – Prins (2004)

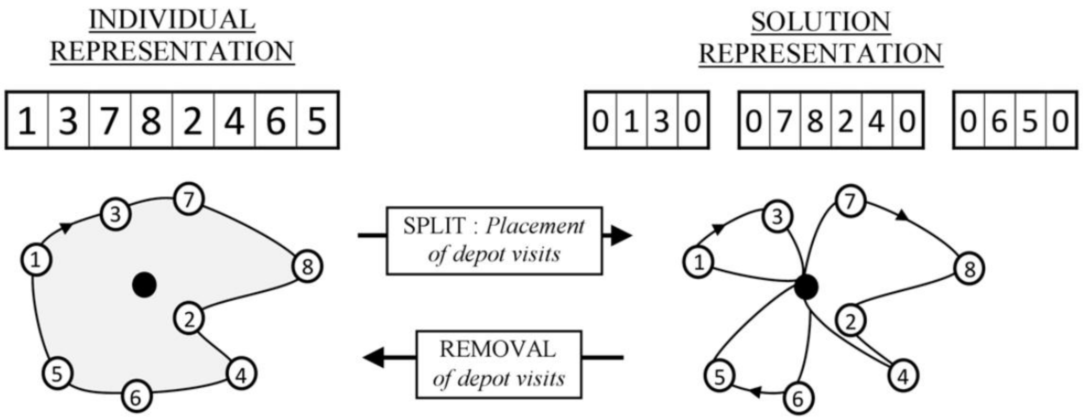
- First **Genetic Algorithm (GA)** to achieve competitive results on some VRP variants.

- **Genetic algorithms** mimic natural evolution
  - Population of solutions
  - Selection
  - Crossover
  - Mutation
    (replaced here by a local search)



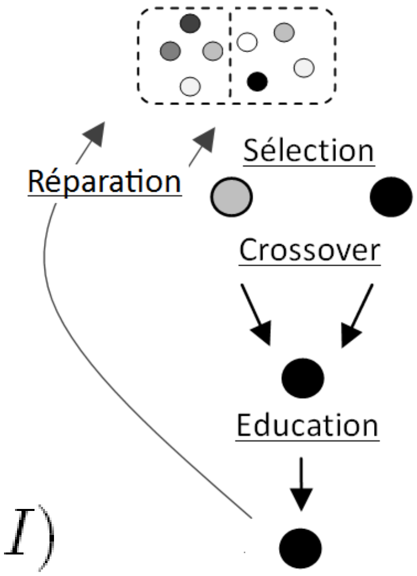
5. HGA – Prins (2004)

- The algorithm of Prins (2004) includes a few important « tricks »:
- Giant-tour solution representation
  - Polynomial Split algorithm to obtain a complete solution
- Simple Crossover
- Local search on the offspring
- Population management (spacing constraint)



6. UHGS – Vidal et al. (2012,2013)

- Classic GA framework with :
  - Giant-tour solution representation (the same as in Prins 2004)
  - Efficient local search
  - Penalized Infeasible Solutions
  - **Promotion of diversity – biased fitness**



$$BF(I) = fit(I) + (1 - \frac{nbElit}{nbIndiv - 1}) \times dc(I)$$

- **Highly flexible method →** applicable to many VRP variants.

Classic Heuristics and Metaheuristics

- **Range of problems addressed with some classic methods:**

Type	Attribute	Acronym	UTS	ALNS	HYB	UHGS
	Multiple depots	MDVRP	X	X	X	X
	Multiple periods	PVRP	X			X
	Heterogeneous fleet	HVRP			X	X
	Site-dependent	SDVRP	X	X		X
	Pickup & deliveries	VRPPD	X	X		
	Backhauls	VRPB		X		X
	Open	OVRP		X	X	X
	Cumulative	CCVRP				X
	Load-dependent costs	LDVRP				X
	Simultaneous P.&D.	VRPSDP		X	X	X
	Vehicle Fleet Mix	VFMP			X	X
	Duration constraints	DurVRP	X			X
	Hard TW	VRPTW	X	X		X
	Soft TW	VRPSTW				X
	Multiple TW	VRPMTW				X
	General TW	VRPGTW				X
	Time-dep. travel time	TDVRP				X
	Lunch breaks	VRPLB				X
	Work hours reg	VRTDSP				X
	Service site choice	GVRP[2]				X

[2] Problem known as "Generalized Vehicle Routing Problem"

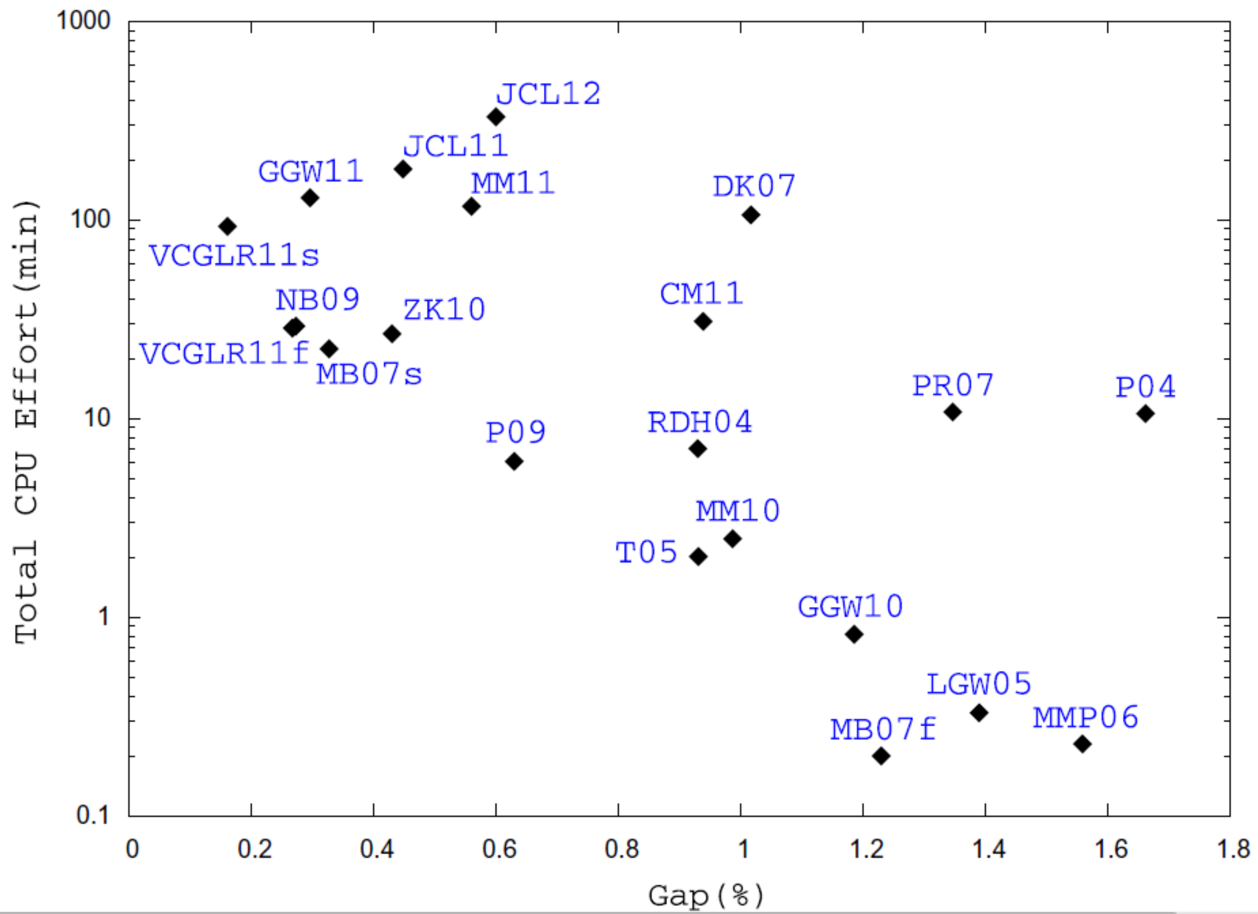
Classic Heuristics and Metaheuristics

- Quick glimpse on some other approaches, CVRP results on Golden et al (1998) instances – many types of successful methods

Acronym	Reference	Approach	Runs	Gap	T(min)	CPU	$T^{\#}$(min)
VCGLR11s	Vidal et al. (2012) slow	Hybrid GA	Avg 10	0.161%	113	Opt 2.4G	92.7
VCGLR11f	Vidal et al. (2012) fast	Hybrid GA	Avg 10	0.267%	34.8	Opt 2.4G	28.5
NB09	Nagata and Bräysy (2009)	Hybrid GA	Avg 10	0.273%	35.6	Opt 2.4G	29.2
GGW11	Groër et al. (2011)	Para. R-to-R	Best 5	0.296%	5.00	8×Xe 2.3G	129
MB07s	Mester and Bräysy (2007) slow	EA+ELS	Single	0.327%	24.4	P IV 2.8G	22.4
ZK10	Zachariadis and K. (2010a)	GLS+Tabu	Avg 10	0.430%	40.5	T5500 1.6G	26.7
JCL11	Jin et al. (2011)	Guided Tabu	Avg 10	0.448%	47.1	5×Xe 2.66G	180
MM11	Marinakis and Marinaki (2011)	Bees mating	Best 50	0.560%	3.96	P-M 1.86G	117
JCL12	Jin et al. (2012)	Coop Tabu	Avg 10	0.600%	41.9	8×Xe 3.0G	330
P09	Prins (2009a)	GRASP+ELS	Single	0.630%	7.27	P-IV 2.8G	6.09
RDH04	Reimann et al. (2004)	ACO	Avg 10	0.930%	49.3	P-III 900M	7.05
T05	Tarantilis (2005)	Ad.M.+Tabu	Single	0.931%	45.5	P-II 400M	2.02
CM11	Cordeau and M. (2012)	Iter. Tabu	Avg 10	0.939%	31.3	Xe 2.93G	30.8
MM10	Marinakis and Marinaki (2010)	GA+PSO	Avg 50	0.987%	4.20	P-M 1.86G	2.48
DK07	Derigs and Kaiser (2007)	ABHC	Single	1.017%	113	Cel 2.4G	106
GGW10	Groër et al. (2010)	R-to-R + EC	Single	1.186%	1.28	Xe 2.3G	0.82
MB07f	Mester and Bräysy (2007) fast	EA+ELS	Single	1.230%	0.22	P-IV 2.8G	0.20
PR07	Pisinger and Ropke (2007)	ALNS	Avg 10	1.347%	10.8	P-IV 3.0G	10.8
LGW05	Li et al. (2005)	R-to-R	Single	1.390%	1.13	Ath 1.0G	0.33
MMP06	Marinakis et al. (2006)	Hybrid GA	Single	1.559%	3.44	P-III 667G	0.23
P04	Prins (2004)	Hybrid GA	Single	1.662%	66.6	P-III 1.0G	10.6

Classic Heuristics and Metaheuristics

- Quick glimpse on some other approaches, CVRP results on Golden et al (1998) instances – many types of successful methods



Classic Heuristics and Metaheuristics

- **A plethora of metaheuristics specific to one or a few variants, often hybrids. Exponential literature growth.**
- Many existing concepts and methods, but… even more questions :
  - Why using a strategy of a given type
  - What is its scope of application, on which range of problems is it successful
  - Method quality = tradeoff between solution quality, speed, flexibility, robustness and simplicity

Presentation outline

PART 1)

- I) Vehicle Routing Problem, and attributes.
- II) Classic Heuristics and metaheuristics for vehicle routing
- III) An analysis of some winning strategies

PART 2)

- IV) A new general-purpose solution approach
  - Attribute-based modular design
  - Unified Local Search
  - Unified Hybrid Genetic Search
  - Computational Experiments
- V) Some application cases

An analysis of winning strategies

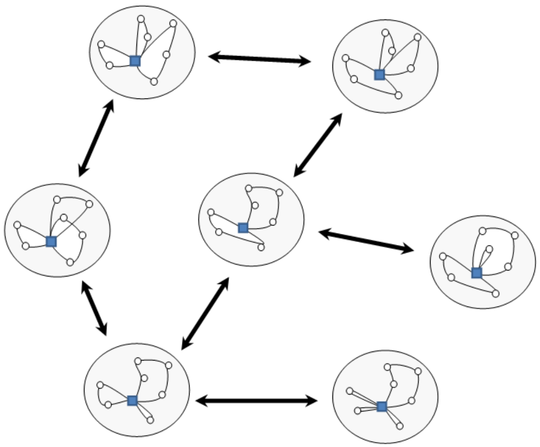
- Analyzing the method concepts, taking a broad perspective detached from problem attributes.

- **Methodology for this survey :**
  - Selecting 14 notable VRP variants. Criteria : classic benchmark instances available + large number of heuristics
  - Identifying the top 3 to 5 best metaheuristics w.r.t. solution quality
  - The resulting 64 methods are ``analyzed” to locate the recurrent successful elements of methodology.

An analysis of winning strategies

- 19 aspects of the methods have been scrutinized :

Search space	1) presence of infeasible solutions
	2) use of indirect representations of solutions
Neighbourhoods	3) presence of multiple neighbourhoods
	4) use of polynomially enumerable neighbourhoods
	5) use of pruning procedures
	6) use of large neighbourhoods
	7) use of solution recombinations
Trajectory	8) presence of random components
	9) continuous aspect of trajectories
	10) discontinuous aspect
	11) mixed aspect
Control and memories	12) use of populations
	13) diversity management
	14) parameter adaptation
	15) advanced guidance mechanisms
Hybrid strategies	16) use of hybridization
	17) matheuristics with integer programming
Parallelism	18) use of parallelism or cooperation concepts
Problem decompositions	19) use of problem decompositions



An analysis of winning strategies

- 19 aspects of the methods have been scrutinized.

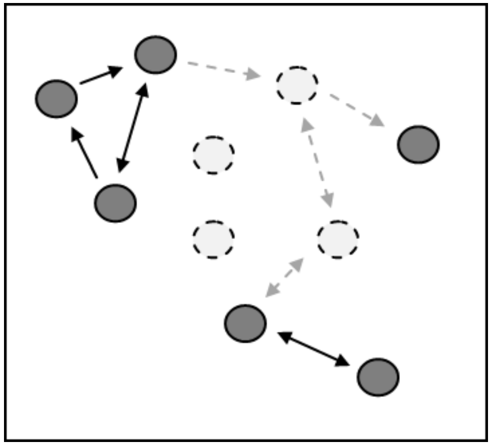
		SP. 1 RELAXATION	SP. 2 SOL. REPRESENT.	NEIGHBOUR. 3 MULT NEIGHB.	NEIGHBOUR. 4 ENUMERATIVE	NEIGHBOUR. 5 PRUNING	NEIGHBOUR. 6 LARGE	NEIGHBOUR. 7 RECOMBIN.	TRAJEC. 8 RANDOMNESS	TRAJEC. 9 CONTINUOUS	TRAJEC. 10 DISCONTINOUS	TRAJEC. 11 MIXED	CONTROL 12 POPULATIONS	CONTROL 13 DIV. MANAG.	CONTROL 14 PARAM. ADAPT.	CONTROL 15 GUIDANCE	16 HYBRIDIZATION	17 MATHEUR.	18 PARALLELISM	19 DECOMPOSITION
BACKHAULS																				
Brandão (2006)	Tabu	X		X	X				X	X					X	X				
Ropke and Pisinger (2006a)	ALNS	X		X			X		X	X					X	X	X			
Gajpal and Abad (2009)	ACO			X	X	X			X		X					X	X			
Zachariadis and K. (2012)	Attrib. driven LS			X	X	X				X						X				
PICK-UP AND DELIVERIES																				
Bent and V.H. (2006)	SA + LNS			X	X	X	X		X	X							X	X		
Ropke and Pisinger (2006b)	ALNS	X		X			X		X	X					X	X	X			
Cordeau and Laporte (2003)	Tabu	X			X	X				X					X	X				
Parragh et al. (2010)	VNS	X		X	X		X		X	X					X					
Nagata and Kobayashi (2011)	HGA			X	X	X	X	X	X		X		X			X	X			
MULTIPLE TRIPS																				
Taillard et al. (1996)	Adapt. M. + Tabu	X		X	X			X	X	X	X	X	X			X	X		X	X
Olivera and Viera (2007)	Adapt. M. + Tabu	X		X	X	X		X	X	X	X	X	X			X	X			
Alonso et al. (2008)	Tabu	X			X					X					X	X				
TIME WINDOWS																				
Hashimoto et Y. (2008)	Path Relinking	X		X	X	X	X	X	X		X		X		X	X				
Repoussis et al. (2009)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
P.-Gagnon et al. (2009)	LNS + Col. Gen.			X			X		X	X							X	X		
Nagata et al. (2010)	HGA	X		X	X	X		X	X		X		X				X			

…

An analysis of winning strategies

- **Search Space : Relaxations (31/64 methods).**

- Most often, relaxations of route constraints. (capacity, duration, time windows…). Relaxing fleet size is usually not very convenient.



- Enables to transition more easily in the search space between feasible solutions.

- Simple procedure for fleet-size minimization.

- Strategic oscillation concept (Glover 1986), good solutions tend to be close to the borders of feasibility. Oscillating around these borders by adapting the penalty coefficients.

An analysis of winning strategies

- **Search Space : Relaxations (31/64 methods).**
- We conducted some experiments on this topic:
  - Solomon VRPTW instances, (several types of) relaxations of time windows, simple LS-improvement procedure.

Inst.	SolI1	NoUnf	Late	Twarp	Flex	NoUnf	Late	Twarp	Flex
		SolomonI1 Initial Solution				Random Initial Solution			
R1	1431.97	1225.25	1219.11	1220.13	**1219.41**	1268.97	1231.70	1226.08	1229.73
R2	1326.64	963.53	957.11	947.77	942.87	982.44	**940.38**	947.41	940.79
C1	936.48	844.00	835.17	835.67	**834.26**	860.82	835.14	840.73	835.32
C2	696.57	605.62	603.62	603.08	**600.59**	709.37	650.52	645.45	649.61
RC1	1578.28	1401.49	1399.11	**1389.83**	1396.54	1482.79	1406.57	1401.53	1404.57
RC2	1653.61	1139.12	1077.93	1093.02	1079.40	1130.21	1072.63	1075.60	**1070.59**
CTD	71633	58067	57319	57275	**57125**	60361	57679	57678	57621
T(sec)	0.03	3.41	20.06	6.59	7.90	6.25	17.55	8.03	10.00

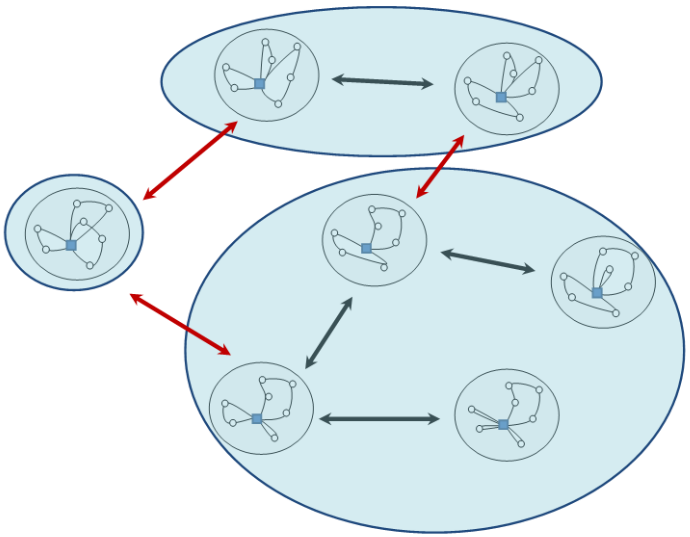
  - Same observations on distance and load relaxations on CVRP, PVRP and MDVRP with advanced metaheuristics (Vidal et al 2012).

An analysis of winning strategies

- **Search Space : Indirect representations of solutions (12/64)**

- Indirect or incomplete representations
  - Giant-tour without trip delimiters (Prins 2004)
  - Only storing customer-to-visit-days choices (PVRP -- Alegre et al 2007)
  - Solution representation as a set of circular sectors (Salhi and Petch 2007)

- Using of a **decoding algorithm** to obtain the **best complete solution** from a **solution representation**
  - 1-to-many relationship
  - Shrinking the search space
  - Target : the *shrinking ratio* aims to be much larger than the additional effort related to decoding.



An analysis of winning strategies

- **Neighborhoods: Polynomially enumerable (almost all methods)**
  - All champion MAVRP metaheuristics use either local or large neighborhood search.
  - LS neighborhoods are usually of quadratic size
  - Ruin-and-recreate is frequently used
- **Neighborhoods: Multiple neighborhoods (60/64)**
  - The richness, variety, of the neighborhoods is determining to achieve high-quality solutions. Trade-off with search speed.
  - Addressing all attributes of the problems (sequencing, assignment to depots, vehicles, days) with purposeful, possibly compound, moves is often a key to success.

An analysis of winning strategies

- **Neighborhoods: Pruning and speed-up techniques (26/64)**

- Indeed : Neighborhood search = bottleneck of most recent metaheuristics (also including population-based methods, which are usually hybrids)

- Speed-up techniques
  - **Neighborhood pruning** : granular or sequential search
  - **Memory structures** : matrices for move evaluations, hashtables for route evaluations.
  - **Information preprocessing** on subsequences to speed move evaluations in presence of complicating attributes (see end of this talk).

An analysis of winning strategies

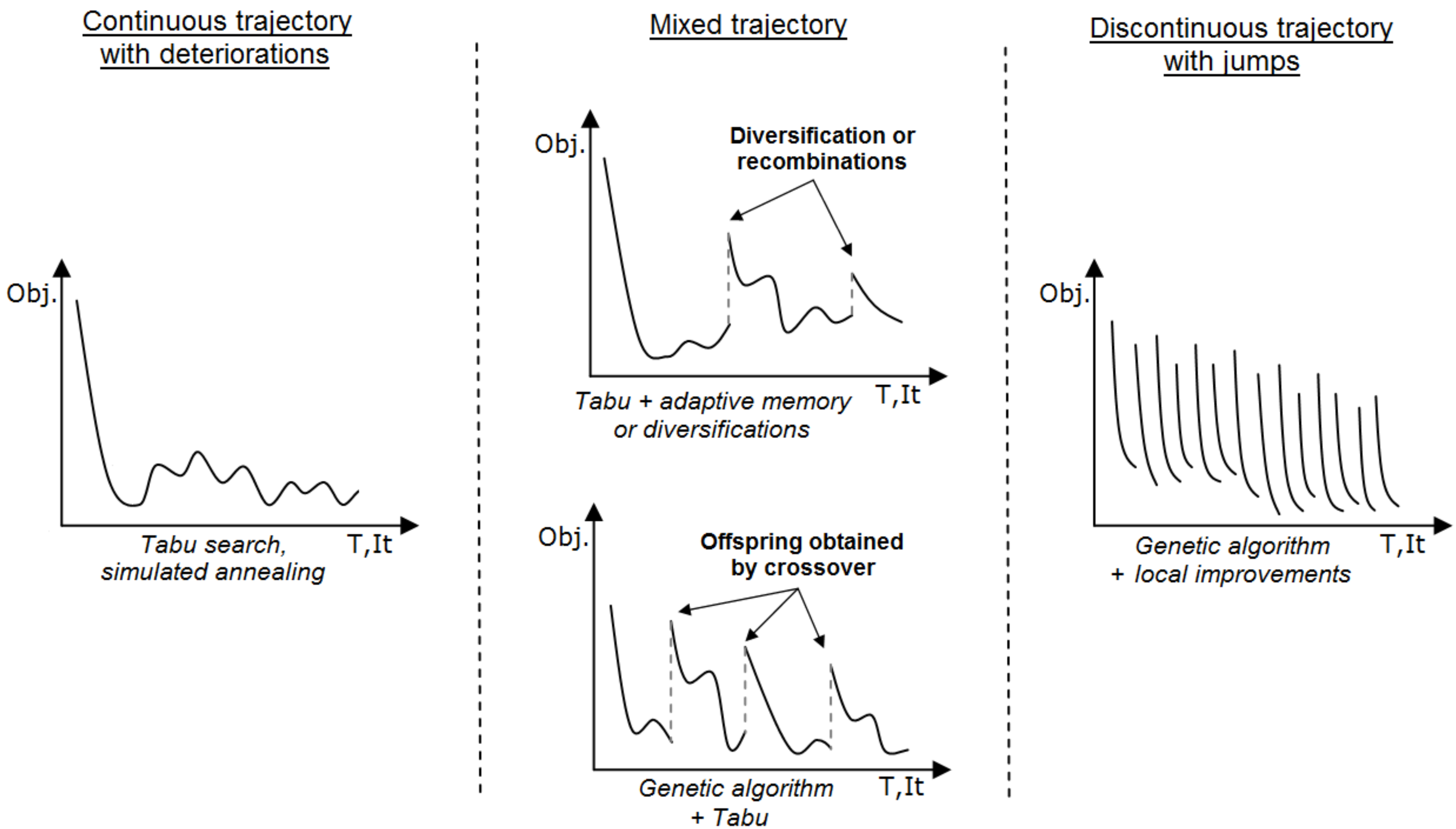
- **Neighborhoods: Solution recombinations (29/64)**
  - Combining fragments of good solutions leads to increased chances of finding new good solutions
    - Related to the building block hypothesis of Holland (1975)
    - MAVRP search landscapes often assimilated to ``big rugged valleys''
  - Not only GA or other population-based methods use recombination → c.f. adaptive memory Tabu search

An analysis of winning strategies

- **Trajectory: Randomization (56/64)**
  - Necessary for asymptotic convergence properties of SA and GA.
  - But, mostly used in recent metaheuristics as a simple way for avoiding cycling and bringing more diversity.
  - *[an intelligent use of randomization, which is not blindly uniform but embedded in probabilities that account for history and measures of attractiveness, offers a useful type of diversification that can substitute for more complex uses of memory] (*Rochat and Taillard 1995)

An analysis of winning strategies

- Trajectory : continuous (42/64), discontinuous (35/64), mixed aspect (12/64)

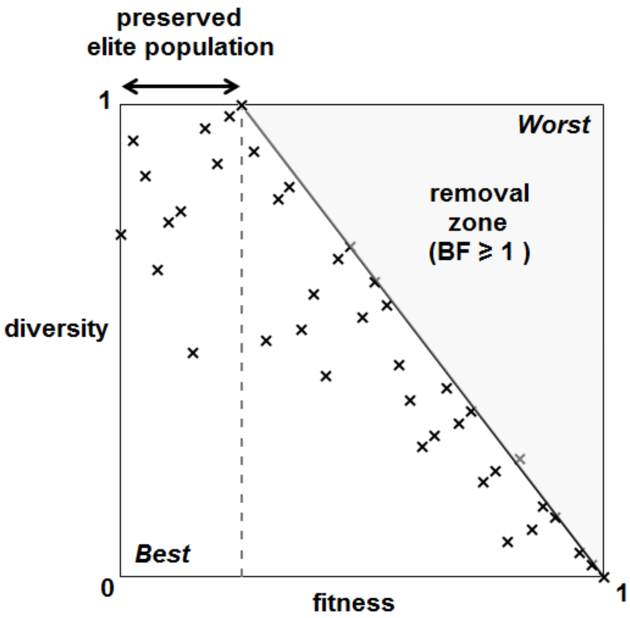


- **Memories and control : populations (28/64)**
- Judicious acquisition, management, and exploitation of problem-knowledge → complex task that belongs to the core of metaheuristics.
- Glover (1975) discern several types of memories
  - Short term memories (e.g. tabu lists) – evade local optima
  - Medium and long-term memories – used to direct the overall exploration
- Standard form of memory : populations (28/64) to store full solutions, solution representants, routes or other kind of fragments of solutions.

- **Memories and control : population management (14/28)**

- Population-based methods : need diverse and high-quality solutions
  - PM is critical to avoid premature convergence, e.g. a state where the population information is poor and redundant. Needed to compensate the aggressive-improvement abilities of LS in hybrid population-based methods.

- Diversity management strategies (Prins 2004, Sörensen and Sevaux 2006)
- Promotion of diversity in the objective (Vidal et al 2012)
- Based on distance measures, in objective or solution space.



An analysis of winning strategies

- **Memories and control : population management (14/28)**

- Some experiments on this topic (Vidal 2012), solution-quality with HGSADC on standard PVRP, MDVRP, and MDPVRP instances.
  - **HGA** : No diversity management method
  - **HGA-DR** : Dispersal rule on objective space (Prins 2004)
  - **HGA-PM** : Dispersal rule on solution space (Sörensen and Sevaux 2006)
  - **HGSADC** : Promotion of diversity in the objective (Vidal et al 2012)

Benchmark		**HGA**	**HGA-DR**	**HGA-PM**	**HGSADC**
PVRP	T	6.86 min	7.01 min	7.66 min	8.17 min
	%	+0.64%	+0.49%	+0.39%	+0.13%
MDVRP	T	7.93 min	7.58 min	9.03 min	8.56 min
	%	+1.04%	+0.87%	+0.25%	-0.04%
MDPVRP	T	25.32 min	26.68 min	28.33 min	40.15 min
	%	+4.80%	+4.07%	+3.60%	+0.44%

An analysis of winning strategies

- **Memories and control : population management (14/28)**

- Some experiments on this topic (Vidal 2012), solution-quality with HGSADC on standard PVRP, MDVRP, and MDPVRP instances.
  - **HGA** : No diversity management method
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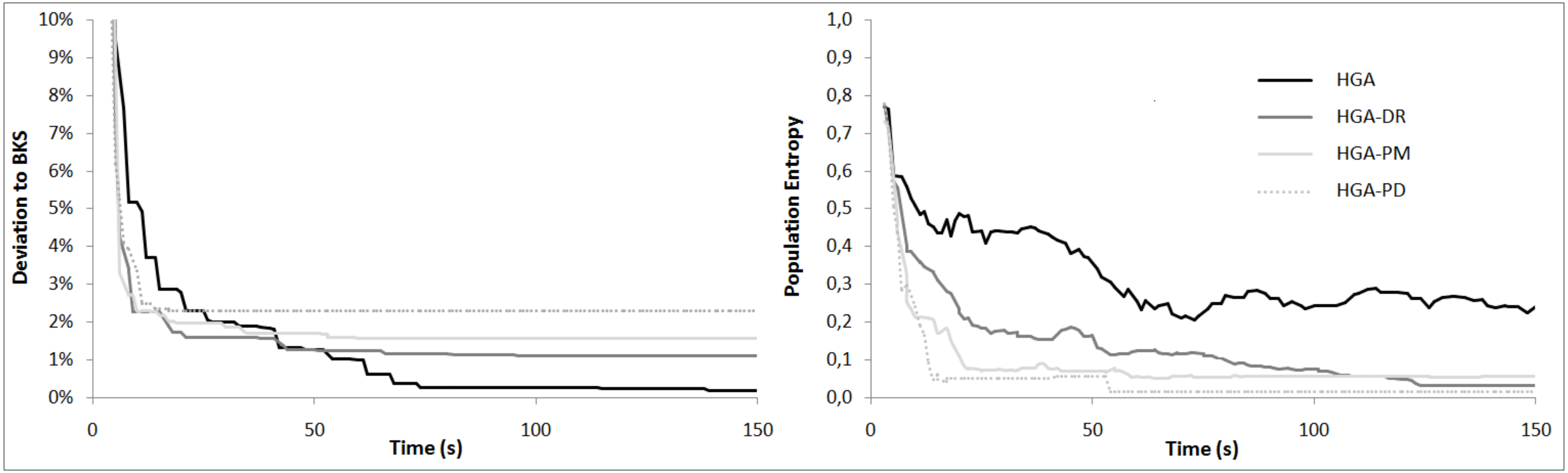


An analysis of winning strategies

- **Control and memories : guidance**

- A very simple form of guidance : parameters adaptation (30/64)
  - Driving infeasibility penalties, mutation and crossover rates, frequency of use of some operators or strategies. Method adaptation is a fundament of hyper-heuristics (Burke 2010)

An analysis of winning strategies

- **Control and memories : guidance**

- More advanced forms of guidance ***:*** explicitly collect, analyze, and exploit knowledge on the past search to orient the future trajectories.

- **Acquisition of guidance information** :
  - Historical statistics on solution features, arcs, sets of arcs, routes, or problem specic attributes.
  - Search context, value of incumbent and best solution
  - Possibly using data mining (Santos et al 2006)

- **Control and memories : guidance**

- **Exploitation of guidance information** :
  - Guidance actions to
  - Either **intensify** the search around promising solution features
  - Or **diversify** the search around promising unexplored areas.
  - Applying penalties or incentives on solution features
  - Jumps toward elite solutions or restarts
  - Target solutions in path relinking
  - Neighborhood choice driven by pheromone matrices in ACO

- Continuous during all the search, or discreet through a purposeful move

An analysis of winning strategies

- **Hybridizations (39/64)**
  - Multiple methods combine different concepts
  - Among the most frequent in the heuristics surveyed : GA+LS, ACO+LS or ACO+LNS, Tabu + recombinations, ILS + VNS…
- On a general level, metaheuristics are inherently hybrids … described sometimes as “heuristics that guide other heuristics”
- **Matheuristics (9/64)**, blending metaheuristics with integer programming components. In the methods surveyed, IP used for
  - Handling problem-attributes (e.g. loading constraints or split deliveries)
  - Exploring large neighborhoods
  - Recombining solution elements.

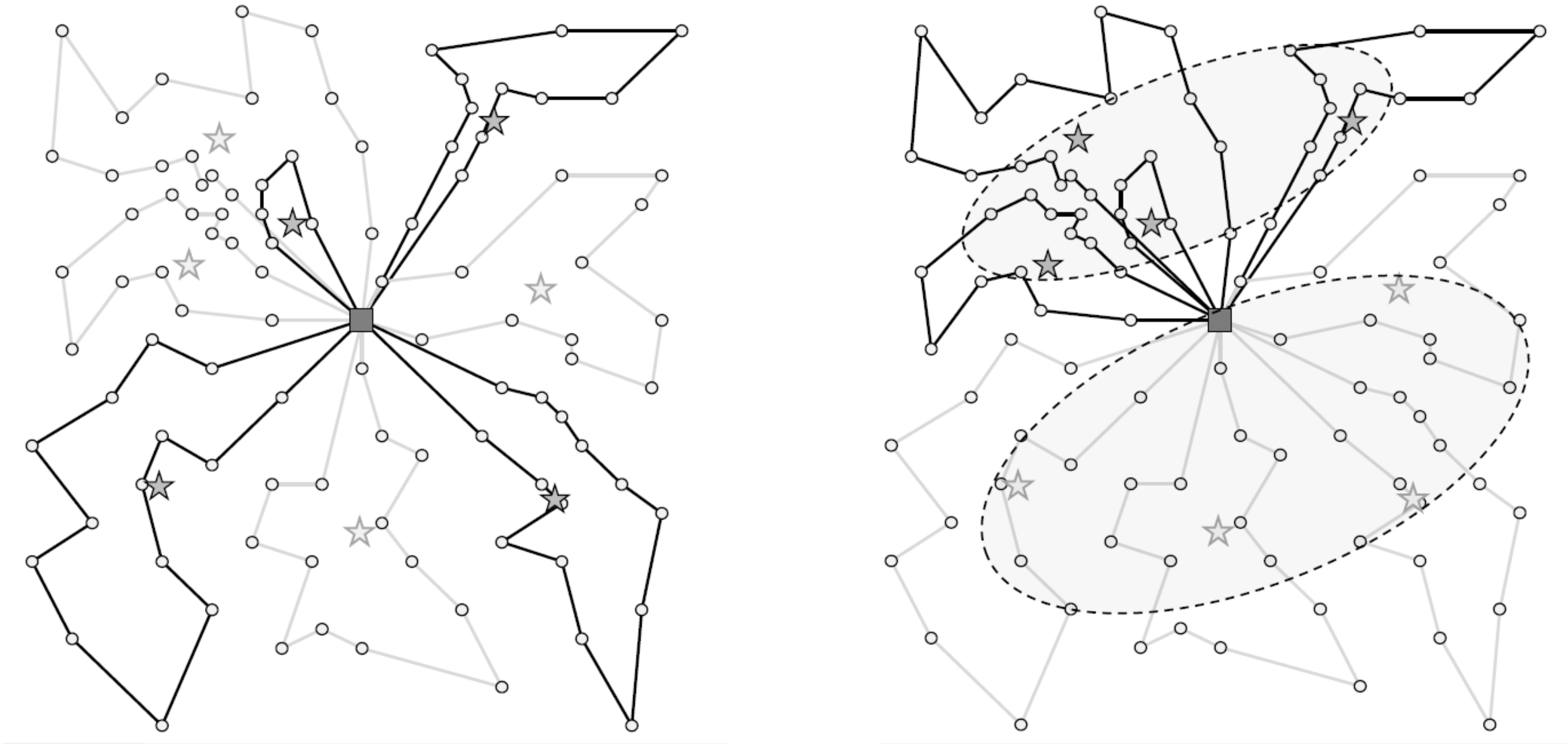
An analysis of winning strategies

- **Parallelism and cooperation (6/64)**
  - Parallel Tabu searches that cooperating through an adaptive memory of solution elements (Ichoua et al 2003), or through a central memory of complete solutions.
  - Cooperation by pheromone exchanges (Balseiro et al 2011)
  - Other works : central memory with heterogeneous methods (Le Bouthillier and Crainic 2005), or Integrative Cooperative Search (Crainic et al 2012).

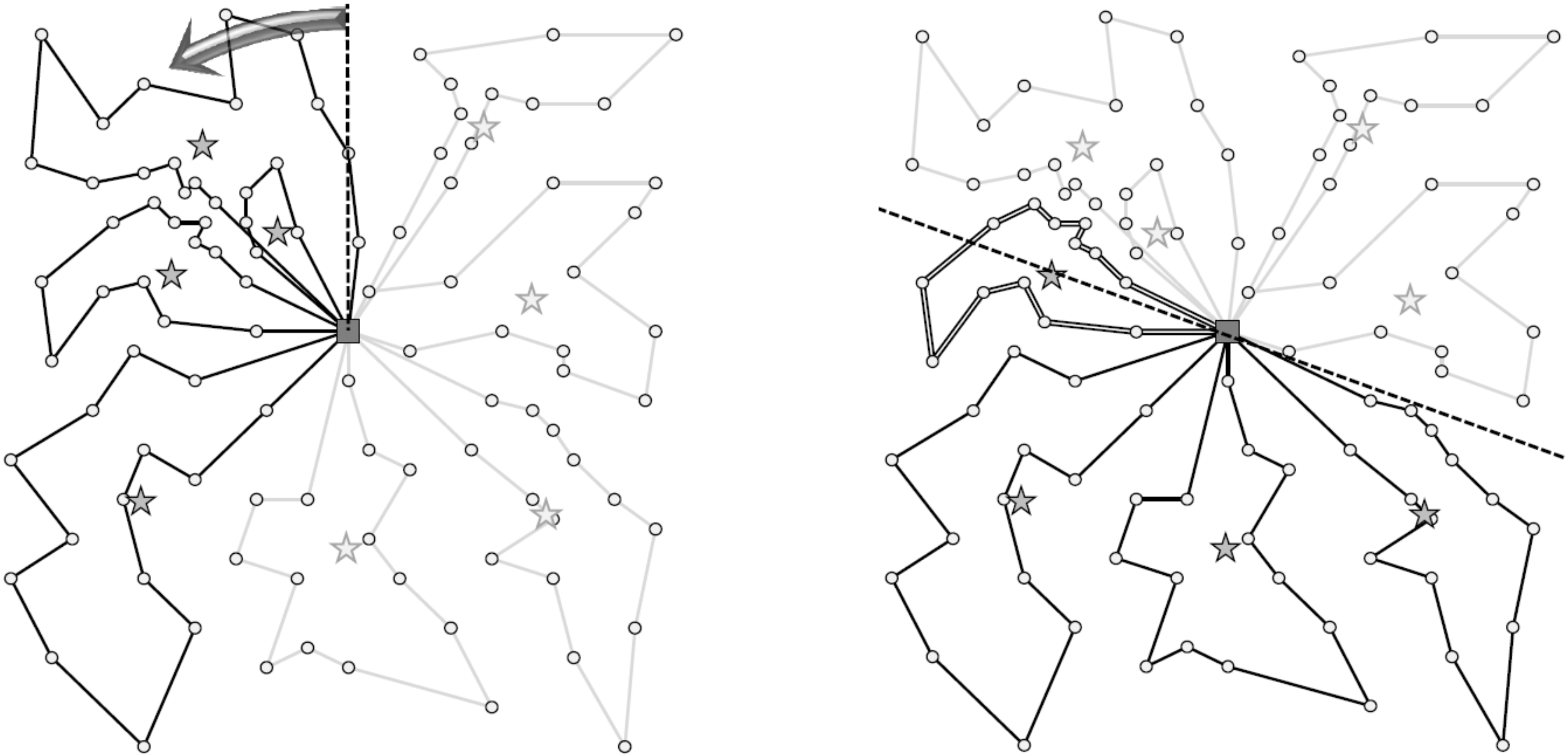
- **Decompositions**
  - MAVRPs lend themselves well to
    - Structural or geometrical problem decompositions (assignment, sequencing, attribute subsets),
    - or based on attribute resources (e.g. time)

An analysis of winning strategies

- Some possible decompositions for the CVRP:



- Some possible decompositions for the CVRP:



An analysis of winning strategies

- **Some conclusions (part I)**:

  - **1.** Recurrent notions such as *mix* , *variability*, *hybridization*, *cooperation, diversity, multiplicity*, as well as *balance*, *equilibrium, trade-off* …
  - **Success is not related a single feature but rather to a combination of concepts.**

An analysis of winning strategies

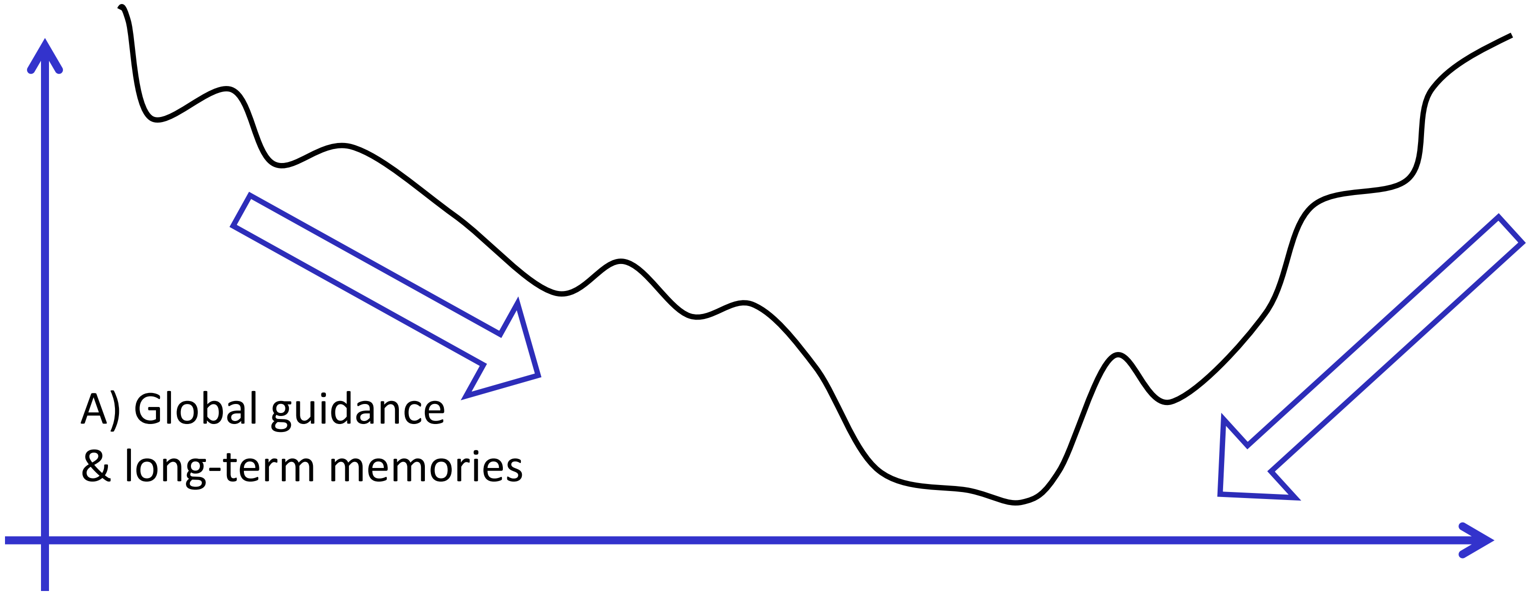
- **Some conclusions (part I)**:

  - **2. Interplay between different search levels**:
  - Long-term memories and guidance provide the necessary diversification to make the search progress in the general “big rugged valley”
  - LNS allow some medium-scale refinements
  - Short and medium-term memories and well-designed LS-improvement methods provide the aggressive search capabilities to refine the solutions.

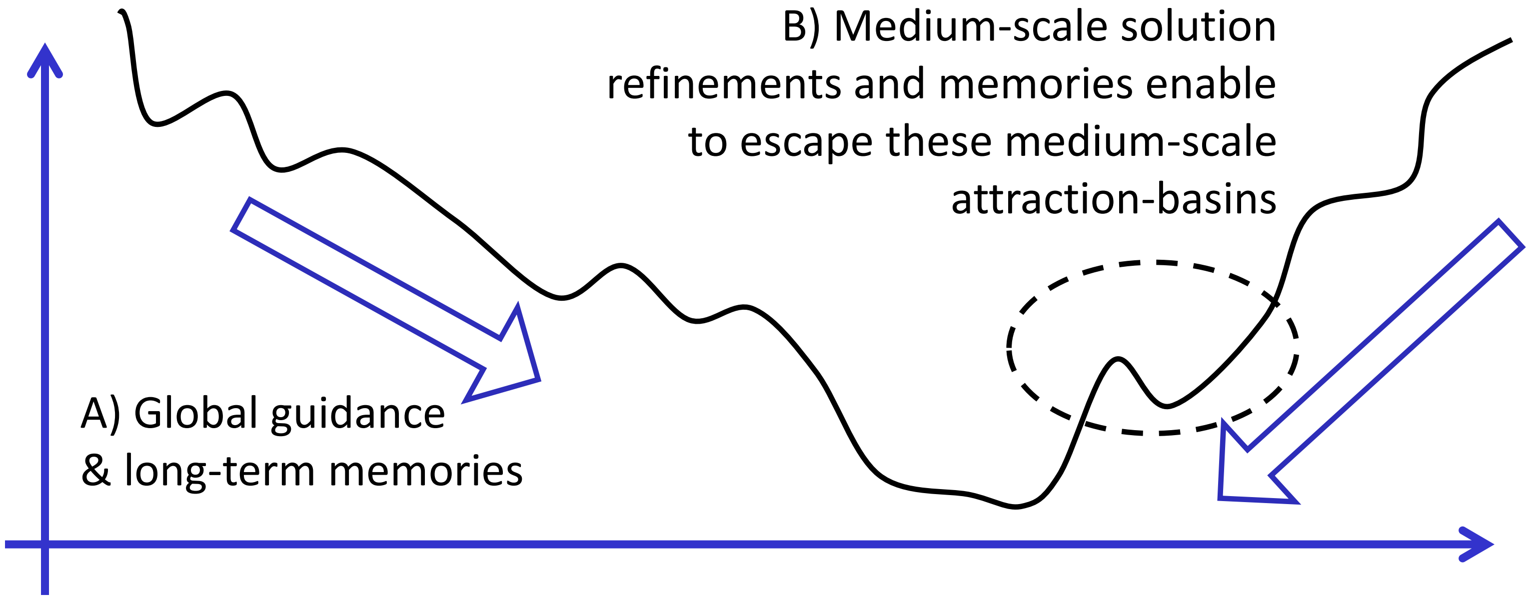
An analysis of winning strategies

- **Some conclusions (part I)**:

  - A personal mind picture :

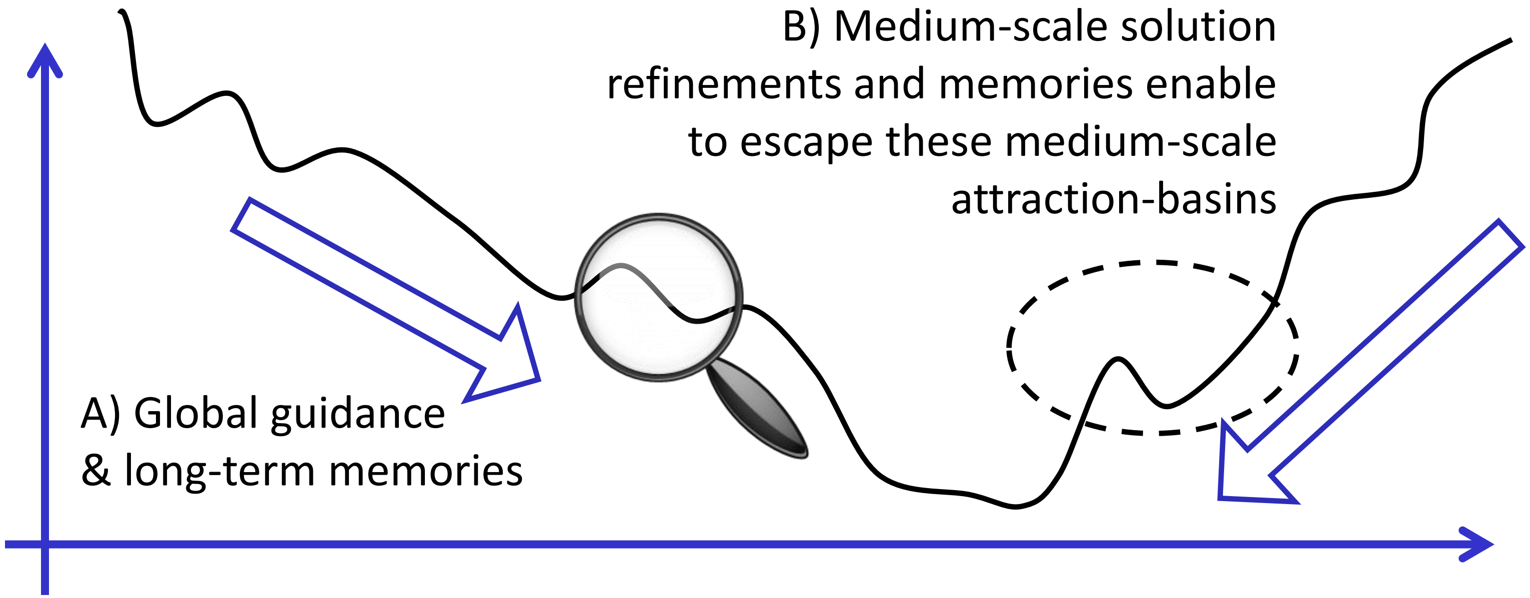


- **Some conclusions (part I)**:
  - A personal mind picture :

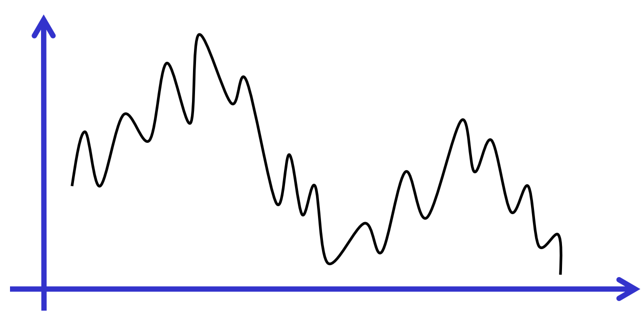


An analysis of winning strategies

- **Some conclusions (part I)**:
  - A personal mind picture :



C) Zooming-in → emphasizes small-scale ruggedness. Need LS to drive down the peaks



An analysis of winning strategies

- **Some conclusions (part I)**:

  - **3. Efficient neighborhood search and clever implementation of algorithms is a prerequisite for high performance** :
  - State-of-the-art move evaluations, reducing the complexity by keeping information on sequences,
  - neighborhood pruning and memories are critical.

  - **4. And good exploitation the search history to keep a suitable balance between intensification and diversification.**

An analysis of winning strategies

- **Some conclusions (part I)**:

  - 5. Finally, **many other components can contribute to increase solution quality**, and there are often ways to improve a method by combining additional successful concept.
  - However, success should be more considered as a good tradeoff between performance and simplicity.
  - Only real critical components should be kept and presented. (it is critical to experimentally assess the impact of each separate component).