# A General-Purpose Heuristic for Multi-Attribute Vehicle Routing Problems

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# Presentation outline

1. General-purpose solvers
2. Multi-attribute vehicle routing problems (MAVRPs)
3. An efficient and unified local search for MAVRPs
    a) Route evaluation operators
    b) Implementation for several attributes
4. A Unified Hybrid Genetic Search (UHGS) for MAVRPs
    a) General framework
    b) Unified solution representation and Split
    c) Generic implementation of other genetic operators
5. Computational experiments

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# General-purpose solvers

- **General-purpose solvers for combinatorial optimization**
  - Solvers that can address a wide range of problems without need for extensive adaptation or user expertise.

- **Necessary tools** for the timely application of current optimization methods to industrial settings.

- Examples of such solvers :
  - Integer & constraint programming solvers
  - Local search-based methods : "*LocalSolver*" (Benoist et al. 2011).
  - Methods designed to address a large compound problem model.

- Libraries of metaheuristic components and classes libraries: Open BEAGLE (Gagné and Parizeau 2002), ParadisEO (Cahon et al. 2004)...

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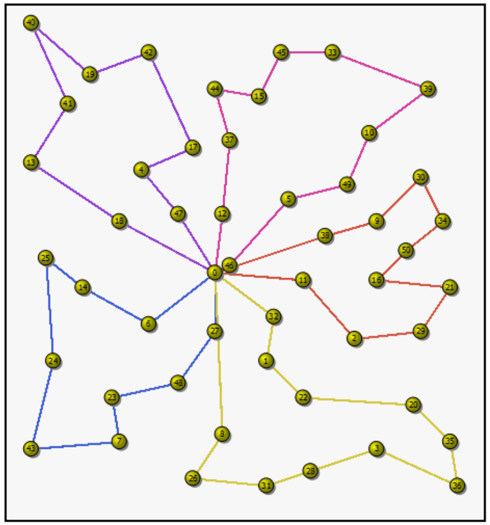
# Multi-attribute vehicle routing problems (MAVRPs)

- Classical "vehicle routing problem (VRP)"
  → wide range of exact and heuristic methods

- **Challenges** related to the resolution of **VRP variants with additional *attributes*** (multi-attribute VRPs, MAVRPs)
  - modeling the specificities of application cases, customers requirements, network and vehicle specificities, operators abilities…
  - Combining **several attributes** together can lead to highly complex **rich VRPs.**
  - Dramatic increase in the literature dedicated to specific VRP variants.



# Multi-attribute vehicle routing problems (MAVRPs)

- Some unified algorithms reporting high quality solutions on several MAVRPs:

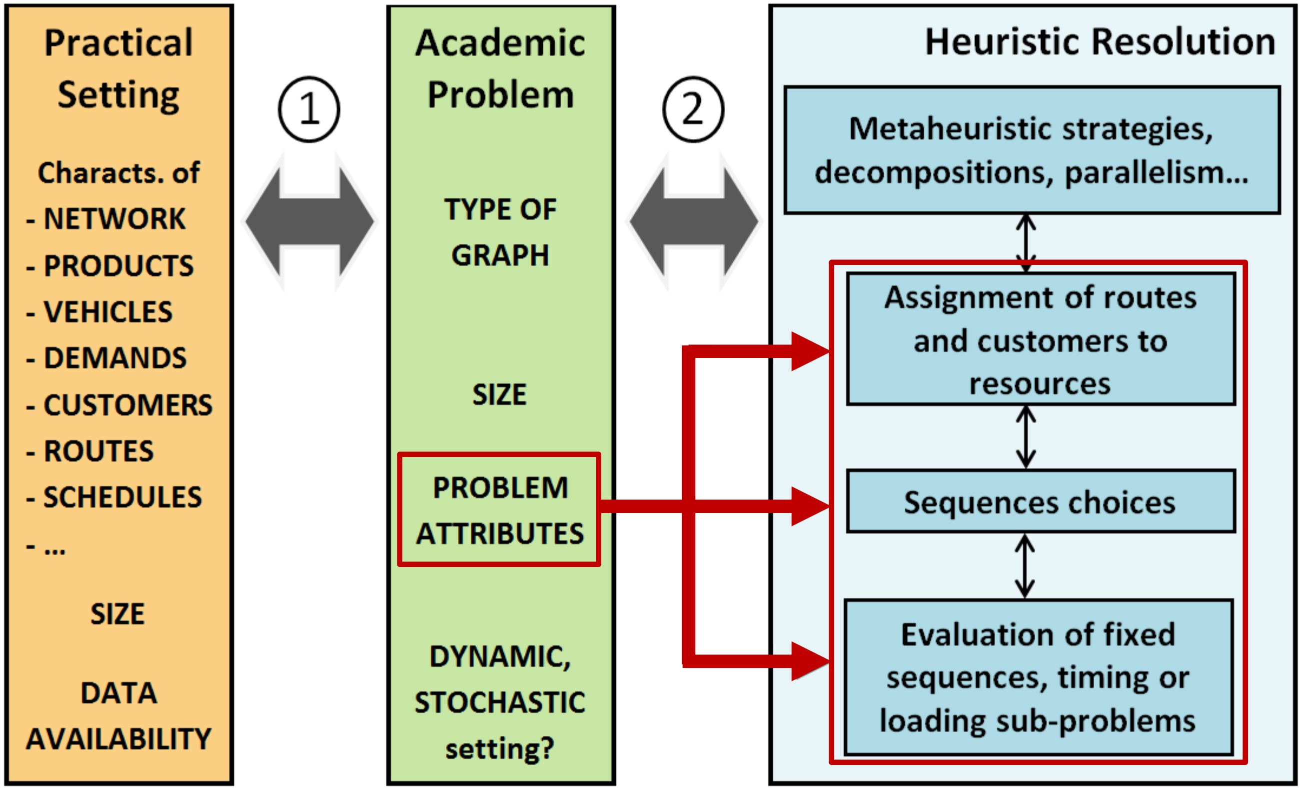
| Type | Attribute | Acronym | UTS | ALNS | ILS | HYB | VNS | IPSP | UHGS |
|---|---|---|---|---|---|---|---|---|---|
| | Multiple depots | MDVRP | X | X | | X | | X | X |
| | Multiple periods | PVRP | X | | | | | X | X |
| | Heterogeneous fleet | HVRP | | | | X | | X | X |
| | Site-dependent | SDVRP | X | X | | | | X | X |
| | Pickup & deliveries | VRPPD | X | X | | | | X | |
| | Backhauls | VRPB | | X | | | | | X |
| | Open | OVRP | | X | | X | X | | X |
| | Cumulative | CCVRP | | | | | | | X |
| | Load-dependent costs | LDVRP | | | | | | | X |
| | Simultaneous P.&D. | VRPSDP | | X | | X | | | X |
| | Vehicle Fleet Mix | VFMP | | | | X | | X | X |
| | Duration constraints | DurVRP | X | | X | | | | X |
| | Hard TW | VRPTW | X | X | X | | X | X | X |
| | Soft TW | VRPSTW | | | X | | X | | X |
| | Multiple TW | VRPMTW | | | X | | | | X |
| | General TW | VRPGTW | | | X | | | | X |
| | Time-dep. travel time | TDVRP | | | X | | X | | X |
| | Lunch breaks | VRPLB | | | | | | | X |
| | Work hours reg | VRTDSP | | | | | | | X |
| | Service site choice | GVRP[2] | | | | | | | X |

[2] Problem known as "Generalized Vehicle Routing Problem"

- Issues: dealing with a rich VRP model that includes several MAVRP as special cases → Still accounting for non-activated attributes

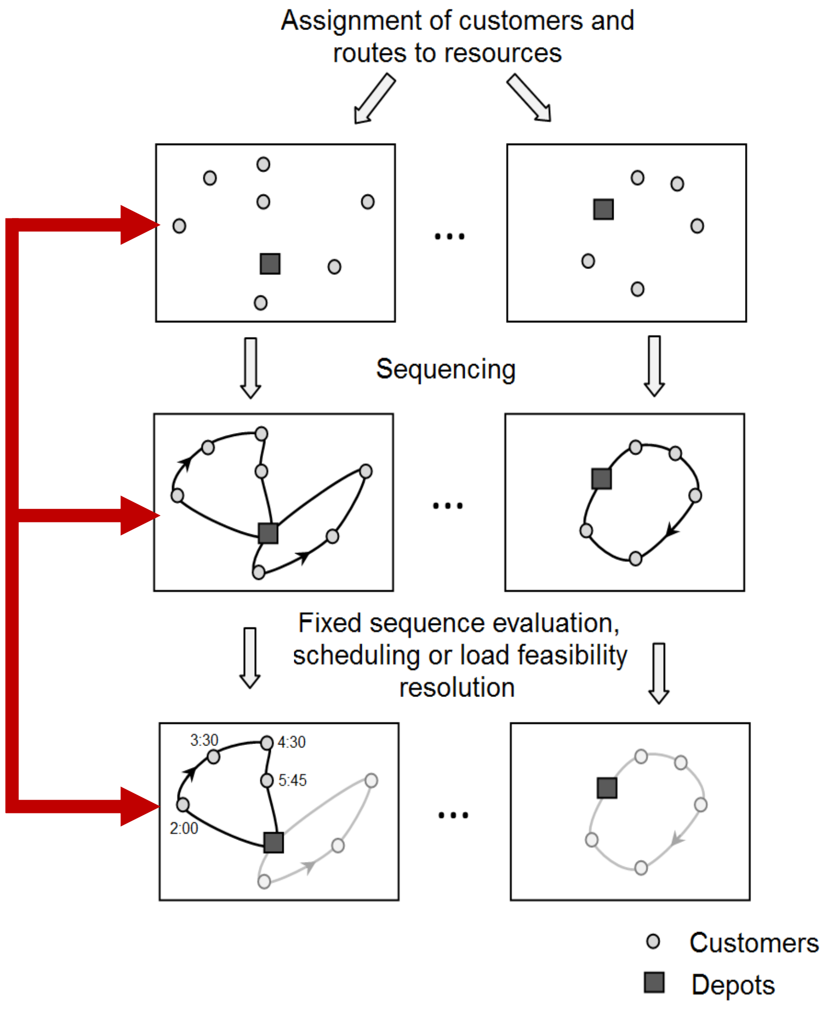
# Classification & Proposed Methodology

- We classified attributes into three categories related to their impact on VRP resolution methods :



# Classification & Proposed Methodology

- **ASSIGN ATTRIBUTES**: impacting the assignment of customers and routes
  - **Periodic, Multi-Depot, Heterogeneous Fleet, Location Routing…**
- **SEQ ATTRIBUTES:** impacting the nature of the network and the sequences
  - **P&D, Backhauls, Two Echelon, Multi Trips, Truck-and-Trailer…**
- **EVAL ATTRIBUTES:** impacting the evaluation of fixed routes
  - **Time windows, Time-dep. travel time, Loading constraints, HOS regulations Lunch breaks, Load-Dependent costs…**

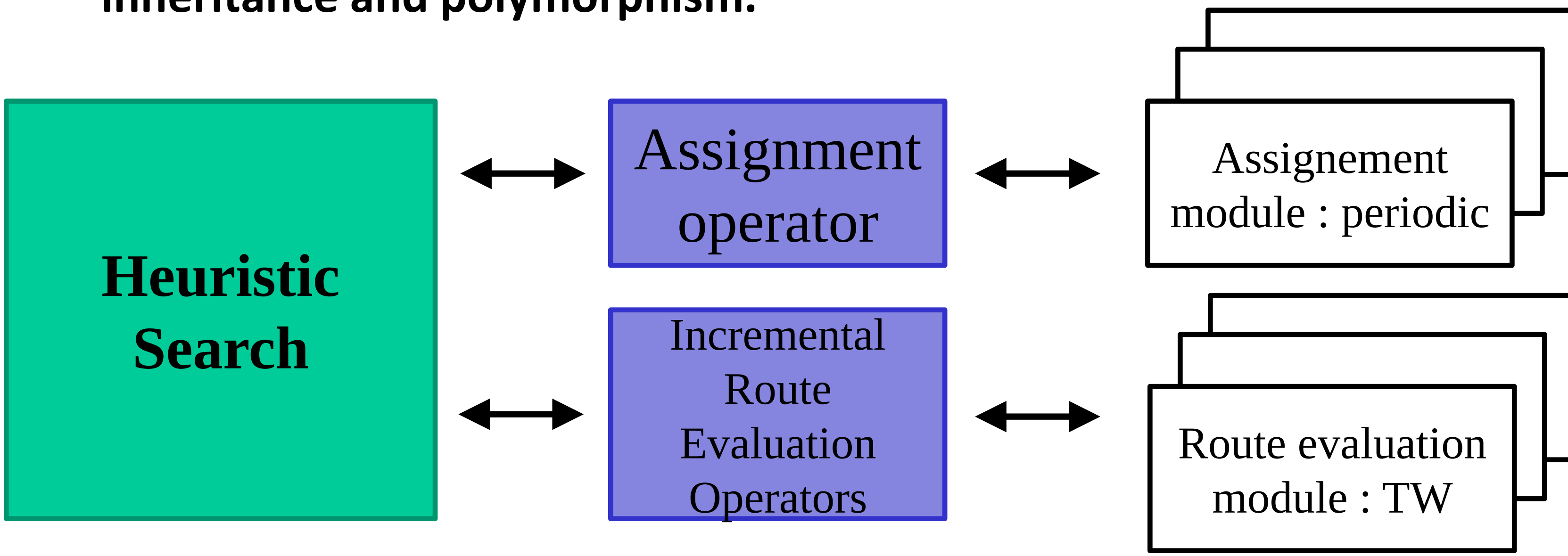


# Classification & Proposed Methodology

- Challenge: **Achieving both genericity and efficiency**
  - Still need to address the problem → but relegating problem-specificities to small modular components
  - Each separate MAVRP shall be still addressed with state-of-the-art solution evaluation and search procedures
  - Not dealing with “dummy” attributes

# Classification & Proposed Methodology

- Proposed Methodology:
  - Relying on assignment, sequencing & route evaluation operators → implemented in a generic way, based on a library of attribute-specific modules
  - **Attribute-dependent modules are automatically selected by the algorithm to serve as the basis for the assignment, sequencing, and route evaluation operators → Object-oriented programming, using inheritance and polymorphism.**



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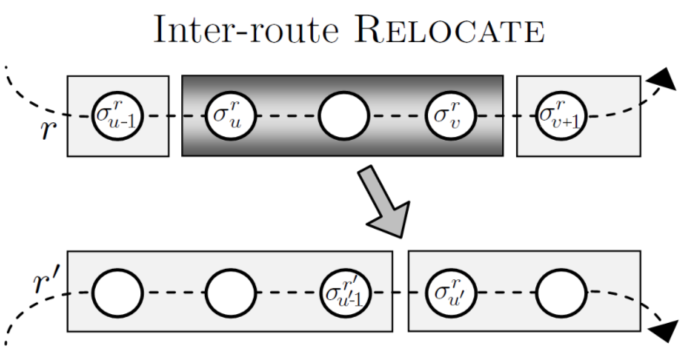
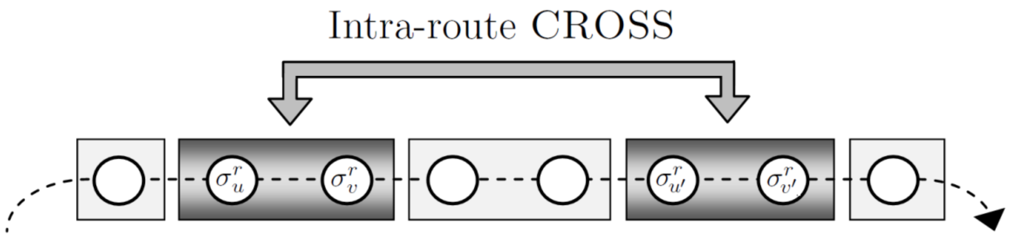
# An efficient and unified local search for MAVRPs

- Route Evaluation Operators based on re-optimization
  - Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
  - The same subsequences appear many times during different moves




  - Data preprocessing on sub-sequences to speed up the search (Savelsbergh 1985,1992 …)
  - The route evaluation modules must allow for such preprocessing.

- <u>Route Evaluation Operators based on re-optimization</u>
  - Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
  - Hence, to manage and exploit information on subsequences, five families of route evaluation operators are used :

| | |
|---|---|
| *Operators for data construction:* | |
| $\textsc{Init}(\sigma)$ | Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit. |
| $\textsc{Forw}(\sigma)$ | Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence $\sigma$ and vertex $v_i$. |
| $\textsc{Back}(\sigma)$ | Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex $v_i$ and sub-sequence $\sigma$. |
| *Operators for route evaluations:* | |
| $\textsc{Eval2}(\sigma_1, \sigma_2)$ | Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \sigma_2$. |
| $\textsc{EvalN}(\sigma_1, \dots, \sigma_n)$ | Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \dots \oplus \sigma_n$. |

# Route evaluation operators examples

- **Example 1)** Route evaluation operators for **distance and capacity constraints**

| | |
|---|---|
| **What is managed ? →** Partial loads $L(\sigma)$ and distance $D(\sigma)$ | |
| **Init →** For a sequence $\sigma_0$ with a single visit $v_i$ , $L(\sigma_0) = q_i$ and $D(\sigma_0) = 0$ | |
| **Forw and Back →** increment $L(\sigma)$ and $D(\sigma)$ | |
| **Eval →** compute the data by induction on the concatenation operator | |

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2)$$
$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

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# Route evaluation operators examples

- **Example 2)** Route evaluation operators for **cumulated arrival time objectives**

**What is managed ?** → Travel time D(σ), Cumulated arrival time C(σ), Delay Cost W(σ) associated to one unit of delay in starting time

**Init** → For a sequence $\sigma_0$ with a single visit $v_i$ , $D(\sigma_0) = 0$ and $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ if $v_i$ is a customer, and $W(\sigma_0) = 0$ if $v_i$ is a depot visit.

**Forw & Back & Eval** → induction on the concatenation operator:

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2)$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2)$$

- **Example 3)** Route evaluation operators for **time windows (and route duration constraints)**

**What is managed ? →** Travel time and service time $T(\sigma)$, earliest feasible completion time $E(\sigma)$, latest feasible starting date $L(\sigma)$, statement of feasibility $F(\sigma)$.

**Init →** For a sequence $\sigma_0$ with a single visit $v_i$ , $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = \textit{true}$.

**Forw & Back & Eval →** induction on the concatenation operator:

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2)$$
$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$
$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$
$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \ \wedge \ F(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$

# Route evaluation operators examples

- **Example 4)** Route evaluation operators for **lunch break positioning in presence of time-window constraints**

**What is managed ? →** Same set of data ($T(\sigma)$, $E(\sigma)$, $L(\sigma)$, and $F(\sigma)$) as in the TW case, and it is duplicated to also provide $T'(\sigma)$, $E'(\sigma)$, $L'(\sigma)$, and $F'(\sigma)$ for the sequence where exactly one lunch break was inserted.

**Init →** As previously for $T(\sigma_0)$, $E(\sigma_0)$, $L(\sigma_0)$, and $F(\sigma_0)$. Furthermore, $T'(\sigma_0) = +\infty$ , $E'(\sigma_0) = +\infty$ , $L'(\sigma_0) = 0$, and $F'(\sigma_0) =$ *false*.

**Forw & Back & Eval →** induction on the concatenation operator, see next page for the equations.

# Route evaluation operators examples

- **Example 4)** Route evaluation operators for **lunch break positioning in presence of time-window constraints**

$$E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case i}} | F'_{\text{case i}} = true\} \cup +\infty)$$
$$L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case i}} | F'_{\text{case i}} = true\} \cup -\infty)$$
$$F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case 1}} \vee F'_{\text{case 2}} \vee F'_{\text{case 3}}$$

$$E'_{\text{case 1}} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$
$$E'_{\text{case 2}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\}$$
$$E'_{\text{case 3}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T'(\sigma_2), E'(\sigma_2)\}$$
$$L'_{\text{case 1}} = \min\{L'(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T'(\sigma_1)\}$$
$$L'_{\text{case 2}} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - s_{\text{LB}} - T(\sigma_1)\}$$
$$L'_{\text{case 3}} = \min\{L(\sigma_1), L'(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$
$$F'_{\text{case 1}} = F'(\sigma_1) \wedge F(\sigma_2) \wedge (E'(\sigma_1) + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$
$$F'_{\text{case 2}} = F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) \leq l_{\text{LB}}) \wedge (E(\sigma_1) + s_{\text{LB}} + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$
$$F'_{\text{case 3}} = F(\sigma_1) \wedge F'(\sigma_2) \wedge (E(\sigma_1) + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L'(\sigma_2))$$

# Route evaluation operators examples

- **Example 5)** Route evaluation operators for **soft and general time windows**

**What is managed ? →** Minimum cost F(σ)(t) to process the sequence σ while starting the last service before time t, minimum cost B(σ)(t) to process the sequence σ after time t.

**Init →** For a sequence $\sigma_0$ with a single visit $v_i$ characterized by a service cost function $c_i(t)$, $F(\sigma_0)(t) = \min_{(x \leq t)} ci(x)$ and $B(\sigma_0)(t) = \min_{(x \geq t)} c_i(x)$.

**Forw & Back →**

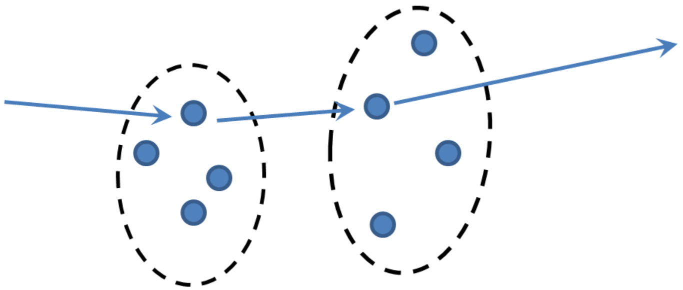
$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t}\{c_i(t) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\}$$

**Eval 2 →** $$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0}\{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\}$$

# Route evaluation operators examples

- **Example 6)** Route evaluation operators for **the generalized VRP :**



**What is managed ? →** The shortest path S(σ)[i,j] inside the sequence σ starting at the location *i* of the starting group and finishing at location *j* of the ending group.

**Init →** For a sequence $\sigma_0$ with a single visit $v_i$ , S(σ)[i,j] = +∞ if i ≠ j, and S(σ)[i,i] = 0.

**Forw & Back & Eval →** induction on the concatenation operator:

$$S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j]$$

$$\forall i \in \{1, \ldots, \lambda_{\sigma_1(1)}\}, \forall j \in \{1, \ldots, \lambda_{\sigma_2(|\sigma_2|)}\}$$

# An efficient and unified local search for MAVRPs

- Generic local-search based on route evaluation operators

**Algorithm 1** Unified local search based on route evaluation operators

Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
**while** some improving moves exist in the neighborhood $\mathcal{N}$ **do**
  **for** each move $\mu_i$ in $\mathcal{N}$ **do**
    **for** each route $r_j^\mu$ produced by the move **do**
      Determine the $k$ sub-sequences $[\sigma_1, \ldots, \sigma_k]$ that are concatenated to produce $r_j^\mu$
      **if** $k = 2$, then NEWCOST$(r)$ = EVAL2$(\sigma_1, \sigma_2)$
      **else if** $k > 2$, then NEWCOST$(r)$ = EVALN$(\sigma_1, \ldots, \sigma_k)$
    **if** ACCEPTCRITERIA$(\mu_i)$ **then** perform the move $\mu$ and update the re-optimization data on for each route $r_j^\mu$ using the INIT, FORW and BACK operators.

- Can serve as the basis to build any neighborhood-based unified solver based on VNS, Tabu, ILS for MAVRPs with EVAL attributes.
- Going one step further, designing a unified hybrid GA.

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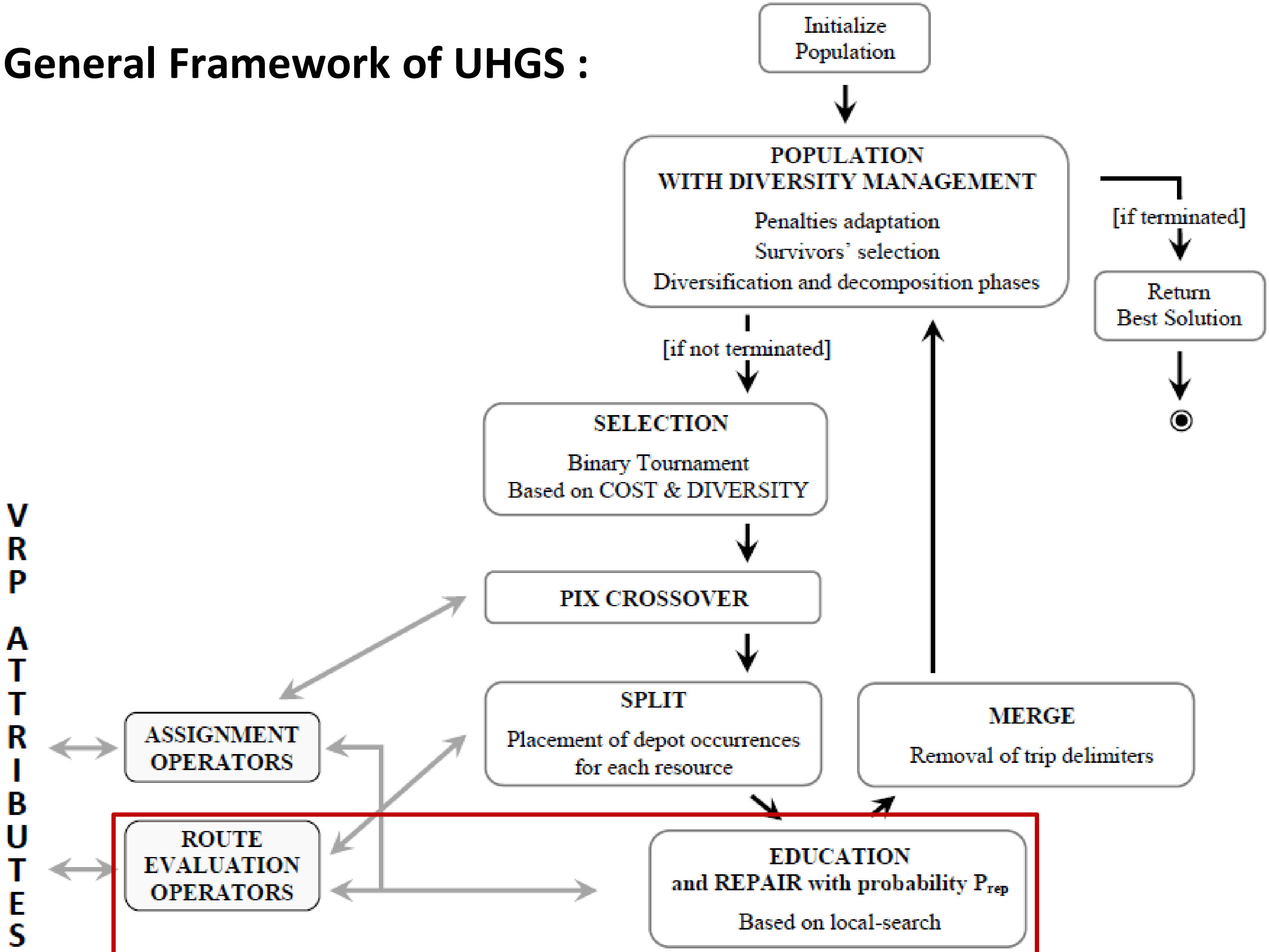
# A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- UHGS = Classic GA framework + 4 main ingredients (Vidal et al. 2010)
  - Management of penalized infeasible solutions in two subpopulations
  - High-performance local search-based *Education* procedure
  - Solution Representation *without trip delimiters*
  - **Diversity & Cost objective for individuals evaluations**

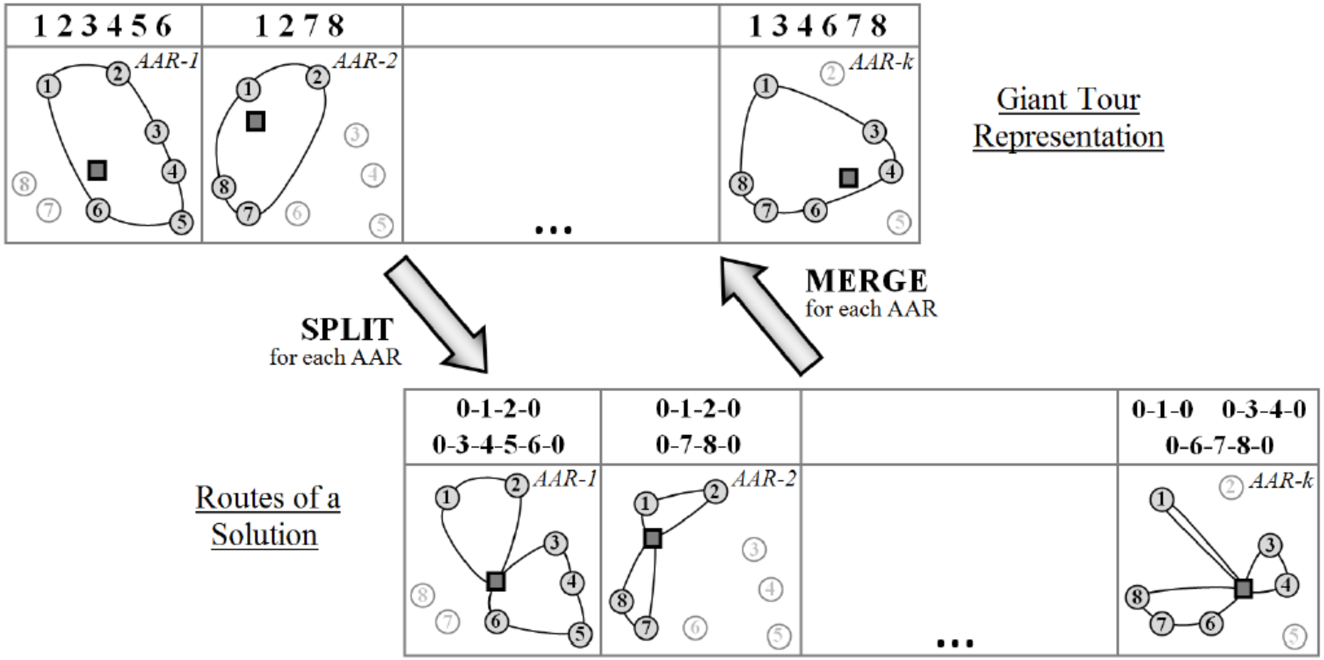
# A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- General Framework of UHGS :



# Unified Solution Representation and Split

- Now dealing with MAVRPs with both ASSIGN and EVAL attributes: Assignment of customer services to some ASSIGN attributes resources (AARs) + separate optimization of routes for each AARs.
  - Solution representation is designed accordingly.
  - Furthermore, representation without trip delimiters for each AAR.



# Unified Solution Representation and Split

- Solution representation as a giant-tour per AAR → requires a Split algorithm (Prins 2004) for optimal segmentation into routes.

- We propose a **unified Split algorithm**.
  - As usual, the problem is solved as a m-shortest path
  - The route evaluation operators are used to build the auxiliary graph

**Algorithm 2** Generic Split

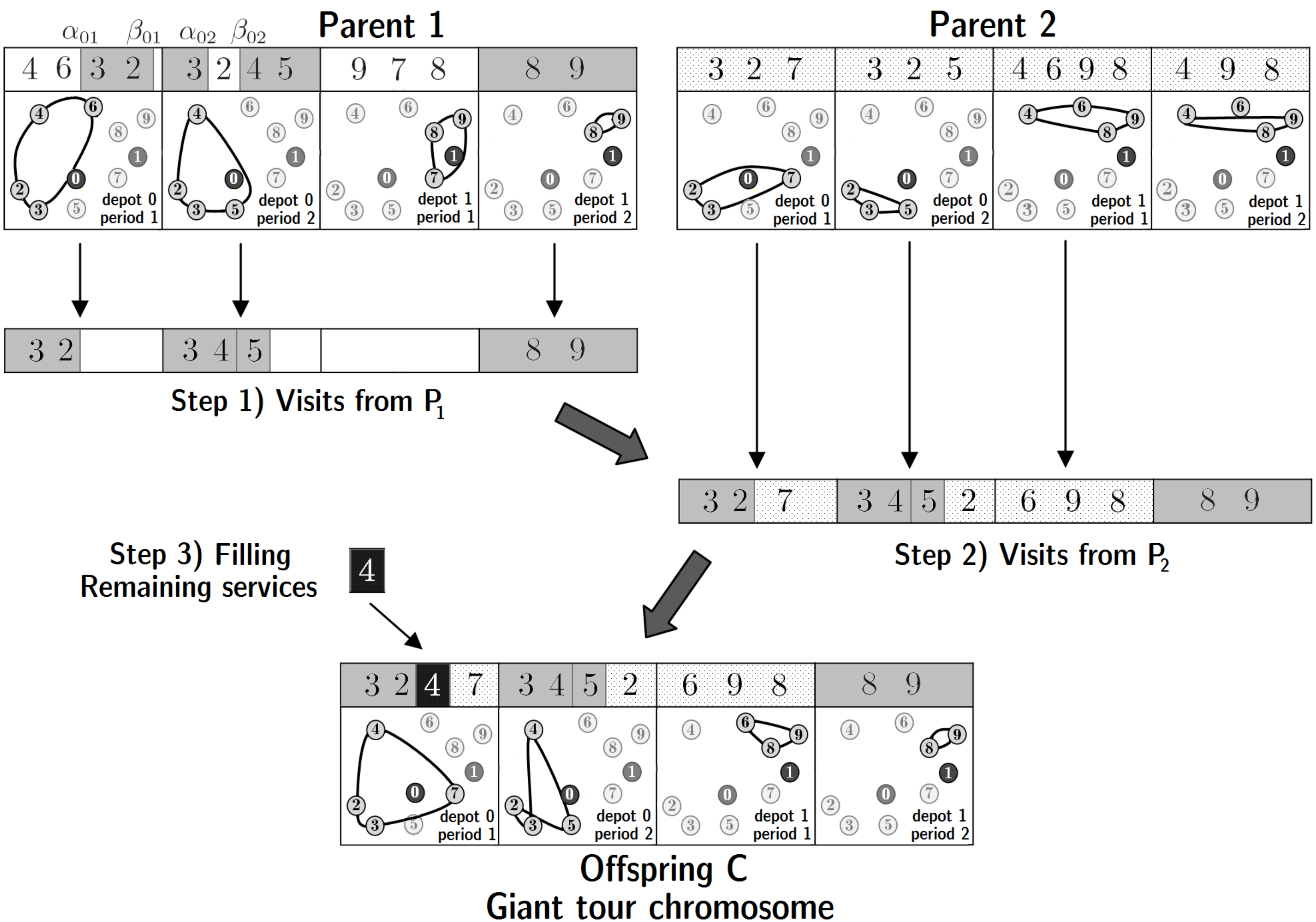
**for** each node $i \in \{0, \ldots, \nu\}$ **do**
 $SeqData(\sigma) = \textsc{Init}(\{v_0\})$ *//Initialize with depot vertex*
 **for** each node $j \in \{i, \ldots, \min(i + \bar{r}, \nu)\}$ **do**
  $\phi(a_{ij}) = \textsc{Eval2}(\sigma,\{v_0\})$ *//Evaluate the route*
  $SeqData(\sigma) = \textsc{Forw}(\sigma,\{\tau_j\})$ *//Append a new customer to the route end*
Solve the shortest path problem on $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc $a_{ij}$
Return the set of routes associated to the set of arcs of the shortest path

# Unified Crossover Operator

- 4 phases **Assignment and Insertion Crossover (AIX),** to produce a single offspring C from two parents P1 and P2.

- Step 1) Choose for each AAR whether the genetic material of P1, P2, or both parents is inherited.
- Step 2) Fully transmit the selected material from P1
- Step 3) Complete with the selected material from P2, check at each step with an Assignment module whether the inheritance respects the ASSIGN attributes specifications.
- Step 4) Perform a best insertion of missing visits.

# Unified Crossover Operator



# Unified Education Procedure

- Based on the previously described Unified Local Search to perform route improvement (RI) on separate AAR.
  - Using CROSS, I-CROSS, Relocate, 2-Opt* and 2-Opt neighborhoods
  - Pruning procedures (granular search)
  - Hybrid acceptance strategy (intermediate between first improvement and best improvement)

- Combined with an assignment-improvement (AI) procedure to re-assign customer visits into different resources and routes.

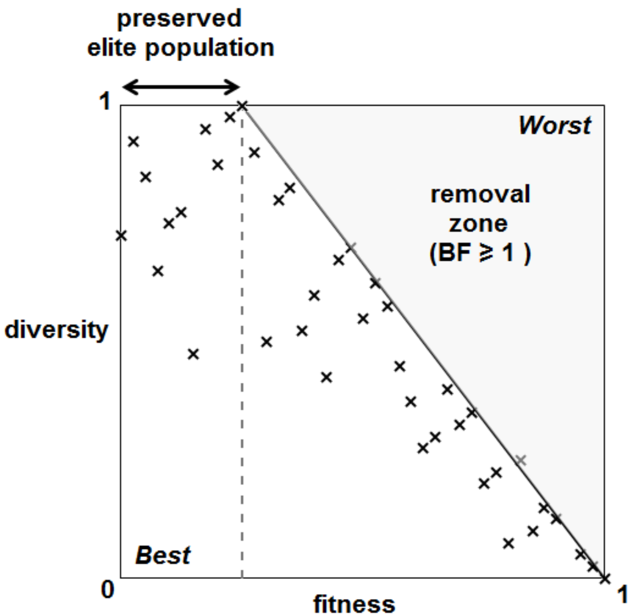
- These two procedures are called in the sequence RI-AI-RI.

# Population management and search guidance

- **Biased Fitness** is a tradeoff between ranks in terms of **solution penalized cost** *cost(I),* and **contribution to the diversity** *dc(I),* measured as a distance to others individuals in the population.

$$BF(I) = fit(I) + (1 - \frac{nbElit}{nbIndiv - 1}) \times dc(I)$$

- Used during selection of the parents
  - Balancing strength with innovation during reproduction, and thus favoring exploration of the search space.
- and during selection of the survivors:
  - Removing the individual *I* with worst *BF(I)* also guarantees some elitism in terms of solution value.



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# Comparison with problem-tailored state-of-the-art methods

- Extensive computational experiments on 26 structurally different VRP variants and 39 sets of benchmark instances.

- Comparing UHGS with the best problem-tailored method for each benchmark.

- In  the following, we indicate for each method
  - % Gap to the BKS of an average run (out of 10 for UHGS).
  - % Gap to the BKS of a best run (out of 10 for UHGS).
  - Computational effort (total work time) for an average run
  - Type of processor used.

# Comparison with problem-tailored state-of-the-art methods

| Variant | Bench. | $n$ | Obj. | State-of-the-art methods Author | Avg.% | Best% | T(min) | CPU |
|---|---|---|---|---|---|---|---|---|
| CVRP | CMT79 | [50,199] | C | GG11: | — | +0.03% | 8×2.38 | 8×Xe 2.3G |
| | | | | MB07: | +0.03% | — | 2.80 | P-IV 2.8G |
| | | | | **UHGS*:** | **+0.02%** | **+0.00%** | 11.90 | Opt 2.4G |
| CVRP | GWKC98 | [200,483] | C | GG11: | — | +0.29% | 8×5 | 8×Xe 2.3G |
| | | | | NB09: | +0.27% | +0.16% | 21.51 | Opt 2.4G |
| | | | | **UHGS*:** | **+0.15%** | **+0.02%** | 71.41 | Opt 2.4G |
| VRPB | GJ89 | [25,200] | C | ZK12: | +0.38% | +0.00% | 1.09 | T5500 1.67G |
| | | | | GA09: | +0.09% | +0.00% | 1.13 | Xe 2.4G |
| | | | | **UHGS:** | **+0.01%** | **+0.00%** | 0.99 | Opt 2.4G |
| CCVRP | CMT79 | [50,199] | C | NPW10: | +0.74% | +0.28% | 5.20 | Core2 2G |
| | | | | RL12: | +0.37% | +0.07% | 2.69 | Core2 2G |
| | | | | **UHGS:** | **+0.01%** | **-0.01%** | 1.42 | Opt 2.2G |
| CCVRP | GWKC98 | [200,483] | C | NPW10: | +2.03% | +1.38% | 94.13 | Core2 2G |
| | | | | RL12: | +0.34% | +0.07% | 21.11 | Core2 2G |
| | | | | **UHGS:** | **-0.14%** | **-0.23%** | 17.16 | Opt 2.2G |
| VRPSDP | SN99 | [50,199] | C | SDBOF10: | +0.16% | +0.00% | 256×0.37 | 256×Xe 2.67G |
| | | | | ZTK10: | — | +0.11% | — | T5500 1.66G |
| | | | | **UHGS:** | **+0.01%** | **+0.00%** | 2.79 | Opt 2.4G |
| VRPSDP | MG06 | [100,400] | C | SDBOF10: | +0.30% | +0.17% | 256×3.11 | 256×Xe 2.67G |
| | | | | UHGS: | +0.20% | +0.07% | 12.00 | Opt 2.4G |
| | | | | **S12 :** | **+0.08%** | **+0.00%** | 7.23 | I7 2.93G |

# Comparison with problem-tailored state-of-the-art methods

| Variant | Bench. | $n$ | Obj. | State-of-the-art methods | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Author | Avg.% | Best% | T(min) | CPU |
| VFMP-F | G84 | [20,100] | C | ISW09: | — | +0.07% | 8.34 | P-M 1.7G |
| | | | | SPUO12: | +0.12% | +0.01% | 0.15 | I7 2.93G |
| | | | | **UHGS:** | **+0.04%** | **+0.01%** | 1.13 | Opt 2.4G |
| VFMP-V | G84 | [20,100] | C | ISW09: | — | +0.02% | 8.85 | P-M 1.7G |
| | | | | SPUO12: | +0.17% | +0.00% | 0.06 | I7 2.93G |
| | | | | **UHGS:** | **+0.03%** | **+0.00%** | 0.85 | Opt 2.4G |
| VFMP-FV | G84 | [20,100] | C | P09: | — | +0.02% | 0.39 | P4M 1.8G |
| | | | | UHGS: | +0.01% | +0.00% | 0.99 | Opt 2.4G |
| | | | | **SPUO12:** | **+0.01%** | **+0.00%** | 0.13 | I7 2.93G |
| LDVRP | CMT79 | [50,199] | C | XZKX12: | +0.48% | +0.00% | 1.3 | NC 1.6G |
| | | | | **UHGS:** | **-0.28%** | **-0.33%** | 2.34 | Opt 2.2G |
| LDVRP | GWKC98 | [200,483] | C | XZKX12: | +0.66% | +0.00% | 3.3 | NC 1.6G |
| | | | | **UHGS:** | **-1.38%** | **-1.52%** | 23.81 | Opt 2.2G |
| PVRP | CGL97 | [50,417] | C | HDH09: | +1.69% | +0.28% | 3.09 | P-IV 3.2G |
| | | | | UHGS*: | +0.43% | +0.02% | 6.78 | Opt 2.4G |
| | | | | **CM12:** | **+0.24%** | **+0.06%** | 64×3.55 | 64×Xe 3G |
| MDVRP | CGL97 | [50,288] | C | CM12: | +0.09% | +0.03% | 64×3.28 | 64×Xe 3G |
| | | | | S12: | +0.07% | +0.02% | 11.81 | I7 2.93G |
| | | | | **UHGS*:** | **+0.08%** | **+0.00%** | 5.17 | Opt 2.4G |
| GVRP | B11 | [16,262] | C | BER11: | +0.06% | — | 0.01 | Opt 2.4G |
| | | | | MCR12: | +0.11% | — | 0.34 | Duo 1.83G |
| | | | | **UHGS:** | **+0.00%** | **-0.01%** | 1.53 | Opt 2.4G |

# Comparison with problem-tailored state-of-the-art methods

| Variant | Bench. | $n$ | Obj. | State-of-the-art methods | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Author | Avg.% | Best% | T(min) | CPU |
| OVRP | CMT79 &F94 | [50,199] | F/C | RTBI10: | 0%/+0.32% | — | 9.54 | P-IV 2.8G |
| | | | | S12: | —/+0.16% | 0%/+0.00% | 2.39 | I7 2.93G |
| | | | | **UHGS:** | **0%/+0.11%** | **0%/+0.00%** | 1.97 | Opt 2.4G |
| OVRP | GWKC98 | [200,480] | F/C | ZK10: | 0%/+0.39% | 0%/+0.21% | 14.79 | T5500 1.66G |
| | | | | S12: | 0%/+0.13% | 0%/+0.00% | 64.07 | I7 2.93G |
| | | | | **UHGS:** | **0%/-0.11%** | **0%/-0.19%** | 16.82 | Opt 2.4G |
| VRPTW | SD88 | 100 | F/C | RTI09: | 0%/+0.11% | 0%/+0.04% | 17.9 | Opt 2.3G |
| | | | | UHGS*: | 0%/+0.04% | 0%/+0.01% | 2.68 | Xe 2.93G |
| | | | | **NBD10:** | **0%/+0.02%** | **0%/+0.00%** | 5.0 | Opt 2.4G |
| VRPTW | HG99 | [200,1000] | F/C | RTI09b: | — | +0.16%/+3.36% | 270 | Opt 2.3G |
| | | | | NBD10: | +0.20%/+0.42% | +0.10%/+0.27% | 21,7 | Opt 2.4G |
| | | | | **UHGS*:** | **+0.18%/+0.11%** | **+0.08%/-0.10%** | 141 | Xe 2.93G |
| OVRPTW | SD88 | 100 | F/C | RTI09a: | +0.89%/+0.42% | 0%/+0.24% | 10.0 | P-IV 3.0G |
| | | | | KTDHS12: | 0%/+0.79% | 0%/+0.18% | 10.0 | Xe 2.67G |
| | | | | **UHGS:** | **+0.09%/-0.10%** | **0%/-0.10%** | 5.27 | Opt 2.2G |
| TDVRPTW | SD88 | 100 | F/C | KTDHS12: | +2.25% | 0% | 10.0 | Xe 2.67G |
| | | | | **UHGS:** | **-3.31%** | **-3.68%** | 21.94 | Opt 2.2G |
| VFMPTW | LS99 | 100 | D | BDHMG08: | — | +0.59% | 10.15 | Ath 2.6G |
| | | | | RT10: | +0.22% | — | 16.67 | P-IV 3.4G |
| | | | | **UHGS:** | **-0.15%** | **-0.24%** | 4.58 | Opt 2.2G |
| VFMPTW | LS99 | 100 | C | BDHMG08: | — | +0.25% | 3.55 | Ath 2.6G |
| | | | | BPDRT09: | — | +0.17% | 0.06 | Duo 2.4G |
| | | | | **UHGS:** | -0.38% | -0.49% | 4.82 | Opt 2.2G |

# Comparison with problem-tailored state-of-the-art methods

| Variant | Bench. | $n$ | Obj. | State-of-the-art methods Author | Avg.% | Best% | T(min) | CPU |
|---|---|---|---|---|---|---|---|---|
| PVRPTW | CL01 | [48,288] | C | PR08: | — | +1.75% | — | Opt 2.2G |
| | | | | CM12: | +1.10% | +0.76% | 64×11.3 | 64×Xe 3G |
| | | | | **UHGS*:** | **+0.63%** | **+0.22%** | 32.7 | Xe 2.93G |
| MDVRPTW | CL01 | [48,288] | C | PBDH08: | — | +1.37% | 147 | P-IV 3.6G |
| | | | | CM12: | +0.36% | +0.15% | 64×6.57 | 64×Xe 3G |
| | | | | **UHGS*:** | **+0.19%** | **+0.03%** | 6.49 | Xe 2.93G |
| SDVRPTW | CL01 | [48,288] | C | B10: | +2.23% | — | 2.94 | Qd 2.67G |
| | | | | CM12: | +0.62% | +0.36% | 64×5.60 | 64×Xe 3G |
| | | | | **UHGS*:** | **+0.36%** | **+0.10%** | 5.48 | Xe 2.93G |
| VRPSTW (type 1, $\alpha$=100) | SD88 | 100 | F/TW/C | F10: | 0% | — | 9.69 | P-M 1.6G |
| | | | | **UHGS:** | **-3.05%** | **-4.42%** | 18.62 | Opt 2.2G |
| VRPSTW (type 1, $\alpha$=1) | SD88 | 100 | C+TW | KTDHS12: | +0.62% | +0.00% | 10.0 | Xe 2.67G |
| | | | | **UHGS:** | **-0.13%** | **-0.18%** | 5.82 | Opt 2.2G |
| VRPSTW (type 2, $\alpha$=100) | SD88 | 100 | F/TW/C | FEL07: | 0% | — | 5.98 | P-II 600M |
| | | | | **UHGS:** | **-13.91%** | **-13.91%** | 41.16 | Opt 2.2G |
| VRPSTW (type 2, $\alpha$=1) | SD88 | 100 | C+TW | **UHGS:** | **+0.26%** | **0%** | 29.96 | Opt 2.2G |
| MDPVRPTW | New | [48,288] | C | **UHGS:** | **+0.77%** | **0%** | 16.89 | Opt 2.2G |
| VRTDSP (E.U. rules) | G09 | 100 | F/C | PDDR10: | 0%/0% | 0%/0% | 88 | Opt 2.3G |
| | | | | **UHGS*:** | **-0.56%/-0.54%** | **-0.85%/-0.70%** | 228 | Xe 2.93G |

# Comparison with problem-tailored state-of-the-art methods

**List of acronyms for benchmarks**

| | | | | | |
|---|---|---|---|---|---|
| B11 | Bektas et al. (2011) | G84 | Golden (1984) | LS99 | Liu and Shen (1999) |
| CGL97 | Cordeau et al. (1997) | G09 | Goel (2009) | MG06 | Montané and Galvão (2006) |
| CL01 | Cordeau and Laporte (2001) | GH99 | Gehring and Homberger (1999) | SD88 | Solomon and Desrosiers (1988) |
| CMT79 | Christofides et al. (1979) | GJ89 | Goetschalckx and J.-B. (1989) | SN99 | Salhi and Nagy (1999) |
| F94 | Fisher (1994) | GWKC98 | Golden et al. (1998) | | |

**List of acronyms for state-of-the-art algorithms**

| | | | | | |
|---|---|---|---|---|---|
| B10 | Belhaiza (2010) | KTDHS12 | Kritzinger et al. (2012) | RT10 | Repoussis and Tarantilis (2010) |
| BDHMG08 | Bräysy et al. (2008a) | MB07 | Mester and Bräysy (2007) | RTBI10 | Repoussis et al. (2010) |
| BER11 | Bektas et al. (2011) | MCR12 | Moccia et al. (2012) | RTI09a | Repoussis et al. (2009a) |
| BLR11 | Balseiro et al. (2011) | NB09 | Nagata and Bräysy (2009) | RTI09b | Repoussis et al. (2009b) |
| BPDRT09 | Bräysy et al. (2009) | NBD10 | Nagata et al. (2010) | S12 | Subramanian (2012) |
| CM12 | Cordeau and M. (2012) | NPW10 | Ngueveu et al. (2010) | SDBOF10 | Subramanian et al. (2010) |
| F10 | Figliozzi (2010) | P09 | Prins (2009) | SPUO12 | Subramanian et al. (2012) |
| FEL07 | Fu et al. (2007) | PBDH08 | Polacek et al. (2008) | XZKX12 | Xiao et al. (2012) |
| GA09 | Gajpal and Abad (2009) | PDDR10 | Prescott-Gagnon et al. (2010) | ZTK10 | Zachariadis et al. (2010) |
| GG11 | Groër and Golden (2011) | PR07 | Pisinger and Ropke (2007) | ZK10 | Zachariadis and Kiranoudis (2010) |
| HDH09 | Hemmelmayr et al. (2009) | PR08 | Pirkwieser and Raidl (2008) | ZK11 | Zachariadis and Kiranoudis (2011) |
| ISW09 | Imran et al. (2009) | RL12 | Ribeiro and Laporte (2012) | ZK12 | Zachariadis and Kiranoudis (2012) |

# Conclusions and Research Perspectives

- A unified hybrid genetic search
  - Using a local-search framework which is generic and computationally efficient.
  - With generalized solution representation, Split procedure, genetic operators (Crossover) and population management methods.
  - State-of-the-art results when compared to each problem-tailored method for 26 VRP variants.
- It appears that **generality does not necessarily impede performance for a wide class of VRP variants.**

# Conclusions and Research Perspectives

- Perspectives :
  - Extend the range of problems (especially SEQ attributes, stochastic and multi-objective settings)
  - Use UHGS to conduct experiments on metaheuristic strategies on a wide range of VRPs
  - Further study of the combinatorial aspect of attributes relatively to UHGS operators.

# Thanks for your attention

## THANK YOU

- For more details on this work:
  - Vidal, T., Crainic, T. G., Gendreau, M., Lahrichi, N., & Rei, W. (2012). A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research (To appear)*.
  - Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2012). *Heuristics for Multi-Attribute Vehicle Routing Problems : A Survey and Synthesis*. Tech Rep. CIRRELT 2012-05.
  - The CIRRELT technical report on the unified algorithm will appear very soon.

# Empirical studies on diversity management methods (1/2)

- Sensitivity analysis on diversity management methods:
  - **HGA** : No diversity management method
  - **HGA-DR** : Dispersal rule on objective space
  - **HGA-PM** : Dispersal rule on solution space
  - **HGSADC** : The proposed approach

| Benchmark | | **HGA** | **HGA-DR** | **HGA-PM** | **HGSADC** |
|---|---|---|---|---|---|
| PVRP | T | 6.86 min | 7.01 min | 7.66 min | 8.17 min |
| | % | +0.64% | +0.49% | +0.39% | +0.13% |
| MDVRP | T | 7.93 min | 7.58 min | 9.03 min | 8.56 min |
| | % | +1.04% | +0.87% | +0.25% | -0.04% |
| MDPVRP | T | 25.32 min | 26.68 min | 28.33 min | 40.15 min |
| | % | +4.80% | +4.07% | +3.60% | +0.44% |

# Empirical studies on diversity management methods (2/2)

- Behavior of HGSADC during a random run:
  - Higher entropy (average distance between two individuals)
  - Better final solution
  - Diversity can increase during run time