

Node, Edge, Arc Routing and Turn Penalties: Multiple problems – One Neighborhood Extension

Thibaut VIDAL

Department of Computer Science, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro 22451-900, Brazil,
vidalt@inf.puc-rio.br

Author Accepted Manuscript, Operations Research

DOI : 10.1287/opre.2017.1595

This article explores a structural neighborhood decomposition for arc routing problems, in which the decisions about traversal orientations during services are made optimally as part of neighbor evaluation procedures. Using memory structures, bidirectional dynamic programming, and lower bounds, we show that a large neighborhood involving classical moves on the sequences of services along with optimal orientation decisions can be searched in amortized $\mathcal{O}(1)$ time per move evaluation instead of $\mathcal{O}(n)$ as in previous works. Because of its generality and now-reduced complexity, this approach can be efficiently applied to several variants of arc routing problems, extended into large neighborhoods such as ejection chains, and integrated into two classical metaheuristics. Our computational experiments lead to solutions of high quality on the main benchmark sets for the capacitated arc routing problem (CARP), the mixed capacitated general routing problem (MCGRP), the periodic CARP, the multi-depot CARP, and the min-max k-vehicles windy rural postman problem, with a total of 1528 instances. We also report sensitivity analyses for new MCGRP instances with turn penalties, which are uncommonly challenging yet critical for many practical applications.

Key words: Arc routing, General routing, Turn penalties, Service clusters, Heuristics, Structural problem decomposition, Local search, Large neighborhoods.

History: Submitted April 2015, revisions received April 2016, October 2016; accepted December 2016.

1. Introduction

Arc routing problems arise in a wide variety of applications, including snow plowing, refuse collection, maintenance, and postal services. Practical issues introduce a variety of supplementary constraints, combined objectives, and decision sets, leading to a large family of problem variants (Corberán and Laporte 2014). Since exact solution methods are often impractical for industrial needs, many researchers have investigated heuristics. The literature on arc routing metaheuristics has thus seen a substantial increase in the last fifteen years, from a review of just four pages in Dror (2000) to more than one hundred articles now.

Arc routing is notably different from node routing, due to the necessity of considering precise network information and deciding on traversal orientations for the services on edges. In node routing algorithms, decisions related to the shortest paths between deliveries are usually concealed within the computation of the distance matrix. In arc routing applications, these path decisions are nontrivial, because they may be conditioned by the choices of service orientations, and they can also depend on other attributes such as delays at intersections and turn restrictions, which are essential features of urban applications. Recent capacitated arc routing (CARP) heuristics (e.g., Lacomme et al. 2004, Brandão and Eglese 2008, Usberti et al. 2013) thus deal with service orientations via additional neighborhoods. However, since the changes of orientation are considered only in specific move subsets, the resulting search spaces may be deceptive.

These distinct decision classes contributed to split the research effort on heuristics, between arc and node routing algorithms and concepts. This contrasts with the goal of several recent contributions on routing optimization, relating to unified metaheuristic strategies rather than problem-tailored developments (Irñich 2008a, Vidal et al. 2014a). Due to their purpose, these approaches could be extended to several variants of arc routing problems, but their success depends on our ability to deal efficiently and systematically with arc-routing-specific decision sets.

In this article, we consider a broad class of arc routing variants, with services on arcs, nodes, and edges as well as possible turn penalties, and formulate them as four main decision sets: ASSIGNMENT of services to routes, SEQUENCING, MODE CHOICES, and PATHS between services. The mode choices represent different ways of fulfilling a service, e.g., on either direction of a street or on a different lane. These choices arise naturally in the CARP, where two service orientations are possible for each edge, as well as in various advanced applications for snow plowing, street sweeping, salt spreading, meter reading, refuse collection, and courier delivery, among others (Corberán and Laporte 2014). Instead of designing separate neighborhoods for each decision class, we perform a heuristic search on a subset of the decisions while deducing exactly the others. A solution is formulated as sequences of services, without knowledge of the mode decisions. Alternative sequence and assignment decisions are heuristically enumerated by means of classical vehicle routing moves, while the optimal mode choices are derived via dynamic programming, using the approach of Beullens et al. (2003).

Such extended neighborhoods, known for a decade in the arc routing literature, have been scarcely used because of their higher computational complexity. In this article, we break this complexity barrier, demonstrating that each move can be evaluated in amortized $\mathcal{O}(1)$ by means of efficient memory structures and bidirectional search. Moreover, by means of lower bounds on moves, we show that each evaluation requires roughly the same average number of elementary operations as those needed to evaluate distances and capacity constraints in a capacitated vehicle routing problem (CVRP). In other words, the mode decisions can be optimally addressed with little or no additional computational effort. The dynamic programming subproblem can also be used within polynomial ejection chains to search, in $\mathcal{O}(n^2)$, a exponential set of solutions obtained from chained service relocations and mode changes. Finally, we show that the approach is systematic: difficult decision sets for arc routing variants, such as turn penalties, can be fully concealed in the dynamic programming subproblems.

To investigate the improvement capabilities of these neighborhoods, we integrated them into two classical vehicle routing metaheuristics: the iterated local search (ILS) of Prins (2009) and the unified hybrid genetic search (UHGS) of Vidal et al. (2012, 2014a). We conducted extensive experiments on several arc routing variants: the CARP and the mixed capacitated general routing problem (MCGRP) – also called the node, edge, and arc routing problem (NEARP) – as well as problem extensions with turn penalties (CARP-TP and MCGRP-TP), multiple service periods (PCARP), multiple depots (MDCARP), and finally the min-max k-vehicles windy rural postman problem (MM-kWRPP). We considered a total of 1528 instances from 18 benchmark sets. With a single parameter setting, the proposed heuristics achieved results of very high quality for all the instance classes, matching or outperforming previous algorithms in the literature (some of which are fine-tuned for specific instances) while remaining conceptually simple and flexible. The key contributions of this article are thus the following:

- A full-fledged local search for the CARP, which evaluates moves on service sequences and jointly optimizes the service mode decisions in amortized $\mathcal{O}(1)$ time per move evaluation;
- Lower bounds on move evaluations, which further reduce the computational effort;
- An extension of this methodology within ejection chains, exploring an exponential number of combinations of sequence and mode changes;
- An application to various arc routing problems via a broader interpretation of *service modes*;
- An integration of this neighborhood search into two state-of-the-art routing metaheuristics, leading to high-quality solutions for six important arc routing problem variants.

2. Problem statement

The CARP can be defined on a connected graph $G = (V, E)$, where V is the set of nodes and E the set of edges. A subset $E_R \subseteq E$ of these edges must be serviced. Let $n = |E_R|$ be the number of services. An edge $(i, j) \in E$ can be traversed any number of times for a cost of c_{ij} each time, and a demand of q_{ij} is associated with any edge $(i, j) \in E_R$. The CARP aims to find a set of vehicle trips

with minimum cost, such that each trip starts and ends at a depot node $v_0 \in V$, each required edge is serviced by a single trip, and the total demand for any vehicle does not exceed a capacity Q .

The literature on CARP heuristics has grown rapidly in recent years. Most classical metaheuristic frameworks have been tested multiple times. Various tabu searches have been proposed (Hertz et al. 2000, Brandão and Eglese 2008), as well as a variable neighborhood descent (Hertz and Mittaz 2001), a guided local search (Beullens et al. 2003), and a GRASP (Usberti et al. 2013). Neighborhood-centered methods are often hybridized with population-based methods such as scatter search (Greistorfer 2003), path relinking (Usberti et al. 2013), genetic algorithms (Lacomme et al. 2001, 2004, Wang et al. 2015), ant colony optimization (Santos et al. 2010), and evolution strategies (Mei et al. 2013, 2014). Almost all these methods combine an aggressive optimization via local searches with diversification techniques such as crossovers, memories for guidance, or restarts from promising initial solutions.

Several variants of arc routing problems have been formulated to meet the needs of practical applications (see, e.g., Corberán and Laporte 2014, chapters 13–16). The MCGRP, in particular, can be defined on a connected multi-graph $G = (V, E, A)$, where V is the set of nodes, E the set of edges, and A the set of arcs. Services are required for a subset of nodes $V_R \subseteq V$, edges $E_R \subseteq E$, and arcs $A_R \subseteq A$ such that $n = |V_R| + |E_R| + |A_R|$. Early research on this problem concerned constructive heuristics. Later, several metaheuristics have been introduced: a giant-tour-based hybrid GA (Prins and Bouchenoua 2005), a simulated annealing (Kokubugata et al. 2007), the *Spider* solver based on ILS and VNS (Hasle et al. 2012), a large neighborhood search and integer programming hybrid (Bosco et al. 2014), and an adaptive ILS (Dell’Amico et al. 2016). This last method generates high-quality results for a wide range of instances. All these metaheuristics use adaptations of classical neighborhoods for the CARP with eventual inversions of service orientations.

Another recurrent challenge of urban networks comes from possible turn restrictions and delays at intersections. These aspects are indispensable for commercial routing solvers, but more scarcely discussed in the academic literature. In arc routing problems with turn penalties, turn costs c_{ijk} are considered for each pair of connected edges and/or arcs (i, j) and (j, k) . Early research on this topic was primarily focused on ad-hoc constructive procedures and real applications. Later, heuristic and exact methods were proposed for single-vehicle settings (Benavent and Soler 1999, Corberán et al. 2002), based on problem reductions to generalized or asymmetric traveling salesman problems. Dussault (2012) and Dussault et al. (2014) consider the optimization of snow-plowing operations, leading to problem variants with multiple vehicles, a min-max objective, precedence constraints for deadheading, and turn penalties. Solution approaches based on integer programming, tour splitting, and cycle permutations were proposed. A few other articles discussed extensions of CARP with turn penalties (Lacomme et al. 2004, Bautista and Pereira 2004). The authors suggest specifying the service orientations in the solution representation, using a shortest path algorithm with turn penalties in the line graph. This approach works for extensions of the CARP but not in the presence of additional services on nodes as in the MCGRP–TP. For this last problem, we are aware of a single heuristic (Bräysy et al. 2011) based on a problem transformation into a generalized CVRP.

To extend the scope of the approach, we finally consider asymmetric travel costs and differentiate the travel (deadheading) cost c_{ij} and the service cost s_{ij} for any arc (i, j) . This allows us to solve *asymmetric* and *windy* arc routing problems (Benavent et al. 2009). Experimental analyses will also be reported for problems with multiple depots and visit periods. For the sake of brevity, these experiments are documented in the electronic companion of this paper.

3. A Question of Search Space

We consider a combinatorial optimization problem of the form $\min_{x \in X} c(x)$, where X is the solution space, and c is a cost function. A neighborhood is defined as a mapping $\mathcal{N}: X \rightarrow 2^X$ that associates to each solution x a set of neighbors $\mathcal{N}(x) \subset X$. In the routing literature, neighborhoods are not directly specified as subsets $\mathcal{N}(x)$, but rather as the byproduct of a move definition. A move ϕ is a local modification that can be applied to a given solution x to generate a neighbor $\phi(x) \in \mathcal{N}(x)$.

Examples of classical moves include the RELOCATE move, which changes the position of a service in the solution, and the SWAP move, which exchanges two services. A local search (LS) progresses iteratively from a solution to an improving neighbor until a local optimum is reached.

In the above definitions, the choice of solution space X appears to be transparent. This is, however, generally not the case, especially when a solution can be characterized by different decision sets. In production scheduling problems, for example, the solutions are often represented as sequences of activities for each machine, without precise information about their starting dates. The missing information is derived during solution evaluations, a task which is straightforward when the objective is *regular* (Giffler and Thompson 1960) but leads to a *timing* optimization problem in other cases (Vidal et al. 2015). For the CARP, a complete solution includes four decision subsets:

- ASSIGNMENT of services to routes,
- SEQUENCING of services for each route,
- MODE CHOICE for each service,
- PATHS between successive services,

and each of these four decision subsets leads to an exponential number of solutions. Now, when some of these decision sets are known (e.g., ASSIGNMENT and SEQUENCING), then the optimal choices for the other sets can be derived via dynamic programming. An incomplete solution representation can thus serve as a structural problem decomposition, dividing the search into the heuristic optimization of some decision sets and a simultaneous exact resolution of the others. By reducing the scope of the heuristic search, we aim to reduce its inherent error and to relegate a larger set of combinatorial decisions in the dynamic programming procedure. The main solution representations and neighborhoods used for the CARP are presented in Table 1.

R1	Explicit representation of ASSIGNMENT, SEQUENCING, MODE, and PATH decision subsets. In the figure, shortest paths between successive services (deadheading arcs) are represented with dashes.	
R2A	Each solution is represented as a set of routes. Each route is represented as a sequence of services with their modes (service orientations in the CARP). Thus, the ASSIGNMENT, SEQUENCING, and MODE decision subsets are explicitly represented.	
R2B	Each solution is represented as a giant tour, specifying a sequence of services with their modes. As such, only the SEQUENCING and MODE decision subsets are explicitly given.	In the representation R2A, each route is represented as illustrated above
R3A	Each solution is represented as a set of routes. Each route is represented as a sequence of services. As such, only the ASSIGNMENT and SEQUENCING decision subsets are explicit. The same representation is classically used for the CVRP.	
R3B	Each solution is represented as a giant tour containing a sequence of services. Hence, only the SEQUENCING decision subset is explicit. The same representation is classically used for the TSP.	In the representation R3A, each route is represented as illustrated above

Table 1 Alternative solution representations for the CARP

First, in the complete solution representation (R1 in Table 1), all edges of the solution are specified, for both the services and the paths between services, thus giving explicitly the ASSIGNMENT,

SEQUENCING, MODE, and PATH decision subsets. Such a representation contains up to $\mathcal{O}(n|V|)$ edges. This representation has been used in some seminal algorithms (e.g., Hertz et al. 2000), but it is now scarcely used in recent metaheuristics, since a preliminary computation of all-pairs distances allows to avoid the heuristic optimization of deadhauling paths. In mathematical programming methods, still, the sparsity of the underlying street network can be advantageously used (Letchford and Oukil 2009, Bode and Irnich 2012).

Representation R2A specifies the sequence of services to edges with their modes. Shortest-paths between edge endpoints have been processed prior to routing resolution. It requires $\mathcal{O}(n)$ space per solution and $\mathcal{O}(|V|^2)$ space for the shortest-path information. This representation, which allows move evaluations in $\mathcal{O}(1)$ time, is used in all recent CARP metaheuristics during the local searches. However, as the SEQUENCING and MODE decisions are tightly correlated, some moves with joint modifications of these two decision sets are needed to progress toward high-quality solutions.

Representation R2B, without trip delimiters, has been frequently used during crossover operations in population metaheuristics (Lacomme et al. 2001, 2004). The $\mathcal{O}(n^2)$ computational effort of the Split algorithm, used to decode the solutions, is affordable when used once before the LS, but not when systematically used during all move evaluations. For this reason, the above metaheuristics revert to R2A during the LS phases.

Solution representations without explicit MODE CHOICES (R3) have been rarely used until now. Beullens et al. (2003) perform mode optimization as a stand-alone procedure, and Irnich (2008b) combines mode optimization with the large neighborhood of Balas and Simonetti (2001), leading to promising results for mail delivery applications. Muyldermans et al. (2005) evaluate 2-OPT moves on R3A in $\mathcal{O}(n)$ time per move. SWAP moves have been used on R3B by Wøhlk (2003, 2004), in a simulated annealing algorithm. However, the dynamic programming decoder, referred to as *Split with flips* in Prins et al. (2009), requires $\mathcal{O}(n^2)$ operations per move evaluation. Ramdane-Cherif (2002) also suggest using R3B during crossover operations and reverting to R2A during LS. Overall, R3A and R3B are desirable since they conceal the MODE CHOICES in the dynamic programming subproblems, and thus guarantee the optimality of these decisions. However, the price to pay was, until now, a higher computational complexity for the move evaluations.

In the next section, we explain how to efficiently evaluate the solutions R3A in $\mathcal{O}(1)$ instead of $\mathcal{O}(n)$ in the context of a local search. As such, the heuristic search can be fully focused on the ASSIGNMENT and SEQUENCING decision subsets, as in CVRP metaheuristics, while the MODE CHOICE and PATH decisions are implicitly and optimally determined with little or no additional computational effort.

4. Neighborhood Search with Optimal Mode Decisions

In the following, a solution $x \in X$ will be represented as a set of routes $\mathcal{R}(x)$. Each route $\sigma \in \mathcal{R}(x)$ corresponds to a sequence of services $\sigma = (\sigma(1), \dots, \sigma(|\sigma|))$, starting and ending at the depot (counted as a dummy service, such that $\sigma(1) = 0$ and $\sigma(|\sigma|) = 0$). Any service i can be performed using one of a subset of modes M_i , with a service cost s_i^k for mode $k \in M_i$. We assume that the number of possible modes per service is bounded by a constant. Two modes are used for the CARP, one for each admissible service orientation. By convention, $|M_0| = 1$ and $s_0^1 = 0$ for the depot. The cost of traveling between services i and j in modes $k \in M_i$ and $l \in M_j$ is given by c_{ij}^{kl} . For the CARP, this corresponds to the distance of the shortest path between the exit node of edge i in mode k , and the entry node of edge j in mode l .

4.1. Optimization of mode choices

For any given sequence of services σ , an optimal selection of modes can be efficiently determined by dynamic programming as in Beullens et al. (2003). This procedure corresponds to the search for a shortest path in an acyclic auxiliary graph $H_\sigma = (V_\sigma, A_\sigma)$ containing $\sum_{i=1}^{|\sigma|} |M_{\sigma(i)}|$ nodes, one for each mode of each service. For each consecutive pair of services $\sigma(i)$ and $\sigma(i+1)$, between each node pair associated with modes $k \in M_{\sigma(i)}$ and $l \in M_{\sigma(i+1)}$, an arc is added in A_σ with cost $c_{\sigma(i)\sigma(i+1)}^{kl} + s_{\sigma(i)}^k$.

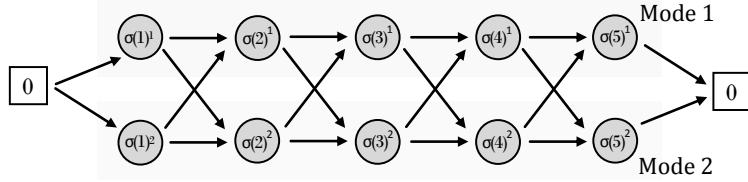


Figure 1 Shortest path subproblem for the evaluation of a route σ with optimal mode choices.

Figure 1 illustrates the graph H_σ for the CARP. The shortest path problem can be solved in $\mathcal{O}(n)$, using the Bellman algorithm in the presence of a topological order. However, even linear-time move evaluations would be a significant computational bottleneck, such that further improvements are needed to efficiently employ this structure.

4.2. Move evaluations by concatenation

To reduce this computational complexity, we propose incremental move evaluations in which the shortest-path subproblems are not solved independently. Instead, all-pairs shortest paths are preprocessed in the incumbent solution and then used for move evaluations. Thus, for each route σ in the incumbent solution, and each subsequence of consecutive services $\bar{\sigma} \subset \sigma$, the method keeps track of the cost of the partial shortest path $C(\bar{\sigma})[k, l]$ in $H_{\bar{\sigma}}$ between the first and last service in the sequence, for any combination of modes k and l , as well as the sum of the service demands $Q(\bar{\sigma})$.

The values $C(\bar{\sigma})[k, l]$ are computed by induction on the operation of concatenation (\oplus) of two sequences. For a sequence $\bar{\sigma} = (v_i)$ containing a single service, $C(\bar{\sigma})[k, l] = s_i^k$ for $k = l$, else $C(\bar{\sigma})[k, l] = +\infty$. Moreover, the following equations allow us to derive C and Q for any sequence $\sigma_1 \oplus \sigma_2$ resulting from a concatenation of two sequences σ_1 and σ_2 , the first service of σ_2 being done immediately after the last service of σ_1 :

$$C(\sigma_1 \oplus \sigma_2)[k, l] = \min_{x \in M_{\sigma_1(|\sigma_1|)}} \left\{ \min_{y \in M_{\sigma_2(1)}} \left\{ C(\sigma_1)[k, x] + c_{\sigma_1(|\sigma_1|)\sigma_2(1)}^{xy} + C(\sigma_2)[y, l] \right\} \right\} \quad (1)$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \quad (2)$$

First, Equations (1–2) are used to preprocess the shortest paths in the incumbent solution in $\mathcal{O}(n^2)$: the services are enumerated according to their visit order, and the subsequences are evaluated. Then, the same equations are used for move evaluations. Any route σ obtained from a classical LS move corresponds to a concatenation of K service sequences from the incumbent solution: $\sigma = \sigma_1 \oplus \dots \oplus \sigma_K$. Evaluating the cost of σ involves evaluating the cost of a shortest path in H_σ , which is obtained via $K - 1$ applications of Equation (1). As illustrated in Figure 2, this is equivalent to contracting known shortest paths in each sequence σ_i of H_σ , and then evaluating the shortest path on a reduced graph using the Floyd–Warshall algorithm.

PROPOSITION 1. *Using the proposed approach, the complexity of move evaluations for classical neighborhoods such as RELOCATE, SWAP, 2-OPT, and 2-OPT* is in $\mathcal{O}(1)$.*

Indeed, any classical move on sequences can be assimilated to a recombination of a bounded number of sequences of services (Vidal et al. 2014a). Thus, each move evaluation is performed with $\mathcal{O}(1)$ calls to Equations (1–2), requiring $\mathcal{O}(1)$ elementary arithmetic operations since the number of alternative modes is bounded.

4.3. Lower bounds on move evaluations

Even with an $\mathcal{O}(1)$ complexity, the use of the \min operator and the enumeration of modes leads to a constant but larger computational effort for move evaluations ($\approx 4\times$ for the CARP when compared to the CVRP). To further reduce the CPU time, we propose a two-step evaluation procedure using

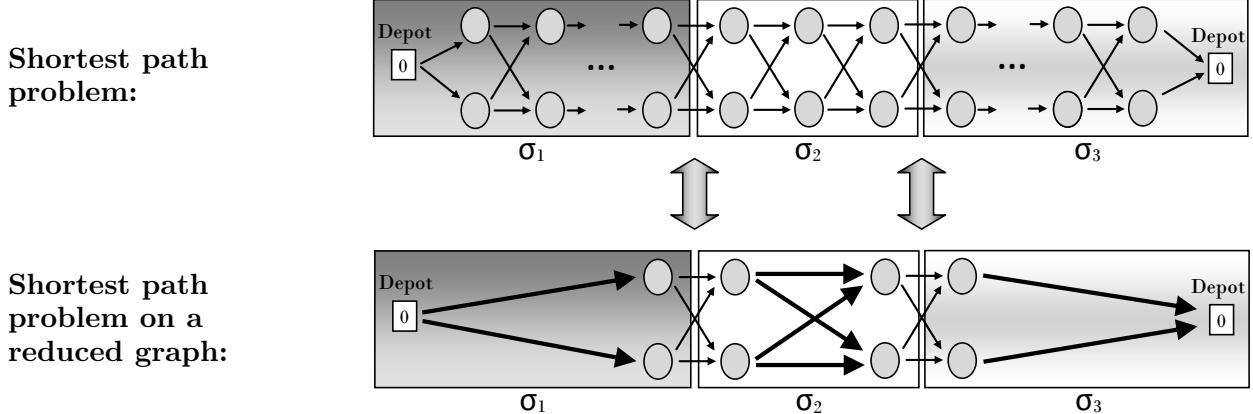


Figure 2 Using preprocessed information on sequences of services to reduce the auxiliary graph H_σ

lower bounds on moves. Consider two routes σ_1 and σ_2 , subject to a move Π that produces two new routes σ'_1 and σ'_2 . The move Π is improving if and only if $\Delta_\Pi = C(\sigma'_1) + C(\sigma'_2) - C(\sigma_1) - C(\sigma_2) < 0$. Let $C_{\text{LB}}(\sigma')$ be a lower bound on the cost of a route σ' ; then any improving move fulfills the following condition, which can be used as a filter during a preliminary evaluation:

$$\Delta_{\Pi}^{\text{LB}} = C_{\text{LB}}(\sigma'_1) + C_{\text{LB}}(\sigma'_2) - C(\sigma_1) - C(\sigma_2) < 0. \quad (3)$$

Let $C_{\text{MIN}}(\sigma) = \min_{k \in M_{\sigma(1)}} \{ \min_{l \in M_{\sigma(|\sigma|)}} \{ C(\sigma)[k, l] \} \}$ be the minimum distance of a shortest path in H_σ between any pair of modes for the first and last service of σ . Let $c_{ij}^{\text{MIN}} = \min_{k \in M_i} \{ \min_{l \in M_j} \{ c_{ij}^{kl} \} \}$ be the minimum cost of a shortest path in G between services i and j , for any mode pair $k \in M_i$ and $l \in M_j$. The following equation provides a lower bound on the cost of a route $\sigma = \sigma_1 \oplus \dots \oplus \sigma_K$ composed of a concatenation of K sequences:

$$C_{\text{LB}}(\sigma_1 \oplus \dots \oplus \sigma_K) = \sum_{j=1}^K C_{\text{MIN}}(\sigma_j) + \sum_{j=1}^{K-1} c_{\sigma_j(|\sigma_j|), \sigma_{j+1}(1)}^{\text{MIN}}. \quad (4)$$

The values $C_{\text{MIN}}(\sigma)$ can be preprocessed simultaneously with $C(\sigma)[k, l]$, and the shortest paths c_{ij}^{MIN} are directly evaluated from the distances c_{ij}^{kl} , prior to the routing optimization. Then, the evaluation of this lower bound takes exactly the same number of elementary operations as a classical move evaluation for the CVRP. Furthermore, most moves for routing problems are far from profitable, even when restricting moves to close services as in Toth and Vigo (2003). In our experiments (Section 5.2), we observed that this simple lower bound helped to discard 90% of the moves on average, and the exact evaluation of the remaining moves is no longer a computational bottleneck.

4.4. Local search

The proposed techniques can serve as a building block for different local search algorithms, depending on the nature of the moves, the move acceptance policy, possible neighborhood restrictions, and the use of infeasible solutions, among other factors. The local search used in our computational experiments is summarized in Algorithm 1 and discussed in details in the rest of this section.

We rely on 2-OPT, 2-OPT*, as well as RELOCATE and SWAP moves of up to $k = 2$ consecutive services with possible reversals (Laporte et al. 2014). All together, these moves define the neighborhood $\mathcal{N}(x^t)$ of an incumbent solution x^t . These moves consider service exchanges and relocations within the same route or between different routes, thus allowing an optimization of both SEQUENCING and ASSIGNMENT decision subsets. To speed up the search, the moves are only attempted between service pairs (i, j) where j belongs to a set $\Gamma(i)$ of $|\Gamma|$ closest services (Toth and Vigo 2003, Vidal et al. 2014a), the distance between services being defined as the shortest distance

```

Input: An initial solution  $x^0$ 
Set  $t = 0$ 
repeat
  Update the auxiliary data structures for the routes of  $x^t$ .           //  $\mathcal{O}(n^2)$  operations
  for each move  $\phi$  in random order, such that  $\phi(x) \in \mathcal{N}(x^t)$            //  $\mathcal{O}(n^2)$  possible moves
    do
      The move  $\phi$  can modify two routes of  $x^t$ , at most. Let  $z_{\text{BEFORE}}$  be the sum of the costs of these two
      routes in  $x^t$ , and let  $(\sigma_1, \dots, \sigma_K)$  and  $(\sigma'_1, \dots, \sigma'_L)$  be the sequences of services which form the
      two new routes in  $\phi(x)$ .
      Evaluate a lower bound on the cost of the new routes:                   //  $\mathcal{O}(1)$  operations
       $z_{\text{LB}} = C_{\text{LB}}(\sigma_1 \oplus \dots \oplus \sigma_K) + C_{\text{LB}}(\sigma'_1 \oplus \dots \oplus \sigma'_L)$ .
      if  $z_{\text{LB}} \geq z_{\text{BEFORE}}$  then continue (to the next move evaluation).
      Evaluate the cost of the new routes with optimal mode choices,           //  $\mathcal{O}(1)$  operations
      using dynamic programming and the known auxiliary data structures:
       $z_{\text{AFTER}} = C(\sigma_1 \oplus \dots \oplus \sigma_K) + C(\sigma'_1 \oplus \dots \oplus \sigma'_L)$ .
      if  $z_{\text{AFTER}} \geq z_{\text{BEFORE}}$  then continue (to the next move evaluation).

      At this stage,  $\phi$  is known to be an improving move:
      Set  $x^{t+1} = \phi(x)$ ,  $t = t + 1$ , and break
    end
  until a local minimum is attained.
Output: A local minimum  $x^t$ 

```

Algorithm 1: Local search using preprocessing, concatenations and lower bounds

between any two edge extremities. Furthermore, a “first improvement” policy is used: the moves are enumerated in random order and any improvement is directly applied.

Capacity constraints. As in previous works, we allow a controlled exploration of penalized infeasible solutions that violate the capacity constraints. Hence, the penalized cost of a route is evaluated as $C_p(\sigma) = C(\sigma) + \omega \max\{0, Q(\sigma) - Q\}$, where ω is a penalty factor. This factor remains constant during one LS, and is self-adapted in the metaheuristic to attain a desired ratio of feasible solutions (Vidal et al. 2014a). The same penalty term is used in the evaluations of the lower bounds. Note that, in a LS variant that does not accept infeasible solutions, a load feasibility check should be included before the lower-bound evaluations.

Preprocessing limitations. Finally, the preprocessing phase represents only a small part of the overall computational effort for most benchmark instances. Still, additional limitation strategies can be used to prevent a larger time consumption in cases with few vehicles and long routes. In particular, Irnich (2008a) proposes limiting the preprocessing to a hierarchy of $\mathcal{O}(n^{4/3})$ or $\mathcal{O}(n^{8/7})$ sequences, at the cost of slightly more expensive but still constant-time move evaluations. To make this even simpler, we limited the preprocessing to sequences that start or end at the depot, or of a size smaller than ten. This still allows to evaluate inter-route moves in constant time (since these moves lead to routes of the form $\sigma = \sigma_1 \oplus \bar{\sigma} \oplus \sigma_2$, where $\sigma_1(1) = 0$, $\sigma_2(|\sigma_2|) = 0$, and $|\bar{\sigma}| \leq 2$), and the remaining intra-route moves, far less numerous, can be evaluated via a concatenation of a small (but possibly linear) number of sequences $\bar{\sigma}$ such that $|\bar{\sigma}| \leq 10$.

4.5. Generalizations – Polynomial ejection chains

The previous sections have introduced a local search which evaluates an exponential number of solutions obtained from combined mode choices and simple service relocations and exchanges. We now generalize this technique into a variant of ejection chains (Glover and Rego 2006) that identifies, in $\mathcal{O}(n^2)$ elementary operations, an improved solution obtained from multiple chained service

relocations among routes as well as combined optimal choices of service modes. The procedure works as follows:

1) Pick a random permutation π of the routes in the current solution.

2) Define the ejection graph $H_\pi = (V_\pi, A_\pi)$ where

- V_π includes one node per service request as well as one *null* node per route, the purpose of which will be described below. Let $R(i)$ be the corresponding route of a node i .
- For any pair of nodes $i \in V_\pi$ and $j \in V_\pi$, A_π contains an arc (i, j) if and only if the route of i precedes the route of j in the permutation π . Note that i, j , or both can be *null* nodes.
- The cost h_{ij} of an arc (i, j) corresponds to the difference in the cost of $R(j)$ when *removing service j* and *inserting service i in its place*. A null node i stands for *inserting nothing*, while a null node j stands for *removing nothing*. In the case where a node is inserted and *nothing* is removed, the best insertion position is used to define the cost. All route costs are evaluated for sequences of services with an optimal choice of service modes. Note that the costs h_{ij} can be negative.
- Complete the graph with a null node standing for a source, connected to all the service nodes, and a null node standing for a sink to which all the null nodes are connected.

3) Find a shortest path in this directed acyclic graph between the source and the sink. This can be done in $\mathcal{O}(|V_\pi| + |A_\pi|) = \mathcal{O}(n^2)$ elementary operations, using Bellman's algorithm in topological order. If the path has a negative cost, apply the corresponding sequence of service removals and insertions to the current solution.

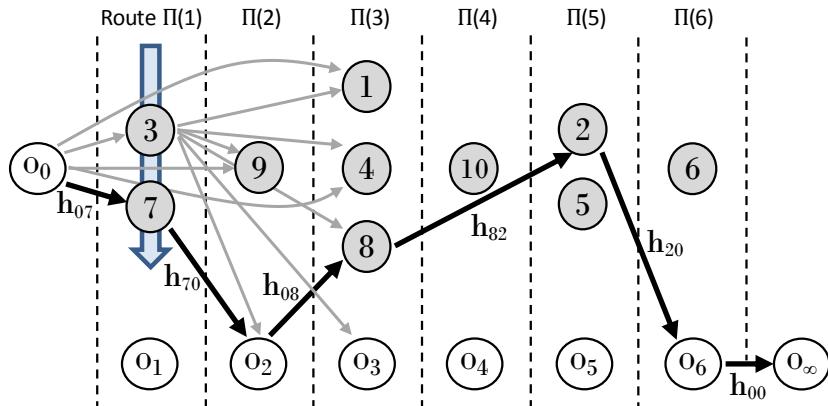


Figure 3 Ejection graph H_π and a possible ejection chain in boldface. Grey and white colors are used for services and null nodes, respectively.

The ejection graph H_π is illustrated in Figure 3 for a problem with ten services in six different routes. The resulting shortest path relocates service 7 from route R1 to R2, relocates service 8 from R3 to R5, and relocates service 2 from R5 to R6.

4.6. Generalizations – Other mode decisions

This section discusses extensions of the proposed methodology to several problem variants, including node, edge, and arc routing, turn penalties, and service clusters.

4.6.1. Services on nodes, edges, and arcs. Most existing metaheuristics for the MCGRP (Prins and Bouchenoua 2005, Kokubugata et al. 2007, Hasle et al. 2012) translate the variety of service types into a larger number of move classes, dealing with multiple cases of service location and orientation changes. Alternatively, Irnich et al. (2015) suggest to transform each service on a node into a service on a loop. In the proposed methodology, we can associate one or two modes for each service depending on its nature:

NODE	$ M_i = 1$	One mode for service;
ARC	$ M_i = 1$	One mode for the only feasible service orientation;
EDGE	$ M_i = 2$	Two modes, one for each service orientation.

Move evaluations are then done exactly as before, the subproblem being even simpler since many services possess a single mode.

4.6.2. Turn restrictions and delays at complex intersections. Turns and delays at intersections can account for 30% of the total transit time in urban networks (Nielsen et al. 1998). As such, considering these features during optimization is a necessity for multiple applications. Shortest path problems with turn restrictions and penalties can be solved efficiently via tailored labeling approaches (Gutiérrez and Medaglia 2008) or graph transformations (line graph or node splitting – Vanhove and Fack 2012). However, representing an MCGRP solution as a permutation of services and using the shortest path with turn restrictions between each pair of services does not necessarily lead to a feasible overall solution. For example, the left part of Figure 4 shows a route with three consecutive services: to an edge i , a node j , and an edge k . The shortest paths between services i and j and between j and k do not contain any forbidden turns. However, the arrival direction at j is the opposite of the starting direction of the next path, leading to an infeasible U-turn.

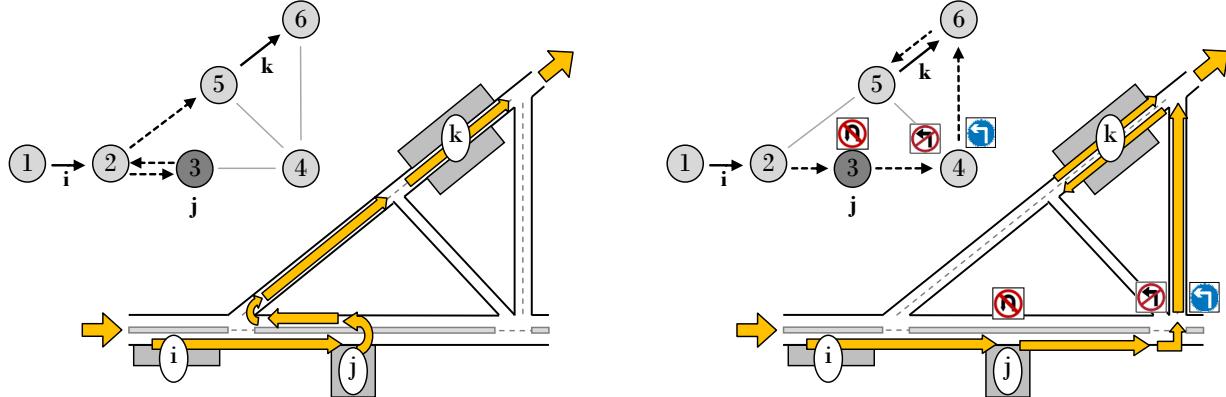


Figure 4 Shortest path for a given sequence of three services. On the left, without turn penalties or constraints; and on the right, in the presence of one forbidden U-turn and other turn limitations.

This inconsistency is due to a lack of characterization of the arrival edge when servicing the node j . The proposed methodology provides a simple solution, by including the arrival edge and driving direction as part of the definition of the mode for a node:

NODE	$ M_i = p_i$	p_i modes to specify the arrival direction, where p_i is the in-degree of v_i ;
ARC	$ M_i = 1$	One mode for the only feasible service orientation;
EDGE	$ M_i = 2$	Two modes, one for each service orientation.

With this definition, the orientation of a service on an edge and the direction of arrival when servicing a node are implied by the mode. The travel times are computed, prior to the optimization, between any pair of (service, mode) couples, i.e., any pairs of arcs in the graph. This is done via an all-pairs shortest path algorithm on the line graph. The other steps are unchanged. The resulting algorithm produces the alternative turn-feasible solution illustrated on the right of Figure 4.

4.6.3. Service clusters. Problem aggregation is a natural way to deal with large-scale applications that contain multiple drop points. A natural aggregation occurs, in arc routing problems, when assimilating drops on the same street to a single edge service. This aggregation implies that these drops are done consecutively, hence forbidding split deliveries. Further aggregations can also be relevant, e.g., considering a geographically delimited group of visits as a service cluster, leading to a *clustered* routing problem, which aims to find routes in which all the services of the same cluster are done consecutively (Battarra et al. 2014).

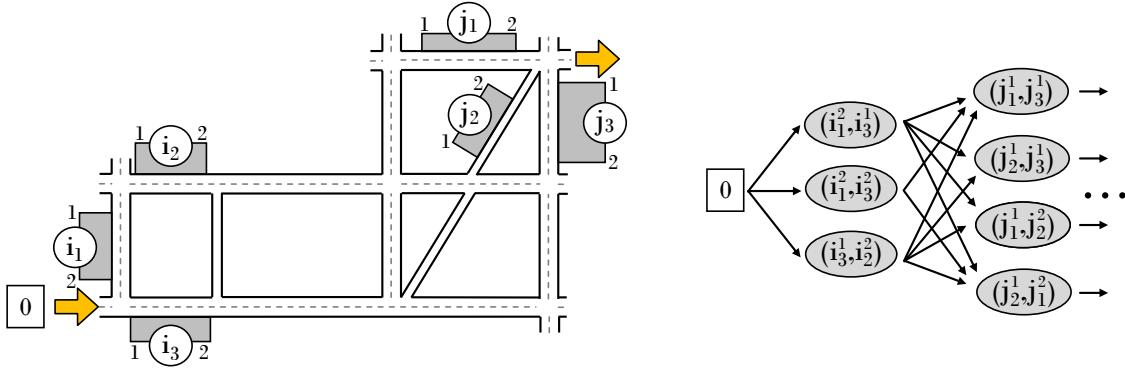


Figure 5 On the left, an example of a street network with two clusters of deliveries: $\{i_1, i_2, i_3\}$ and $\{j_1, j_2, j_3\}$. Each delivery on an edge is represented by a gray rectangle, with two possible orientations (1 and 2). On the right, the associated shortest path problem for mode optimization.

This generalization can be handled via a broader definition of the service mode. A service now represents a cluster of visits, and the service mode of a cluster corresponds to one (entry, exit) direction pair for the cluster, as illustrated in Figure 5. The set of possible (entry, exit) pairs may be exhaustive, limited by practical constraints, or restricted to nondominated choices. The associated service costs are obtained by solving a variant of rural postman problem in the cluster where we impose to start and end at prescribed nodes. This can be done before starting the routing optimization, either exactly in the presence of small clusters, or via efficient heuristics. The other components of the approach are unchanged.

Finally, note that any CARP instance can be reduced to a clustered vehicle routing instance with two nodes in each cluster, one for each end of the associated edge. The visit-order choices in the cluster, i.e., the modes, are then equivalent to the service orientation in the original problem. This immediate reduction highlights the close connections between these problem classes.

4.6.4. Other characteristics. Service-mode choices arise in many other problem applications. The generalized VRP, for example, considers for any service i a set of g_i delivery locations, and one location per group must be visited. The proposed neighborhoods can be applied to this setting by assimilating a service to a group of locations and considering g_i modes per service. The generalized VRP may be viewed as the archetype of a routing problem with mode choices. Each delivery location is an alternative mode with a distinct cost. In this context, problem reductions from arc routing to generalized VRP arise naturally (Baldacci et al. 2009, Micó and Soler 2011).

Some other problems involve generalized concepts of the service mode. These include decisions about visiting an intermediate facility (Polacek et al. 2008), returning to a depot (Vidal et al. 2014b), selecting customers (Vidal et al. 2016), moderating speed (Kramer et al. 2015), or doing a preliminary stop at a charging station (Schneider et al. 2014). In some cases, the mode choices may involve other resources such as time, capacity, and service levels, as objectives or constraints. The methods of this paper can be further generalized to these applications, but leading to resource constrained shortest path subproblems (RCSPP) in cases where several resources are involved. The RCSPP is NP-hard in general, but many specific variants can be efficiently addressed (Irñich and Desaulniers 2005, Lozano and Medaglia 2013).

5. Computational Experiments

We conducted extensive experimental analyses to investigate the effectiveness of the proposed neighborhoods. This section describes the metaheuristic frameworks used for these tests, the benchmark instances, a comparison of the resulting solutions with previous state-of-the-art algorithms, an analysis of method scalability, and finally new benchmark instances and analyses for CARP and MCGRP with turn penalties.

5.1. An extension of two classical metaheuristics

The extended neighborhoods were included in two classical metaheuristic frameworks for vehicle routing: the multi-start iterated local search (ILS) of Prins (2009), and the unified hybrid genetic search (UHGS) of Vidal et al. (2012, 2014a). These metaheuristics were then applied to the CARP without any other problem-specific adaptation or tuning.

ILS starts from a random initial solution. Subsequently, n_C solutions are iteratively generated by applying a shaking operator and a local search, the best solution being kept as the new incumbent solution. The shaking operator applies $k = 2 + \lfloor n/200 \rfloor$ random exchanges of services on a representation of the solution without trip delimiters, and a Split algorithm is used to reinsert the delimiters. The search is restarted n_P times, each run ending when n_I consecutive iterations have been performed without improvement of the best solution or when a time T_{MAX} is reached. The best overall solution is returned.

UHGS follows the methodology of Vidal et al. (2012) with the original binary tournament selection, crossover, and advanced population-diversity management operators. The population is managed to contain between μ^{MIN} and $\mu^{\text{MIN}} + \mu^{\text{GEN}}$ solutions, and each new individual is generated by an OX crossover followed by the local search procedure. The method terminates when I_{MAX} individual generations have been performed without improvement, or a time limit T_{MAX} is reached.

Both methods rely on the local search described in Section 4.4 for solution improvement, considering possible infeasible solutions, followed by one iteration of ejection chains (construction of the ejection graph and resolution). Mode decisions are optimally taken within route evaluations, such that the classical Split algorithm is also implicitly extended into a variant that optimizes the mode choices in $\mathcal{O}(n^2)$, as in Ramdane-Cherif (2002) and Wøhlk (2004). Each metaheuristic is used with its original parameter setting: $(n_I, n_C) = (100, 50)$ for ILS and $(\mu^{\text{EL}}, \mu^{\text{MIN}}, \mu^{\text{GEN}}) = (12, 25, 40)$ for UHGS. The termination criterion is set to $(n_P = 5, T_{\text{MAX}} = 1\text{h})$ for ILS and $(I_{\text{MAX}} = 20,000, T_{\text{MAX}} = 1\text{h})$ for UHGS to produce results in a time comparable to that of other algorithms.

5.2. Comparison with previous literature

The CARP and MCGRP literature contains several well-known sets of benchmark instances, and some of the largest, such as EGL, DI-NEARP, and CBMix, are still challenging for heuristics. We use these instances to assess the performance of the new metaheuristics with extended neighborhoods in comparison with previous algorithms. Table 2 gives the instance characteristics; the files can be found at <http://logistik.bwl.uni-mainz.de/benchmarks.php>, <http://www.sintef.no/Projectweb/TOP/nearp/>.

We ran the ILS and UHGS with extended neighborhoods ten times for each instance with different random seeds, on a single Xeon 3.07 GHz CPU with 16 GB of RAM. Note that some previous CARP algorithms were calibrated for limited groups of instances and frequently tested with a single run, leading to possible *over-tuning* effects. To avoid this, we 1) report the average results from several runs, 2) consider all available nontrivial sets of instances, 3) use new seeds for the final experiments, and 4) disregard any benchmark-set-specific parameter calibration. Finally, some previous algorithms terminate when a known optimal solution is reached. This approach is dependent on exogenous information, leading to inaccurate CPU time comparisons. Hence, we do not trigger the termination of ILS and UHGS when reaching a known optimal solution.

Table 3 gives a summary of the results. It lists for each set of instances the results of the best three methods (selected independently for each set) in terms of the average percentage gap,

Table 2 Benchmark instances

#	Reference	$ N_R $	$ E_R $	$ A_R $	n	Specificities
CARP						
GDB	(23) Golden et al. (1983)	0	[11,55]	0	[11,55]	Random graphs; Only required edges
VAL	(34) Benavent et al. (1992)	0	[39,97]	0	[39,97]	Random graphs; Only required edges
BMCV	(100) Beullens et al. (2003)	0	[28,121]	0	[28,121]	Intercity road network in Flanders
EGL	(24) Li and Eglese (1996)	0	[51,190]	0	[51,190]	Winter-gritting application in Lancashire
EGL-L	(10) Brandão and E. (2008)	0	[347,375]	0	[347,375]	Larger winter-gritting application
MCGRP						
MGGDB	(138) Bosco et al. (2012)	[3,16]	[1,9]	[4,31]	[8,48]	From CARP instances GBD
MGVAL	(210) Bosco et al. (2012)	[7,46]	[6,33]	[12,79]	[36,129]	From CARP instances VAL
CBMix	(23) Prins and B. (2005)	[0,93]	[0,94]	[0,149]	[20,212]	Randomly generated planar networks
BHW	(20) Bach et al. (2013)	[4,50]	[0,51]	[7,380]	[20,410]	From CARP instances GDB, VAL, & EGL
DI-NEARP	(24) Bach et al. (2013)	[120,347]	[120,486]	0	[240,833]	Newspaper and media product distribution

computed as $100(z - z_{\text{BKS}})/z_{\text{BKS}}$, where z is the solution value obtained by the method and z_{BKS} is the best known solution (BKS) value for the instance. This measure is an estimate of the solution quality of a single run. The table also indicates, where available, the gap associated with the best solution of several runs, in which case the number of runs is indicated; the average CPU time “T” from the start of the method to the end (in minutes); the average time “T*” to attain the final solution of the run; and the type of processor used. The new methods are highlighted on a grey background, and the best method, in terms of average gap, is highlighted in bold. The acronyms for the algorithms are listed in Table 4. Finally, the detailed results for each instance are given in Tables EC.1 to EC.15, available in the electronic companion of this article, and at <https://w1.cirrelt.ca/~vidalt/en/VRP-resources.html>.

Table 3 gives an overview of the good performance of the two proposed methods. The new neighborhoods lead to better solutions, attained by either ILS or UHGS, that have not been achieved until now with simpler neighborhoods. The inclusion of more advanced diversification principles, as in UHGS, helps to improve the solution quality even further: UHGS outperforms the current approaches by 0.523% and 1.142% on the largest instances (CBMix and EGL-L), and it outperforms ILS by up to 0.239%. Some of the BKSs for large instances (see Table EC.6) have been improved by up to 2.275%, which is a large difference given the research effort devoted to these problems. The proposed methods also produce solutions of consistent quality, with the average standard deviation for each set ranging from 0.000% to 0.243%. Overall, the impact of the extended neighborhoods and that of using UHGS rather than ILS are greater for larger instances.

We conducted paired-sample Wilcoxon tests to compare the performance of the proposed UHGS to other algorithms in terms of the gap for each instance set. The results of this analysis are illustrated in the boxplots in Figures 6 and 7, which give the gaps for the larger sets, as well as the p-values and significant effects detected by the tests. UHGS performs significantly better ($p < 0.003$ in all cases) than the other methods on the large test sets (EGL, EGL-L, CBMix, BHW, and DI-NEARP). From these experiments, we also observed that the smaller sets have lost their discriminating power, since recent algorithms find the best solutions for the vast majority of instances, leading to many inconclusive observations. We thus encourage future contributors to investigate the performance of heuristics on the larger instances.

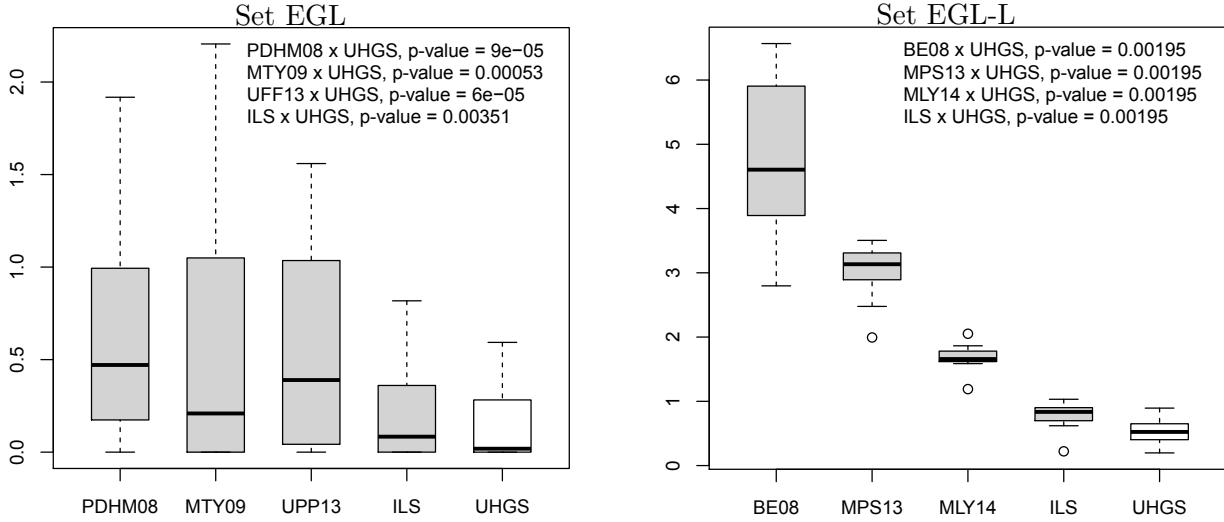
The CPU time of the proposed algorithms ranges from a fraction of a second on the smaller instances to 60 minutes for problems with 833 services. In the MCGRP experiments, this CPU time is smaller than other methods, which were tested on processors of a similar generation. For the CARP, some comparisons involve older processors. Since different CPU speed conversion techniques can give very different results, we opted to present the raw time data and processor information, letting the readers choose their preferred approach. Figure 8 displays the computational effort of

Table 3 Results for CARP and MCGRP benchmark instances.

Variant	Bench.	<i>n</i>	Author	Runs	Avg.	Best	T	T*	CPU
CARP	GDB	[11,55]	TMY09	30	0.009%	0.000%	0.11	—	Xe 2.0G
			BMCV03	1	0.000%	—	—	0.03	P-II 500M
			MTY09	1	0.000%	—	—	0.01	Xe 2.0G
			ILS	10	0.000%	0.000%	0.30	< 0.01	Xe 3.07G
			UHGS	10	0.000%	0.000%	0.31	< 0.01	Xe 3.07G
	VAL	[39,97]	MTY09	1	0.142%	—	—	0.11	Xe 2.0G
			LPR01	1	0.126%	—	2.00	—	P-III 500M
			BMCV03	1	0.060%	—	—	1.36	P-II 500M
			ILS	10	0.044%	0.021%	1.10	0.07	Xe 3.07G
			UHGS	10	0.041%	0.013%	1.13	0.07	Xe 3.07G
	BMCV	[28,121]	BE08	1	0.158%	—	—	1.08	P-M 1.4G
			MTY09	1	0.075%	—	—	0.35	Xe 2.0G
			BMCV03	1	0.038%	—	2.57	—	P-II 450M
			ILS	10	0.020%	0.000%	1.26	0.10	Xe 3.07G
			UHGS	10	0.013%	0.003%	1.19	0.09	Xe 3.07G
	EGL	[51,190]	PDHM08	10	0.625%	—	30.0	8.39	P-IV 3.6G
			UFF13	15	0.562%	0.207%	13.3	—	I4 3.0G
			MTY09	1	0.555%	—	—	2.10	Xe 2.0G
			ILS	10	0.209%	0.088%	3.45	1.47	Xe 3.07G
			UHGS	10	0.141%	0.049%	5.59	3.46	Xe 3.07G
	EGL-L	[347,375]	BE08	1	4.749%	—	—	17.0	P-M 1.4G
			MPS13	10	3.018%	2.591%	20.7	—	I5 3.2G
			MLY14	30	1.671%	0.962%	33.4	—	I7 3.4G
			ILS	10	0.768%	0.468%	28.4	15.8	Xe 3.07G
			UHGS	10	0.529%	0.206%	46.1	36.5	Xe 3.07G
MCGRP	MGGDB	[8,48]	BLMV14	1	1.342%	—	0.31	—	Xe 3.0G
			DHDI14	1	0.018%	—	60.0	0.86	CPU 3G
			ILS	10	0.006%	0.000%	0.22	0.01	Xe 3.07G
			UHGS	10	0.006%	0.000%	0.27	0.01	Xe 3.07G
	MGVAL	[36,129]	BLMV14	1	2.621%	—	16.7	—	Xe 3.0G
			DHDI14	1	0.072%	—	60.0	3.69	CPU 3G
			ILS	10	0.047%	0.015%	1.41	0.15	Xe 3.07G
			UHGS	10	0.047%	0.013%	1.65	0.20	Xe 3.07G
	CBMix	[20,212]	HKSG12	2	—	3.083%	120	56.9	CPU 3G
			BLMV14	1	2.705%	—	44.7	—	Xe 3.0G
			DHDI14	1	0.891%	—	60.0	19.6	CPU 3G
			ILS	10	0.574%	0.287%	3.13	1.54	Xe 3.07G
			UHGS	10	0.361%	0.088%	5.09	3.09	Xe 3.07G
	BHW	[20,410]	HKSG12	2	—	1.976%	120	60.1	CPU 3G
			DHDI14	1	0.581%	—	60.0	21.4	CPU 3G
			ILS	10	0.338%	0.126%	6.91	4.00	Xe 3.07G
			UHGS	10	0.230%	0.084%	9.95	7.12	Xe 3.07G
	DI-NEARP	[240,833]	HKSG12	2	—	1.640%	120	93.0	CPU 3G
			DHDI14	1	0.537%	—	60.0	36.3	CPU 3G
			ILS	10	0.159%	0.074%	58.2	23.6	Xe 3.07G
			UHGS	10	0.106%	0.041%	53.5	30.8	Xe 3.07G

Table 4 Current state-of-the-art methods for the considered benchmark instances, and their acronyms

BCS10	Benavent et al. (2010)	HKSG12	Hasle et al. (2012)	MPY11	Mei et al. (2011)
BE08	Brandão and Eglese (2008)	KY10	Kansou and Yassine (2010)	MTY09	Mei et al. (2009)
BLMV14	Bosco et al. (2014)	LPR01	Lacomme et al. (2001)	PDHM08	Polacek et al. (2008)
BMCV03	Beullens et al. (2003)	LPR05	Lacomme et al. (2005)	TMY09	Tang et al. (2009)
CLP06	Chu et al. (2006)	MLY14	Mei et al. (2014)	UFF13	Usberti et al. (2013)
DHDI14	Dell'Amico et al. (2016)	MPS13	Martinelli et al. (2013)		

**Figure 6** Boxplots of the percentage gap of recent algorithms on the CARP sets EGL and EGL-L. Paired-sample Wilcoxon tests between UHGS and the other methods are also reported. The boxplot associated with a method X is shaded in grey when the solutions of UHGS are of significantly better quality than those of X.

UHGS, in log-log scale, as a function of the number of customer services. The CPU time appears to rise in $\mathcal{O}(n^2)$. Measuring the respective share of computational effort on the EGL instances, we observed that the local search and ejection chains use 60% and 13% of the CPU time of UHGS, respectively. Within the LS, the preprocessing phases use 13% of the time, while the lower bounds and exact move evaluations consume 34% and 6%, respectively.

On the CARP benchmark sets, 187 of 191 BKS have been matched or improved, and 18 of these solutions have been strictly improved. Moreover, 153 solutions out of the 155 known optimal solutions were found. For the last two instances, val9D and val10D, the results are only one unit of distance away from the optimum. For the MCGRP, 408 of 409 BKS have been matched or improved, including 80 new BKS. All 217 known optimal solutions were found. The detailed results are reported in Tables EC.1 to EC.17.

Finally, to complement the experimental comparisons, we considered three additional problems variants: with multiple delivery periods, multiple depots, and the min-max k-vehicles windy rural postman problem. The UHGS with extended neighborhoods also produced solutions of high quality for these problems. All BKS were either retrieved or improved, and for some PCARP instances, the improvements over previous BKS reached up to 10%. These additional experiments, summarized in Table 5, are thoroughly documented in Tables EC.18 to EC.25 of the E-companion.

5.3. Direct approach or problem transformation

The previous sections have investigated whether methods using extended neighborhoods – with optimal choices of service orientations – can outperform methods based on more traditional neighborhoods. This section analyzes whether relying on a problem reduction from CARP to CVRP (Baldacci and Maniezzo 2006) with a classical routing metaheuristic can be profitable. The reduction increases the number of services by a factor of two. Half of the edges of a CVRP solution, with a large fixed negative cost, directly determine the service orientations in the associated CARP solution.

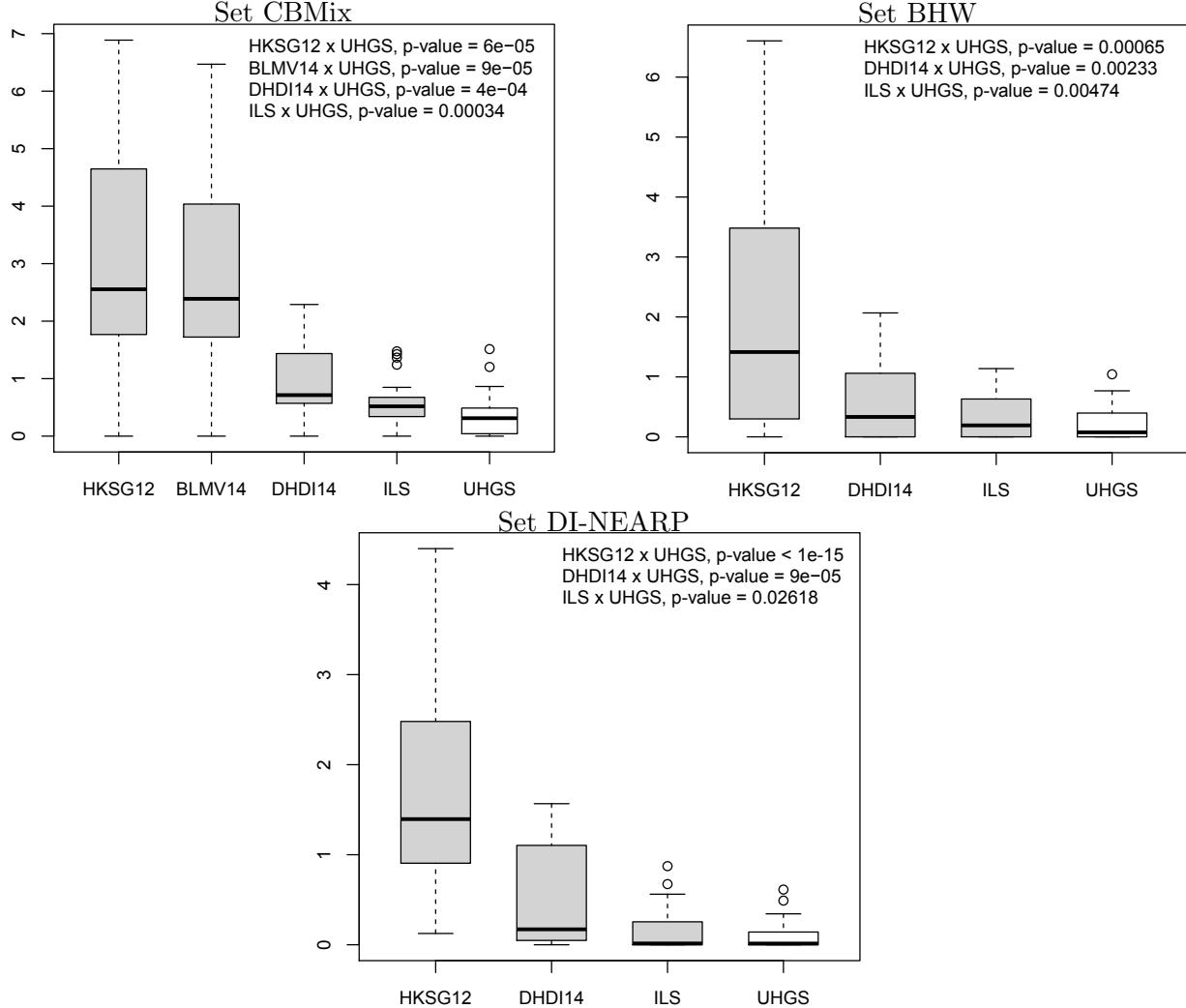


Figure 7 Boxplots of the percentage gap of recent algorithms on the MCGRP sets CBMix, BHW, and DI-NEARP. Same conventions as in Figure 6.

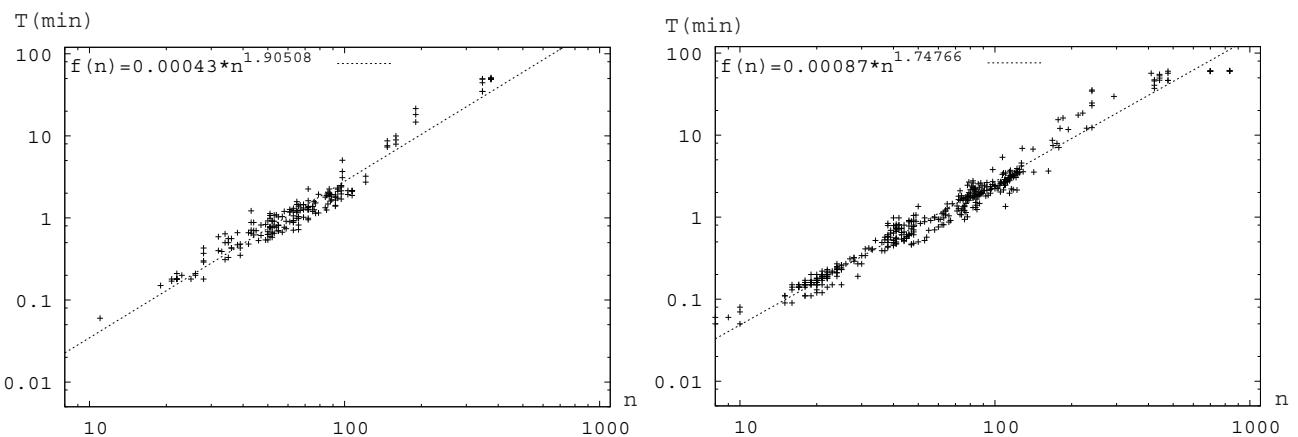


Figure 8 Growth of the CPU time of UHGS as a function of the number of services, for the CARP instances (left figure) and MCGRP instances (right figure). Log-log scale. A linear fit, with a least square regression, has been performed on the sample after logarithmic transformation.

Table 5 Results of UHGS on other CARP variants

Variant	Bench.	<i>n</i>	Author	Runs	Avg.	Best	T	T*	CPU
PCARP	PGDB	[65,165]	LPR05	1	9.448%	—	12.5	—	P-IV 1.4G
			CLP06	1	7.741%	—	1.86	—	P-IV 2.4G
			MPY11	30	3.900%	1.951%	0.20	—	Xe 2.0G
			UHGS [†]	10	0.730%	0.217%	0.14	0.09	Xe 3.07G
			UHGS	10	0.256%	0.071%	0.91	0.41	Xe 3.07G
	PVAL	[94,300]	CLP06	1	16.494%	—	7.38	—	P-IV 2.4G
			MPY11	30	8.691%	6.317%	0.87	—	Xe 2.0G
			UHGS [†]	10	1.614%	0.721%	0.82	0.61	Xe 3.07G
			UHGS	10	0.636%	0.161%	4.91	3.15	Xe 3.07G
MDCARP	GDB	[8,48]	KY10	1	2.041%	—	0.02	—	P-IV 1.4G
			UHGS [†]	10	0.296%	0.104%	0.01	0.01	Xe 3.07G
			UHGS	10	0.017%	0.000%	0.37	0.04	Xe 3.07G
MM-kWRPP	2V	[7,78]	BCS10	1	0.103%	—	0.94	—	I2 2.4G
			UHGS	10	0.008%	0.002%	0.18	0.07	Xe 3.07G
	3V	[7,78]	BCS10	1	0.230%	—	0.41	—	I2 2.4G
			UHGS	10	0.008%	0.000%	0.18	0.07	Xe 3.07G
	4V	[7,78]	BCS10	1	0.303%	—	0.29	—	I2 2.4G
			UHGS	10	0.014%	0.000%	0.18	0.06	Xe 3.07G
	5V	[7,78]	BCS10	1	0.392%	—	0.24	—	I2 2.4G
			UHGS	10	0.021%	0.000%	0.19	0.07	Xe 3.07G

†: A shorter termination criteria has been used (Section EC.3).

We thus applied the same ILS and UHGS to the transformed instances, now using a classical move evaluation for the CVRP. Table 6 compares the results of these approaches, ILS_{CVRP} and $\text{UHGS}_{\text{CVRP}}$, with those of the proposed direct methods for the CARP.

Table 6 Results of ILS_{CVRP} and $\text{UHGS}_{\text{CVRP}}$ on the transformed CVRP. Comparison with the direct approaches.

	Gap(%)		T(min)			Gap(%)		T(min)	
	ILS	ILS_{CVRP}	ILS	ILS_{CVRP}		UHGS	$\text{UHGS}_{\text{CVRP}}$	UHGS	$\text{UHGS}_{\text{CVRP}}$
GDB	0.000%	0.000%	0.30	0.59	GDB	0.000%	0.000%	0.31	0.72
VAL	0.044%	0.061%	1.10	2.39	VAL	0.041%	0.048%	1.13	2.98
BMCV	0.020%	0.047%	1.26	2.79	BMCV	0.013%	0.017%	1.19	3.02
EGL	0.209%	0.346%	3.45	8.50	EGL	0.141%	0.202%	5.59	12.65
EGL-L	0.768%	1.478%	28.4	60.0	EGL-L	0.529%	1.067%	46.1	59.7

Table 6 shows that a transformation into CVRP leads to solutions of lower quality, for a twofold increase in CPU time. The impact is stronger for larger instances, for which significant gap differences can be observed, e.g., 1.067% versus 0.529% for $\text{UHGS}_{\text{CVRP}}$ and 1.478% versus 0.768% for ILS_{CVRP} on set EGL-L. The increased time is a natural consequence of the larger number of services resulting from the transformation. Hence, a direct approach with the extended neighborhoods appears to be a much better alternative than a problem transformation and CVRP resolution.

5.4. Problem settings with turn penalties

Finally, a significant contribution of the proposed methodology is its ability to efficiently handle CARP and MCGRP with turn penalties. These problem classes have been little studied in the

operations research literature, and no benchmark instances for the direct problem formulation subsist nowadays. To fill this gap, we generated two benchmark sets.

The first set, CMMS, extends the 21 mixed rural postman problem instances with turn penalties from Table 3 of Corberán et al. (2002). No services on nodes are required, leading to instances of CARP with turn penalties. The demand quantities were selected with uniform probability in [5, 50], and the vehicle capacities were scaled so as to satisfy 10 or 20 services on average, leading to a total of 42 instances based on random graphs. U-turns are not allowed.

To progress toward more realistic instances, we also extended the DI-NEARP instances of Bach et al. (2013) by adding turn penalties, leading to a set of 28 instances named DI-TP. These instances include services on nodes and edges. They are larger, and they correspond to an application of newspaper and media product delivery in the Nordic countries. Although the graph structure and the edge distances were given, this information was insufficient to identify left or right turns. Thus, we had to reconstruct a plausible planar layout for each instance. This was done with the FM³ algorithm of Hachul and Jünger (2005).

The turn penalties were produced using the following rules. Let T_j be the sets of turns at a node j , ordered by increasing polar angle, $T_j = \{(i, j, k) \mid i \neq k, (i, j) \in E \text{ and } (j, k) \in E\}$.

- If $|T_j| = 1$, then the unique turn $x \in T_j$ is not penalized: $p_x = 0$.
- Otherwise, if $|T_j| \geq 2$,
 1. The turn \bar{x} with angle closest to 180° is the *straight* direction. It receives a penalty $p_{\bar{x}} = \gamma$.
 2. Any turn x with smaller angle than \bar{x} is a *right turn*, and receives a penalty $p_x = 0$.
 3. Any turn x with greater angle than \bar{x} is a *left turn*, and receives a penalty $p_x = 3\gamma$.
- The penalty of any U-turn $x = (i, j, k)$ such that $i = k$ is set to $p_x = 5\gamma$.

The coefficient γ was set, for each instance, to 10% of the average edge length. With this value, the turn costs amount to approximately 25% of the total objective function, a realistic estimate in light of the study of Nielsen et al. (1998).

We investigated the performance of the proposed methods on these sets of instances, also diminishing the population size by two to allow for a faster convergence. Since no solutions are available in the literature, we compare the results obtained with a termination criterion of ($I_{\text{MAX}} = 10,000$; $T_{\text{MAX}} = 1\text{h}$) and ($I_{\text{MAX}} = 20,000$; $T_{\text{MAX}} = 2\text{h}$), to observe to what extent the solution quality could be improved with additional computational effort. A small standard deviation in solution quality and a small gap with respect to the best solution of several runs are usually good indications of performance.

Tables EC.16 and EC.17 give the main characteristics of the new instances and the results of these experiments. The gap and standard deviation over 10 runs remain moderate: 0.526% for UHGS (with $\sigma = 0.227\%$), and 0.626% for ILS (with $\sigma = 0.249\%$), over the whole instance set. The best solutions of the longer runs are, on average, 0.184% better on set CMMS and 0.130% better on set DI-TP. This gap is larger than that for MCGRP instances without turn penalties.

To study further the impact of the turn penalties on the algorithm performance and the structure of the solutions, we conducted additional test runs on the 14 DI-TP instances with vehicle capacities $Q \in \{2000, 4000\}$, multiplying the penalties by a factor $f_{\text{TP}} \in \{0, 0.25, 0.5, 1, 2, 5, 10\}$. The average results for each level of γ are given in Table 7, while Figure 9 displays boxplots of the percentage gap of UHGS as a function of f_{TP} as well as the number of U-turns, left turns, and right turns observed in the solutions.

We observe a correlation between the weight of the turn penalties and the difficulty of the resulting instances, as reflected by a slightly larger gap (up to 0.752%) and standard deviation (up to $\sigma = 0.366\%$) for higher values of f_{TP} . High penalties lead to more deceptive search spaces, possibly because the quality of a sequence of services now depends more strongly on the route context, i.e., the previous services and their impact on the best choices of arrival direction.

Higher turn penalty values lead to significantly fewer left turns in the solutions: from 170.85 turns on average for $f_{\text{TP}} = 0$ to 73.91 for $f_{\text{TP}} = 1$ and 42.01 for $f_{\text{TP}} = 10$. The number of U-turns also decreases until it reaches a plateau, due to some nodes with an out-degree of 1, where a U-turn may

Table 7 Impact of the scale of the turn penalties on algorithm performance and solution characteristics.

γ	Gap (%)	T	Cost	Distance	No. Turns			
					U-turns	Left	Right	All
0	0.141%	50.68	25076.61	25076.61	126.24	170.85	172.35	469.44
0.25	0.280%	51.32	27500.70	25164.44	119.40	91.72	241.98	453.10
0.5	0.281%	51.65	29806.22	25250.74	116.79	82.77	250.17	449.73
1	0.373%	51.74	34339.29	25451.40	113.87	73.91	261.63	449.41
2	0.511%	51.77	43103.49	25986.19	109.84	62.54	282.69	455.06
5	0.607%	51.90	68258.91	27243.48	106.31	48.52	314.51	469.34
10	0.752%	51.92	109011.41	28534.13	105.23	42.01	336.76	484.00

be unavoidable. Decreasing the number of turns naturally comes at the expense of a greater total distance. As Table 7 shows, moderate penalty values below $\gamma = 1$ significantly reduce left turns and U-turns and lead to an increase of just 2% in the distance. This compromise is likely to be acceptable in practice. Finally, as illustrated in Figure 10, significant changes in the structure of the solutions occur as the turn penalties grow. The goal of minimizing left turns leads to clockwise cycles in some areas of the solutions. Despite these changes in solution structure, the proposed metaheuristic with extended neighborhoods still produces high-quality solutions.

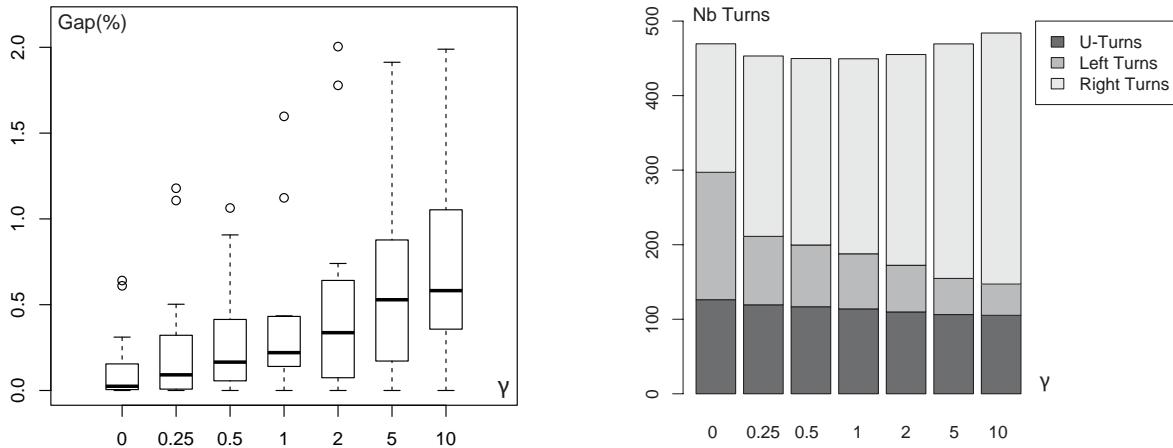


Figure 9 On the left, percentage gap of UHGS for different values of the turn penalties. On the right, average number of U-turns, left turns, and right turns in the solutions as a function of the turn penalties.

6. Conclusions and Future Research

In this article, we have investigated a family of extended neighborhoods for arc routing problems, which involve service relocations and exchanges with combined optimal mode choices on the routes. We have shown that a complete exploration of this extended neighborhood can be done in $\mathcal{O}(n^2)$, the same complexity as a classical neighborhood search without mode optimization. Moreover, as the new approach relegates service mode choices in a dynamic programming subproblem, it guarantees their optimality and helps to concentrate the heuristic search on a more limited decision set. We have generalized this methodology to multiple arc routing variants, extended it into large-scale neighborhood searches such as ejection chains, and integrated it into two classical metaheuristic frameworks. Our experiments led to solutions of high quality on 18 benchmark sets for these problems, with a total of 1528 instances. We also conducted experiments on the CARP–TP and MCGRP–TP, which are uncommonly challenging and critical for practical applications.

This work will help to build more connections between the arc routing and vehicle routing communities and take research into heuristics in new directions. Current research is often excessively

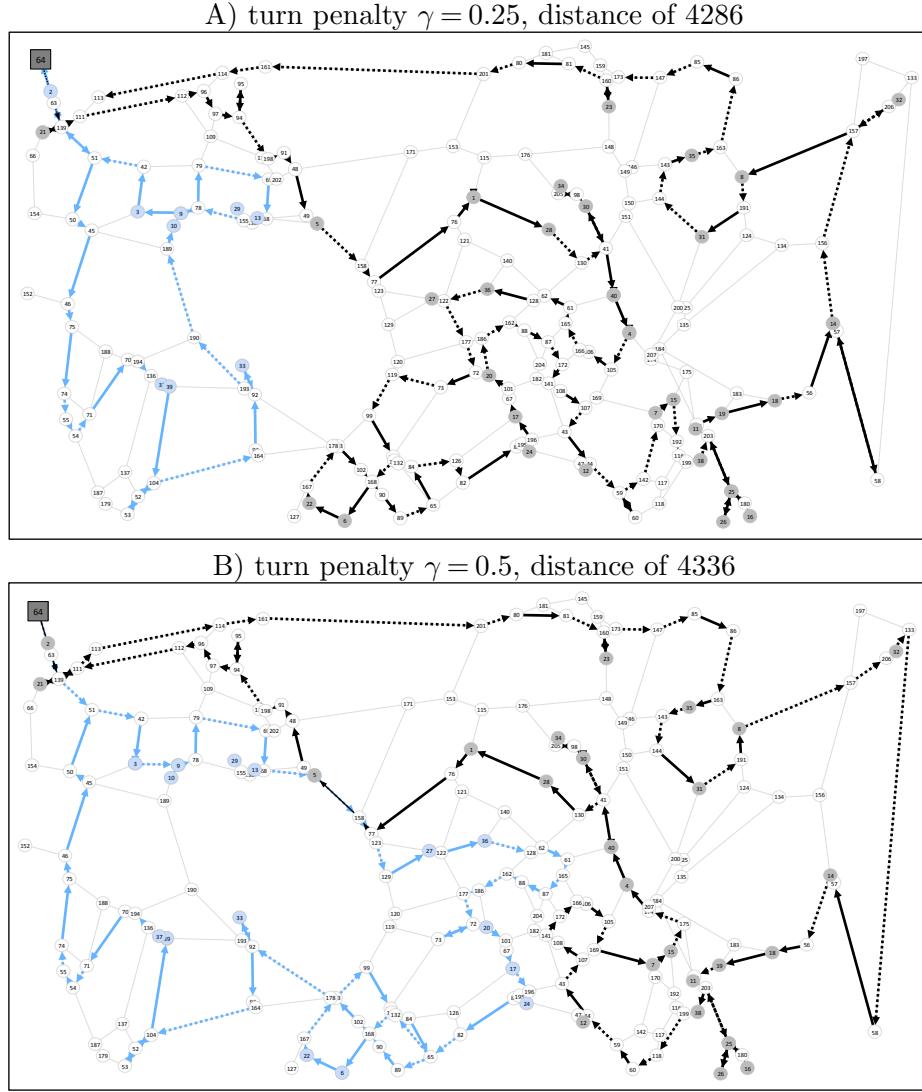


Figure 10 Comparison of solutions for the smallest DI-TP instance, n80-Q4k, when doubling the turn penalties.

concerned with the design of *better* high-level metaheuristic frameworks rather than the study of alternative neighborhood structures. Significant future breakthroughs may arise from a better understanding of problem structure and *neighborhood search* – as attempted here – rather than brute-force *neighborhood enumeration*.

Future research into arc routing could also extend these neighborhoods to an even wider class of problems with different resources, such as time, duration, workload, and energy, as well as possible time-dependent resource consumptions (e.g., congestion aspects in urban environments). In the presence of additional resources and constraints, move evaluations would be formulated as resource-constrained shortest paths problems, and additional move lower bounds might be needed. Finally, similar neighborhood decompositions and dynamic-programming-based move evaluations should be investigated for other difficult combinatorial optimization problems, such as scheduling, allocation, and packing problems.

Revision Note

An article of Chen et al. (2016) appeared online during our final revision steps. It presents an hybrid GA with tabu search, merging moves, and infeasible descents as an intensification procedure. This approach also

produces solutions of high quality, albeit in higher CPU time, via more dedicated metaheuristic strategies rather than extended neighborhoods. This leads to two state-of-the-art methods, which are both interesting in their own right since they bring significant improvements to two symbiotic aspects of the search: the low-level neighborhoods (this paper), and the higher level metaheuristic strategies (the mentioned article). Future research should gather key elements from these methods to progress on a new generation of simple and efficient heuristics.

Acknowledgments

The author would like to thank David Soler Fernandez, Geir Hasle, and Anand Subramanian for their extensive help with the benchmark instances, as well as three anonymous referees for their detailed reports, which significantly contributed to improve this paper.

Bibliography

Thibaut Vidal is professor at the department of computer science of the Pontifical Catholic University of Rio de Janeiro, Brazil. His research interests include combinatorial optimization, metaheuristics, integer and dynamic programming, with applications to logistics and distribution systems design, vehicle routing and scheduling, resource allocation and signal processing.

References

- Bach, L., G. Hasle, S. Wøhlk. 2013. A lower bound for the node, edge, and arc routing problem. *Computers & Operations Research* **40**(4) 943–952.
- Balas, E., N. Simonetti. 2001. Linear time dynamic-programming algorithms for new classes of restricted TSPs: A computational study. *INFORMS Journal on Computing* **13**(1) 56–75.
- Baldacci, R., E. Bartolini, G. Laporte. 2009. Some applications of the generalized vehicle routing problem. *Journal of the Operational Research Society* **61**(7) 1072–1077.
- Baldacci, R., V. Maniezzo. 2006. Exact methods based on node-routing formulations for undirected arc-routing problems. *Networks* **47**(1) 52–60.
- Battarra, M., G. Erdogan, D. Vigo. 2014. Exact algorithms for the clustered vehicle routing problem. *Operations Research* **60**(1) 58–71.
- Bautista, J., J. Pereira. 2004. Ant algorithms for urban waste collection routing. M. Dorigo, M. Birattari, C. Blum, L.M. Gambardella, F. Mondada, T. Stützle, eds., *Ant Colony Optimization and Swarm Intelligence*. LNCS, Springer Berlin Heidelberg, 302–309.
- Benavent, E., V. Campos, A. Corberán, E. Mota. 1992. The capacitated arc routing problem: Lower bounds. *Networks* **22**(7) 669–690.
- Benavent, E., A. Cobéran, I. Plana, J.M. Sanchis. 2009. Min-max K-vehicles windy rural postman problem. *Networks* **54**(4) 216–226.
- Benavent, E., Á. Corberán, J.M. Sanchis. 2010. A metaheuristic for the minmax windy rural postman problem with K vehicles. *Computational Management Science* **7**(3) 269–287.
- Benavent, E., D. Soler. 1999. The directed rural postman problem with turn penalties. *Transportation Science* **33**(4) 408–418.
- Beullens, P., L. Muyldermans, D. Cattrysse, D. Van Oudheusden. 2003. A guided local search heuristic for the capacitated arc routing problem. *European Journal of Operational Research* **147**(3) 629–643.
- Bode, C., S. Irnich. 2012. Cut-first branch-and-price-second for the capacitated arc-routing problem. *Operations Research* **60**(5) 1167–1182.
- Bode, C., S. Irnich. 2015. In-depth analysis of pricing problem relaxations for the capacitated arc-routing problem. *Transportation Science* **49**(2) 369–383.
- Bosco, A., D. Laganà, R. Musmanno, F. Vocaturo. 2012. Modeling and solving the mixed capacitated general routing problem. *Optimization Letters* **7**(7) 1451–1469.

- Bosco, A., D. Laganà, R. Musmanno, F. Vocaturo. 2014. A matheuristic algorithm for the mixed capacitated general routing problem. *Networks* **64**(4) 262–281.
- Brandão, J., R. Eglese. 2008. A deterministic tabu search algorithm for the capacitated arc routing problem. *Computers & Operations Research* **35**(4) 1112–1126.
- Bräysy, O., E. Martínez, Y. Nagata, D. Soler. 2011. The mixed capacitated general routing problem with turn penalties. *Expert Systems with Applications* **38**(10) 12954–12966.
- Chen, Y., J.-K. Hao, F. Glover. 2016. A hybrid metaheuristic approach for the capacitated arc routing problem. *European Journal of Operational Research* **253**(1) 25–39.
- Chu, F., N. Labadi, C. Prins. 2006. A scatter search for the periodic capacitated arc routing problem. *European Journal of Operational Research* **169**(2) 586–605.
- Corberán, Á., G. Laporte, eds. 2014. *Arc Routing: Problems, Methods, and Applications*. Society for Industrial and Applied Mathematics, Philadelphia, PA.
- Corberán, A., R. Martí, E. Martínez, D. Soler. 2002. The rural postman problem on mixed graphs with turn penalties. *Journal of the Operational Research Society* **29**(7) 887–903.
- Dell'Amico, M., J.C.D. Díaz, G. Hasle, M. Iori. 2016. An adaptive iterated local search for the mixed capacitated general routing problem. *Transportation Science, Articles in Advance*.
- Dror, M., ed. 2000. *Arc routing: Theory, Solutions, and Applications*. Springer, New York, USA.
- Dussault, B. 2012. Modeling and solving arc routing problems in street sweeping and snow plowing. Ph.D. thesis, University of Maryland.
- Dussault, B., B. Golden, E. Wasil. 2014. The downhill plow problem with multiple plows. *Journal of the Operational Research Society* **65**(10) 1465–1474.
- Giffler, B., G.L. Thompson. 1960. Algorithms for solving production-scheduling problems. *Operations Research* **8**(4) 487–503.
- Glover, F., C. Rego. 2006. Ejection chain and filter-and-fan methods in combinatorial optimization. *4OR* **4**(4) 263–296.
- Golden, B.L., J.S. DeArmon, E.K. Baker. 1983. Computational experiments with algorithms for a class of routing problems. *Computers & Operations Research* **10**(1) 47–59.
- Greistorfer, P. 2003. A tabu scatter search metaheuristic for the arc routing problem. *Computers & Industrial Engineering* **44**(2) 249–266.
- Gutiérrez, E., A.L. Medaglia. 2008. Labeling algorithm for the shortest path problem with turn prohibitions with application to large-scale road networks. *Annals of Operations Research* **157**(1) 169–182.
- Hachul, S., M. Jünger. 2005. Drawing large graphs with a potential-field-based multilevel algorithm. J. Pach, ed., *Graph Drawing, LNCS*, vol. 3383. Springer, 285–295.
- Hasle, G., O. Kloster, M. Smetsrud, K. Gaze. 2012. Experiments on the node, edge, and arc routing problem. Tech. rep., SINTEF, Oslo, Norway.
- Hertz, A., G. Laporte, M. Mittaz. 2000. A tabu search heuristic for the capacitated arc routing problem. *Operations Research* **48**(1) 129–135.
- Hertz, A., M. Mittaz. 2001. A variable neighborhood descent algorithm for the undirected capacitated arc routing problem. *Transportation Science* **35**(4) 425–434.
- Irnich, S. 2008a. A unified modeling and solution framework for vehicle routing and local search-based metaheuristics. *INFORMS Journal on Computing* **20**(2) 270–287.
- Irnich, S. 2008b. Solution of real-world postman problems. *European Journal of Operational Research* **190**(1) 52–67.
- Irnich, S., G. Desaulniers. 2005. Shortest path problems with resource constraints. G. Desaulniers, J. Desrosiers, M.M. Solomon, eds., *Column Generation*. Springer, New York, 33–65.
- Irnich, S., D. Laganà, C. Schlebusch, F. Vocaturo. 2015. Two-phase branch-and-cut for the mixed capacitated general routing problem. *European Journal of Operational Research* **243**(1) 17–29.

- Kansou, A., A. Yassine. 2010. New upper bounds for the multi-depot capacitated arc routing problem. *International Journal of Metaheuristics* **1**(1) 81–95.
- Kokubugata, H., A. Moriyama, H. Kawashima. 2007. A practical solution using simulated annealing for general routing problems with nodes, edges, and arcs. LNCS, Springer Berlin Heidelberg, 136–149.
- Kramer, R., A. Subramanian, T. Vidal, L.A.F. Cabral. 2015. A matheuristic approach for the pollution-routing problem. *European Journal of Operational Research* **243**(2) 523–539.
- Krushinsky, D., T. Van Woensel. 2015. An approach to the asymmetric multi-depot capacitated arc routing problem. *European Journal of Operational Research* **244**(1) 100–109.
- Lacomme, P., C. Prins, W. Ramdane-Chérif. 2001. A genetic algorithm for the capacitated arc routing problem and its extensions. *Applications of Evolutionary Computing* 473–483.
- Lacomme, P., C. Prins, W. Ramdane-Chérif. 2004. Competitive memetic algorithms for arc routing problems. *Annals of Operations Research* **131**(1-4) 159–185.
- Lacomme, P., C. Prins, W. Ramdane-Chérif. 2005. Evolutionary algorithms for periodic arc routing problems. *European Journal of Operational Research* **165**(2) 535–553.
- Laporte, G., S. Ropke, T. Vidal. 2014. Heuristics for the vehicle routing problem. P. Toth, D. Vigo, eds., *Vehicle Routing: Problems, Methods, and Applications*, chap. 4. Society for Industrial and Applied Mathematics, 87–116.
- Letchford, A.N., A. Oukil. 2009. Exploiting sparsity in pricing routines for the capacitated arc routing problem. *Computers and Operations Research* **36**(7) 2320–2327.
- Li, L.Y.O., R.W. Eglese. 1996. An interactive algorithm for vehicle routeing for winter-gritting. *Journal of the Operational Research Society* **47**(2) 217–228.
- Lozano, L., A.L. Medaglia. 2013. On an exact method for the constrained shortest path problem. *Computers & Operations Research* **40**(1) 378–384.
- Martinelli, R., M. Poggi, A. Subramanian. 2013. Improved bounds for large scale capacitated arc routing problem. *Computers & Operations Research* **40**(8) 2145–2160.
- Mei, Y., X. Li, X. Yao. 2013. Decomposing large-scale capacitated arc routing problems using a random route grouping method. *IEEE Congress on Evolutionary Computation*. IEEE, Cancun, Mexico, 1013–1020.
- Mei, Y., X. Li, X. Yao. 2014. Cooperative co-evolution with route distance grouping for large-scale capacitated arc routing problems. *IEEE Transactions on Evolutionary Computation* **18**(3) 435–449.
- Mei, Y., K. Tang, X. Yao. 2009. A global repair operator for capacitated arc routing problem. *IEEE Transactions on Systems, Man, and Cybernetics. Part B, Cybernetics* **39**(3) 723–734.
- Mei, Y., K. Tang, X. Yao. 2011. A memetic algorithm for periodic capacitated arc routing problem. *IEEE Transactions on Systems, Man, and Cybernetics. Part B: Cybernetics* **41**(6).
- Micó, J.C., D. Soler. 2011. The capacitated general windy routing problem with turn penalties. *Operations Research Letters* **39**(4) 265–271.
- Muyldermans, L., P. Beullens, D. Cattrysse, D. Van Oudheusden. 2005. Exploring variants of 2-opt and 3-opt for the general routing problem. *Operations Research* **53**(6) 982–995.
- Nielsen, O.A., R.D. Frederiksen, N. Simonsen. 1998. Using expert system rules to establish data for intersections and turns in road networks. *International Transactions in Operational Research* **5**(6) 569–581.
- Polacek, M., K.F. Doerner, R.F. Hartl, V. Maniezzo. 2008. A variable neighborhood search for the capacitated arc routing problem with intermediate facilities. *Journal of Heuristics* **14**(5) 405–423.
- Prins, C. 2009. A GRASP - evolutionary local search hybrid for the vehicle routing problem. F.B. Pereira, J. Tavares, eds., *Bio-inspired Algorithms for the Vehicle Routing Problem*. Springer, 35–53.
- Prins, C., S. Bouchenoua. 2005. A memetic algorithm solving the VRP, the CARP, and more general routing problems with nodes, edges and arcs. W. Hart, N. Krasnogor, J. Smith, eds., *Recent advances in memetic algorithms*. Springer, 65–85.
- Prins, C., N. Labadi, M. Reghioui. 2009. Tour splitting algorithms for vehicle routing problems. *International Journal of Production Research* **47**(2) 507–535.

- Ramdane-Cherif, W. 2002. Problèmes d'optimisation en tournées sur arcs. Ph.D. thesis, Université de Technologie de Troyes, France.
- Santos, L., J. Coutinho-Rodrigues, J.R. Current. 2010. An improved ant colony optimization based algorithm for the capacitated arc routing problem. *Transportation Research Part B: Methodological* **44**(2) 246–266.
- Schneider, M., A. Stenger, D. Goeke. 2014. The electric vehicle routing problem with time windows and recharging stations. *Transportation Science* **48**(4) 500–520.
- Tang, K., Y. Mei, X. Yao. 2009. Memetic algorithm with extended neighborhood search for capacitated arc routing problems. *IEEE Transactions on Evolutionary Computation* **13**(5) 1151–1166.
- Toth, P., D. Vigo. 2003. The granular tabu search and its application to the vehicle-routing problem. *INFORMS Journal on Computing* **15**(4) 333–346.
- Usberti, F.L., P.M. França, A.L.M. França. 2013. GRASP with evolutionary path-relinking for the capacitated arc routing problem. *Computers & Operations Research* **40**(12) 3206–3217.
- Vanhove, S., V. Fack. 2012. Route planning with turn restrictions: A computational experiment. *Operations Research Letters* **40**(5) 342–348.
- Vidal, T., T.G. Crainic, M. Gendreau, N. Lahrichi, W. Rei. 2012. A hybrid genetic algorithm for multidepot and periodic vehicle routing problems. *Operations Research* **60**(3) 611–624.
- Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2014a. A unified solution framework for multi-attribute vehicle routing problems. *European Journal of Operational Research* **234**(3) 658–673.
- Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2014b. Implicit depot assignments and rotations in vehicle routing heuristics. *European Journal of Operational Research* **237**(1) 15–28.
- Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2015. Timing problems and algorithms: Time decisions for sequences of activities. *Networks* **65**(2) 102–128.
- Vidal, T., N. Maculan, L.S. Ochi, P.H.V. Penna. 2016. Large neighborhoods with implicit customer selection for vehicle routing problems with profits. *Transportation Science* **50**(2) 720–734.
- Wang, Z., H. Jin, M. Tian. 2015. Rank-based memetic algorithm for capacitated arc routing problems. *Applied Soft Computing* **37**(1) 572–584.
- Wøhlk, S. 2003. Simulated annealing for the capacitated arc routing problem, using an online formulation. Tech. rep., University of Southern Denmark.
- Wøhlk, S. 2004. Combining dynamic programming and simulated annealing. Tech. rep., University of Southern Denmark.
- Xing, L., P. Rohlfsagen, Y. Chen, X. Yao. 2010. An evolutionary approach to the multidepot capacitated arc routing problem. *IEEE Transactions on Evolutionary Computation* **14**(3) 356–374.

Node, Edge, Arc Routing and Turn Penalties: Multiple Problems – One Neighborhood Extension

Electronic Companion

This E-Companion provides the detailed results, on individual instances, for all the benchmark sets considered in this paper.

EC.1. Conventions on fleet size limits

In the usual CARP definition, no limit is imposed on the number of vehicles used (see, e.g., Corberán and Laporte 2014). We use this convention when testing the algorithms. Note that all the solutions found by ILS and UHGS for CARP instances use the minimum number of vehicles, so these solutions remain valid in the presence of the fleet size constraint.

The situation is different for problem extensions with services on nodes, edges, and arcs. The NEARP, as defined by Prins and Bouchenoua (2005), does not include a limit on the number of vehicles; whereas the MCGRP (the same problem), as defined by Bosco et al. (2012), includes a maximum fleet size limit. We respected the fleet size conventions specified by the authors for each benchmark set. Note that for mggdb-0.xx-13 and mgval-0.50-1C, slightly better solutions may be obtained if the fleet size is relaxed. Finally, the solution of mgval-0.50-1C from Dell’Amico et al. (2016) appears to include an extra vehicle. For this reason, it was not considered a BKS.

EC.2. Format of the tables

Tables EC.1 to EC.17 present the detailed results for ILS and UHGS with extended neighborhoods as well as other state-of-the-art methods for the CARP and MCGRP benchmark instances, with possible turn penalties. The first group of columns presents the instance identifiers and the number of services on nodes $|N_R|$, edges $|E_R|$, and arcs $|A_R|$. The next group of columns presents the results. When available, we report both average and best results over several runs (the number of runs is specified in the table headings). The best solution among the methods is highlighted in bold. In addition, the two last columns indicate the previous and new BKS. The new BKS were collected from all the runs, including the preliminary tests. New BKS are underlined. Known optimal solutions from the literature, gathered at <http://logistik.bwl.uni-mainz.de/benchmarks.php> and <http://www.sintef.no/Projectweb/TOP/nearp/>, are indicated with an asterisk. For three benchmark instances (BMCV-E9, mgval-0.25-4D, and mgval-0.25-10D) our new BKS matches the best known lower bound from Bode and Irnich (2015) and Irnich et al. (2015), so these three solutions are also optimal. Finally, the last lines of the tables report average measures over the sets of instances: the CPU time for each method, the time to reach the final solution, the average percentage gap relative to the new BKS, and the processor.

EC.3. Additional experiments on CARP variants

To complement these experimental analyses and provide more elements of comparison with current and future state-of-the-art methods for arc routing problems, we considered three additional problem settings, which are the arc-routing counterparts of important problem variants addressed by the hybrid genetic search of Vidal et al. (2012), used in this paper. As such, we can apply the same metaheuristic with the generalized move evaluations.

- In the **periodic capacitated arc routing problem (PCARP)**, the deliveries are performed in several time periods. The objective is to choose, for each service, a combination of periods in a set of allowed patterns and to design the routing plan for each period. The hybrid genetic search of Vidal et al. (2012) already contains an additional local search phase (called pattern improvement – PI) and a crossover to explore alternative customer-pattern assignment choices. Our only adaptation concerns the evaluations of the routes, which now involve an optimal choice of service orientations and serve as a building block for all other search components. Moreover, the classical objective for the PCARP minimizes the fleet size, and then the travel distance. This is done by iteratively decrementing the fleet size and performing shorter runs of UHGS ($I_{\text{MAX}} = 5,000$) until no solution is found. Then, a final run is performed ($I_{\text{MAX}} = 10,000$) with the smallest feasible fleet size. Tables EC.18 and EC.19 report the performance of UHGS on the benchmark instances of Lacomme et al. (2005) and Chu et al. (2006), in comparison with the current state-of-the-art methods for the PCARP: the improved periodic memetic algorithm of Lacomme et al. (2005) (LPR05), the scatter search of Chu et al. (2006) (CLP06), and the memetic algorithm with route merging of Mei et al. (2011) (MPY11). To allow a better comparison with MPY11, which produces results in very short CPU time, we report additional computational results of UHGS with a short termination criterion, $I_{\text{MAX}} = 1,000$.
- In the **multi-depot capacitated arc routing problem (MDCARP)**, the vehicles are stationed at several depot locations. The objective is to optimize the depot-to-service assignments and the routing plans. Despite the importance of the multi-depot attribute in the vehicle routing literature, few articles have investigated it in the arc routing domain. In Krushinsky and Van Woensel (2015), all services are requested on (oriented) arcs. This problem, however, can be reduced to an asymmetric MDVRP. Xing et al. (2010) consider an extension of MDCARP with prohibited turns and duration constraints. Finally, Kansou and Yassine (2010) address the MDCARP, reporting computational results for extensions of the GDB benchmark, obtained by considering depot locations at the vertices 1 and $|V|$. We followed this latter approach, also generalizing the VAL and EGL instances with $d = 3$ and $d = 4$ depots, respectively, located at the vertices 1 and $x \lfloor \frac{|V|}{d-1} \rfloor$ for $x \in \{1, \dots, d-1\}$. The results for these three sets of instances are reported in Tables EC.20 to EC.22. To allow a better comparison with the algorithm of Kansou and Yassine (2010), on the set GDB, we report additional computational results with $I_{\text{MAX}} = 500$.
- Finally, in the **min-max k-vehicles windy rural postman problem (MM-kWRPP)**, the travel and service costs are asymmetric, and the objective is to minimize the distance of the longest route. We addressed this objective with UHGS by setting a distance constraint D for the routes, and iteratively setting $D = D_S - 1$ when a feasible solution S is found, where D_S represents the maximum distance of a route in S . The termination criterion was set to $I_{\text{MAX}} = 2,000$ at each iteration. Experiments were conducted on the benchmark of Benavent et al. (2009), which includes 24 classes of 6 instances, with 2 to 6 vehicles. The UHGS results are compared with those of the multi-start ILS of Benavent et al. (2010) (detailed solution values were obtained by contacting the authors), and with the BKS from subsequent works on exact methods, available at <http://www.uv.es/corberan/instancias.htm>. The total number of instances (720) is too large for a display of all the individual results. Hence, Tables EC.23 to EC.25 report detailed results for the larger instances of classes C20 to C24, as well as C16 to C19 for problems with six vehicles. For the remaining instances, the optimal solutions are known, and these solutions were found by UHGS on all runs. The average results, at the bottom of Table EC.25, are for the whole set of instances. Note that, for the 76 currently open instances, 71 upper bounds have been improved. This will be helpful for future research into exact methods.

EC.4. Open source code library

Finally, to help reproducing and extending this work, we provide at <https://github.com/vidalthi/HGS-CARP> the source code of the proposed methods under the GPL 3.0 license as well as the data sets considered in this article.

Table EC.1 Results for the CARP – GDB instances

Inst	$ E_R $	BMCV03		TMY09		MTY09		ILS				UHGS				BKS
		Single	Avg-30	Best-30	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Opt		
gdb1	22	316	316.0	316	316	316.0	316	0.14	0.00	316.0	316	0.18	0.00	316*		
gdb2	26	339	339.0	339	339	339.0	339	0.19	0.00	339.0	339	0.21	0.00	339*		
gdb3	22	275	275.0	275	275	275.0	275	0.15	0.00	275.0	275	0.18	0.00	275*		
gdb4	19	287	287.0	287	287	287.0	287	0.12	0.00	287.0	287	0.15	0.00	287*		
gdb5	26	377	377.0	377	377	377.0	377	0.18	0.00	377.0	377	0.20	0.00	377*		
gdb6	22	298	298.0	298	298	298.0	298	0.14	0.00	298.0	298	0.18	0.00	298*		
gdb7	22	325	325.0	325	325	325.0	325	0.14	0.00	325.0	325	0.18	0.00	325*		
gdb8	46	348	348.7	348	348	348.0	348	0.58	0.05	348.0	348	0.53	0.01	348*		
gdb9	51	303	303.0	303	303	303.0	303	0.59	0.02	303.0	303	0.59	0.02	303*		
gdb10	25	275	275.0	275	275	275.0	275	0.20	0.00	275.0	275	0.18	0.00	275*		
gdb11	45	395	395.0	395	395	395.0	395	0.62	0.00	395.0	395	0.70	0.00	395*		
gdb12	23	458	458.0	458	458	458.0	458	0.16	0.00	458.0	458	0.20	0.00	458*		
gdb13	28	536	536.0	536	536	536.0	536	0.28	0.01	536.0	536	0.30	0.01	536*		
gdb14	21	100	100.0	100	100	100.0	100	0.14	0.00	100.0	100	0.18	0.00	100*		
gdb15	21	58	58.0	58	58	58.0	58	0.16	0.00	58.0	58	0.17	0.00	58*		
gdb16	28	127	127.0	127	127	127.0	127	0.28	0.00	127.0	127	0.29	0.00	127*		
gdb17	28	91	91.0	91	91	91.0	91	0.20	0.00	91.0	91	0.18	0.00	91*		
gdb18	36	164	164.0	164	164	164.0	164	0.43	0.00	164.0	164	0.43	0.00	164*		
gdb19	11	55	55.0	55	55	55.0	55	0.05	0.00	55.0	55	0.06	0.00	55*		
gdb20	22	121	121.0	121	121	121.0	121	0.23	0.00	121.0	121	0.21	0.00	121*		
gdb21	33	156	156.0	156	156	156.0	156	0.38	0.00	156.0	156	0.39	0.00	156*		
gdb22	44	200	200.0	200	200	200.0	200	0.62	0.01	200.0	200	0.62	0.01	200*		
gdb23	55	233	233.0	233	233	233.0	233	0.81	0.02	233.0	233	0.81	0.02	233*		
Gap (%)		0.000%	0.009%	0.000%	0.000%	0.000%	0.000%			0.000%	0.000%					
T(min)		—	0.11	—	—	—	0.30			—	—	0.31	—			
T*(min)		0.03	—	—	0.01	—	—	<0.01		—	—	—	<0.01			
CPU	P-II 500M	Xe 2.0G	Xe 2.0G	Xe 2.0G	Xe 3.07G	Xe 3.07G	Xe 3.07G			Xe 3.07G	Xe 3.07G	Xe 3.07G	Xe 3.07G			

Table EC.2 Results for the CARP – VAL instances

Inst	$ E_R $	LPR01	BMCV03	MTY09	ILS				UHGS				BKS
		Single	Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Opt
1A	39	173	173	173	173.0	173	0.43	0.00	173.0	173	0.35	0.00	173*
1B	39	173	173	173	173.0	173	0.62	0.01	173.0	173	0.48	0.01	173*
1C	39	245	245	245	245.0	245	0.38	0.00	245.0	245	0.43	0.00	245*
2A	34	227	227	227	227.0	227	0.55	0.00	227.0	227	0.64	0.00	227*
2B	34	259	259	259	259.0	259	0.44	0.00	259.0	259	0.50	0.00	259*
2C	34	457	457	457	457.0	457	0.33	0.01	457.0	457	0.31	0.01	457*
3A	35	81	81	81	81.0	81	0.53	0.00	81.0	81	0.56	0.00	81*
3B	35	87	87	87	87.0	87	0.46	0.00	87.0	87	0.50	0.00	87*
3C	35	138	138	138	138.0	138	0.30	0.00	138.0	138	0.33	0.01	138*
4A	69	400	400	400	400.0	400	1.36	0.01	400.0	400	1.46	0.01	400*
4B	69	412	412	412	412.0	412	1.19	0.01	412.0	412	1.32	0.01	412*
4C	69	428	428	428	428.0	428	1.28	0.03	428.0	428	1.26	0.02	428*
4D	69	530	530	530	530.1	528	1.12	0.26	529.8	528	1.02	0.06	528*
5A	65	423	423	423	423.0	423	1.33	0.01	423.0	423	1.46	0.01	423*
5B	65	446	446	446	446.0	446	1.11	0.01	446.0	446	1.24	0.01	446*
5C	65	474	474	474	474.0	474	1.06	0.01	474.0	474	1.17	0.01	474*
5D	65	581	579	583	575.8	575	1.14	0.40	575.6	575	1.58	0.64	575*
6A	50	223	223	223	223.0	223	0.74	0.00	223.0	223	0.63	0.00	223*
6B	50	233	233	233	233.0	233	0.65	0.01	233.0	233	0.68	0.00	233*
6C	50	317	317	317	317.0	317	0.54	0.01	317.0	317	0.54	0.01	317*
7A	66	279	279	279	279.0	279	0.84	0.01	279.0	279	0.72	0.00	279*
7B	66	283	283	283	283.0	283	1.01	0.00	283.0	283	1.04	0.00	283*
7C	66	334	334	334	334.0	334	0.86	0.02	334.0	334	0.82	0.02	334*
8A	63	386	386	386	386.0	386	0.96	0.01	386.0	386	0.88	0.00	386*
8B	63	395	395	395	395.0	395	1.15	0.01	395.0	395	1.23	0.01	395*
8C	63	527	521	524	521.0	521	1.16	0.36	521.0	521	1.06	0.19	521*
9A	92	323	323	323	323.0	323	1.65	0.02	323.0	323	1.39	0.05	323*
9B	92	326	326	326	326.0	326	1.87	0.02	326.0	326	1.78	0.03	326*
9C	92	332	332	332	332.0	332	1.76	0.01	332.0	332	1.91	0.04	332*
9D	92	391	391	391	390.8	390	1.43	0.29	390.6	389	1.65	0.19	388*
10A	97	428	428	428	428.0	428	2.45	0.06	428.0	428	2.45	0.03	428*
10B	97	436	436	436	436.0	436	2.36	0.03	436.0	436	2.38	0.08	436*
10C	97	446	446	446	446.0	446	2.14	0.07	446.0	446	2.14	0.06	446*
10D	97	530	526	534	526.3	526	2.03	0.83	526.4	526	2.48	0.82	525*
Gap (%)		0.126%	0.060%	0.142%	0.044%	0.021%			0.041%	0.013%			
T(min)		2.00	—	—			1.10				1.13		
T*(min)		—	1.36	0.11				0.07				0.07	
CPU	P-III 500M	P-II 500M	Xe 2.0G	Xe 3.07G					Xe 3.07G				

Table EC.3 Results for the CARP – BMCV instances

Inst	$ E_R $	BMCV03	BE08	MTY09	ILS				UHGS				BKS	
		Single	Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
C01	79	4150	4150	4150	4154.5	4150	1.33	0.33	4150.0	4150	1.15	0.08	4150*	4150
C02	53	3135	3135	3135	3135.0	3135	0.65	0.01	3135.0	3135	0.65	0.01	3135*	3135
C03	51	2575	2575	2575	2575.0	2575	0.85	0.03	2575.0	2575	0.78	0.09	2575*	2575
C04	72	3510	3510	3510	3510.0	3510	1.04	0.01	3510.0	3510	0.99	0.01	3510*	3510
C05	65	5370	5365	5365	5365.0	5365	0.99	0.08	5365.0	5365	0.99	0.18	5365*	5365
C06	51	2535	2535	2535	2535.0	2535	0.78	0.02	2535.0	2535	0.76	0.02	2535*	2535
C07	52	4075	4075	4075	4075.0	4075	0.61	0.01	4075.0	4075	0.58	0.01	4075*	4075
C08	63	4090	4090	4090	4090.0	4090	0.78	0.02	4090.0	4090	0.71	0.03	4090*	4090
C09	97	5265	5270	5270	5279.0	5260	1.41	0.28	5260.0	5260	1.70	0.43	5260	5260
C10	55	4720	4700	4700	4700.0	4700	0.81	0.05	4700.0	4700	0.69	0.08	4700*	4700
C11	94	4640	4640	4630	4634.0	4630	1.94	0.74	4636.0	4630	1.74	0.36	4630	4630
C12	72	4240	4240	4240	4240.0	4240	0.98	0.02	4240.0	4240	0.95	0.02	4240*	4240
C13	52	2955	2955	2955	2955.0	2955	0.77	0.02	2955.0	2955	0.67	0.02	2955*	2955
C14	57	4030	4030	4030	4030.0	4030	0.80	0.03	4030.0	4030	0.67	0.02	4030*	4030
C15	107	4940	4945	4940	4940.5	4940	2.32	0.57	4940.0	4940	1.88	0.36	4940	4940
C16	32	1475	1475	1475	1475.0	1475	0.45	0.00	1475.0	1475	0.40	0.00	1475*	1475
C17	42	3555	3555	3555	3555.0	3555	0.53	0.02	3555.0	3555	0.48	0.02	3555*	3555
C18	121	5645	5650	5660	5621.0	5605	2.54	0.80	5625.5	5620	3.22	1.37	5620	5605
C19	61	3115	3120	3115	3115.0	3115	0.97	0.02	3115.0	3115	0.90	0.07	3115*	3115
C20	53	2120	2120	2120	2120.0	2120	0.89	0.01	2120.0	2120	0.79	0.01	2120*	2120
C21	76	3970	3970	3970	3970.0	3970	1.27	0.02	3970.0	3970	1.26	0.02	3970*	3970
C22	43	2245	2245	2245	2245.0	2245	0.70	0.00	2245.0	2245	0.71	0.00	2245*	2245
C23	92	4085	4095	4095	4085.0	4085	1.79	0.18	4085.0	4085	1.42	0.14	4085	4085
C24	84	3400	3400	3400	3400.0	3400	1.51	0.10	3400.0	3400	1.33	0.05	3400*	3400
C25	38	2310	2310	2310	2310.0	2310	0.46	0.01	2310.0	2310	0.47	0.01	2310*	2310
D01	79	3215	3230	3230	3215.0	3215	1.95	0.32	3218.0	3215	1.94	0.44	3215*	3215
D02	53	2520	2520	2520	2520.0	2520	0.84	0.00	2520.0	2520	0.91	0.00	2520*	2520
D03	51	2065	2065	2065	2065.0	2065	1.03	0.00	2065.0	2065	0.99	0.00	2065*	2065
D04	72	2785	2785	2785	2785.0	2785	1.44	0.01	2785.0	2785	1.38	0.00	2785*	2785
D05	65	3935	3935	3935	3935.0	3935	1.14	0.01	3935.0	3935	1.21	0.01	3935*	3935
D06	51	2125	2125	2125	2125.0	2125	1.15	0.01	2125.0	2125	1.07	0.00	2125*	2125
D07	52	3115	3115	3115	3115.0	3115	0.92	0.03	3115.0	3115	0.77	0.02	3115*	3115
D08	63	3045	3045	3045	3045.0	3045	1.05	0.02	3045.0	3045	0.89	0.02	3045*	3045
D09	97	4120	4120	4120	4120.0	4120	1.87	0.02	4120.0	4120	1.96	0.03	4120*	4120
D10	55	3340	3340	3340	3340.0	3340	0.81	0.00	3340.0	3340	0.82	0.00	3340*	3340
D11	94	3755	3785	3755	3745.0	3745	2.72	0.59	3745.0	3745	2.24	0.24	3745*	3745
D12	72	3310	3310	3310	3310.0	3310	1.29	0.02	3310.0	3310	1.28	0.01	3310*	3310
D13	52	2535	2540	2535	2535.0	2535	1.09	0.05	2535.0	2535	0.92	0.03	2535*	2535
D14	57	3280	3290	3280	3280.0	3280	1.09	0.01	3280.0	3280	1.04	0.01	3280*	3280
D15	107	3990	4030	4000	3990.0	3990	2.65	0.22	3990.0	3990	2.16	0.17	3990*	3990
D16	32	1060	1060	1060	1060.0	1060	0.55	0.00	1060.0	1060	0.59	0.00	1060*	1060
D17	42	2620	2620	2620	2620.0	2620	0.65	0.00	2620.0	2620	0.66	0.00	2620*	2620
D18	121	4165	4165	4185	4165.0	4165	3.00	0.09	4165.0	4165	2.73	0.10	4165*	4165
D19	61	2400	2410	2400	2400.0	2400	1.33	0.00	2400.0	2400	1.27	0.00	2400*	2400
D20	53	1870	1870	1870	1870.0	1870	1.07	0.00	1870.0	1870	1.08	0.00	1870*	1870
D21	76	3050	3070	3055	3050.0	3050	1.95	0.31	3050.0	3050	1.58	0.07	3050	3050
D22	43	1865	1865	1865	1865.0	1865	1.17	0.00	1865.0	1865	1.22	0.00	1865*	1865
D23	92	3130	3130	3130	3130.0	3130	2.01	0.06	3130.0	3130	1.81	0.04	3130	3130
D24	84	2710	2710	2710	2710.0	2710	1.92	0.02	2710.0	2710	1.79	0.02	2710*	2710
D25	38	1815	1815	1815	1815.0	1815	0.63	0.00	1815.0	1815	0.66	0.00	1815*	1815

Table EC.4 Results for the CARP – BMCV instances (continued)

Inst	E _R	BMCV03	BE08	MTY09	ILS				UHGS				BKS	
		Single	Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
E01	85	4915	4910	4910	4910.0	4910	1.46	0.13	4910.0	4910	1.25	0.11	4910	4910
E02	58	3990	3990	3990	3990.0	3990	0.87	0.03	3990.0	3990	0.77	0.02	3990*	3990
E03	47	2015	2015	2015	2015.0	2015	0.69	0.01	2015.0	2015	0.61	0.01	2015*	2015
E04	77	4155	4160	4155	4155.0	4155	1.44	0.07	4155.0	4155	1.14	0.08	4155*	4155
E05	61	4595	4585	4610	4592.5	4585	1.20	0.56	4585.0	4585	1.19	0.37	4585*	4585
E06	43	2055	2055	2055	2055.0	2055	0.62	0.00	2055.0	2055	0.65	0.00	2055*	2055
E07	50	4155	4155	4155	4155.0	4155	0.65	0.02	4155.0	4155	0.63	0.00	4155*	4155
E08	59	4710	4715	4710	4710.0	4710	0.87	0.05	4710.0	4710	0.74	0.02	4710*	4710
E09	103	5835	5885	5870	5840.0	5810	2.14	0.60	5810.0	5810	1.93	0.41	5820	5810
E10	49	3605	3605	3605	3605.0	3605	0.64	0.01	3605.0	3605	0.54	0.01	3605*	3605
E11	94	4670	4675	4670	4658.0	4650	2.06	0.65	4655.0	4650	2.31	0.89	4650*	4650
E12	67	4195	4215	4200	4185.5	4180	1.23	0.52	4190.0	4180	1.44	0.53	4180*	4180
E13	52	3345	3345	3345	3345.0	3345	0.68	0.02	3345.0	3345	0.64	0.01	3345*	3345
E14	55	4115	4115	4115	4115.0	4115	0.80	0.02	4115.0	4115	0.70	0.02	4115*	4115
E15	107	4225	4225	4225	4214.0	4205	1.96	0.63	4219.0	4205	2.12	0.57	4205*	4205
E16	54	3775	3775	3775	3775.0	3775	0.93	0.04	3775.0	3775	0.77	0.04	3775*	3775
E17	36	2740	2740	2740	2740.0	2740	0.51	0.02	2740.0	2740	0.43	0.01	2740*	2740
E18	88	3835	3835	3835	3835.0	3835	1.59	0.03	3835.0	3835	1.61	0.07	3835	3835
E19	66	3235	3235	3235	3235.0	3235	1.11	0.03	3235.0	3235	0.98	0.02	3235*	3235
E20	63	2825	2825	2825	2825.0	2825	0.96	0.01	2825.0	2825	0.93	0.00	2825*	2825
E21	72	3730	3730	3730	3730.0	3730	1.57	0.15	3730.0	3730	1.26	0.15	3730*	3730
E22	44	2470	2470	2470	2470.0	2470	0.73	0.01	2470.0	2470	0.70	0.01	2470*	2470
E23	89	3710	3725	3710	3710.0	3710	2.01	0.20	3713.0	3710	1.92	0.67	3710	3710
E24	86	4020	4020	4020	4020.0	4020	1.48	0.05	4020.0	4020	1.48	0.06	4020*	4020
E25	28	1615	1615	1615	1615.0	1615	0.34	0.00	1615.0	1615	0.37	0.00	1615*	1615
F01	85	4040	4060	4040	4040.0	4040	2.05	0.07	4040.0	4040	1.88	0.06	4040*	4040
F02	58	3300	3300	3300	3300.0	3300	1.22	0.01	3300.0	3300	1.20	0.01	3300*	3300
F03	47	1665	1665	1665	1665.0	1665	0.84	0.00	1665.0	1665	0.82	0.00	1665*	1665
F04	77	3485	3505	3495	3485.0	3485	1.77	0.33	3485.0	3485	1.52	0.05	3485*	3485
F05	61	3605	3605	3605	3605.0	3605	1.25	0.02	3605.0	3605	1.18	0.01	3605*	3605
F06	43	1875	1875	1875	1875.0	1875	0.82	0.00	1875.0	1875	0.88	0.00	1875*	1875
F07	50	3335	3335	3335	3335.0	3335	1.00	0.02	3335.0	3335	0.92	0.01	3335*	3335
F08	59	3705	3705	3705	3705.0	3705	0.97	0.01	3705.0	3705	0.91	0.00	3705*	3705
F09	103	4730	4755	4730	4730.0	4730	2.33	0.11	4730.0	4730	2.15	0.06	4730*	4730
F10	49	2925	2925	2925	2925.0	2925	0.84	0.00	2925.0	2925	0.83	0.00	2925*	2925
F11	94	3835	3835	3835	3835.0	3835	2.22	0.05	3835.0	3835	2.11	0.07	3835*	3835
F12	67	3395	3395	3395	3395.0	3395	1.24	0.03	3395.0	3395	1.23	0.03	3395*	3395
F13	52	2855	2855	2855	2855.0	2855	0.93	0.00	2855.0	2855	0.91	0.00	2855*	2855
F14	55	3330	3340	3340	3330.0	3330	1.14	0.02	3330.0	3330	1.03	0.02	3330*	3330
F15	107	3560	3605	3560	3560.0	3560	2.53	0.06	3560.0	3560	2.12	0.05	3560*	3560
F16	54	2725	2725	2725	2725.0	2725	1.14	0.00	2725.0	2725	1.09	0.00	2725*	2725
F17	36	2055	2080	2055	2055.0	2055	0.52	0.00	2055.0	2055	0.56	0.00	2055*	2055
F18	88	3075	3075	3075	3075.0	3075	2.04	0.02	3075.0	3075	1.94	0.03	3075	3075
F19	66	2525	2540	2525	2525.0	2525	1.51	0.01	2525.0	2525	1.39	0.01	2525	2525
F20	63	2445	2445	2445	2445.0	2445	1.42	0.04	2445.0	2445	1.31	0.02	2445*	2445
F21	72	2930	2930	2930	2930.0	2930	1.70	0.01	2930.0	2930	1.62	0.01	2930*	2930
F22	44	2075	2075	2075	2075.0	2075	0.91	0.00	2075.0	2075	0.88	0.00	2075*	2075
F23	89	3005	3010	3005	3005.0	3005	2.11	0.22	3005.0	3005	1.66	0.06	3005	3005
F24	86	3210	3245	3240	3210.0	3210	2.24	0.05	3210.0	3210	2.06	0.07	3210*	3210
F25	28	1390	1390	1390	1390.0	1390	0.43	0.00	1390.0	1390	0.43	0.00	1390*	1390
Gap (%)		0.038%	0.158%	0.075%	0.020%	0.000%					0.013%	0.003%		
T(min)		2.57	—	—					1.26					1.19
T*(min)		—	1.08	0.35					0.10					0.09
CPU	P-II 450M	P-M 1.4G	Xe 2.0G	Xe 3.07G									Xe 3.07G	

Table EC.5 Results for the CARP – EGL instances

Inst	E_R	PDHM08		MTY09		UFF13		ILS				UHGS				BKS	
		Avg-10	Single	Avg-15	Best-15	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New		
egl-e1-A	51	3548.0	3548	3548.0	3548	3548.0	3548	0.75	0.00	3548.0	3548	0.76	0.00	3548*	3548		
egl-e1-B	51	4522.2	4498	4508.6	4498	4498.0	4498	1.07	0.18	4498.0	4498	1.13	0.26	4498*	4498		
egl-e1-C	51	5608.0	5595	5615.3	5595	5595.0	5595	0.77	0.03	5595.0	5595	0.78	0.02	5595*	5595		
egl-e2-A	72	5023.8	5018	5018.0	5018	5018.0	5018	1.13	0.02	5018.0	5018	1.26	0.02	5018*	5018		
egl-e2-B	72	6335.4	6317	6330.7	6317	6317.2	6317	1.65	0.55	6321.2	6317	2.25	0.98	6317	6317		
egl-e2-C	72	8355.9	8335	8335.8	8335	8335.0	8335	1.20	0.09	8335.0	8335	1.34	0.18	8335*	8335		
egl-e3-A	87	5898.0	5898	5898.0	5898	5898.0	5898	1.79	0.03	5898.0	5898	1.99	0.10	5898*	5898		
egl-e3-B	87	7806.4	7787	7787.3	7777	7775.6	7775	1.96	0.61	7776.4	7775	1.83	0.44	7775	7775		
egl-e3-C	87	10322.3	10305	10296.5	10292	10292.0	10292	1.84	0.78	10292.0	10292	2.25	0.77	10292	10292		
egl-e4-A	98	6459.4	6461	6461.1	6444	6458.0	6446	2.22	0.17	6444.0	6444	3.68	1.80	6444	6444		
egl-e4-B	98	9016.3	9026	9037.1	9002	8996.8	8987	2.25	1.11	8985.3	8961	3.10	1.47	8961	8961		
egl-e4-C	98	11750.1	11598	11670.0	11626	11563.7	11529	2.66	1.33	11562.8	11529	5.04	3.35	11561	<u>11529</u>		
egl-s1-A	75	5018.0	5018	5038.9	5018	5018.0	5018	1.30	0.02	5018.0	5018	1.38	0.02	5018*	5018		
egl-s1-B	75	6388.0	6394	6388.4	6388	6388.0	6388	1.49	0.27	6388.0	6388	1.38	0.14	6388*	6388		
egl-s1-C	75	8518.2	8518	8521.5	8518	8518.0	8518	1.14	0.06	8518.0	8518	1.20	0.08	8518*	8518		
egl-s2-A	147	9997.9	9970	9980.5	9903	9893.2	9875	4.86	2.16	9886.5	9875	8.70	5.65	9884	<u>9875</u>		
egl-s2-B	147	13176.0	13345	13240.6	13169	13125.6	13095	4.92	2.25	13101.9	13081	7.59	4.54	13100	<u>13057</u>		
egl-s2-C	147	16551.6	16600	16539.9	16442	16451.2	16425	4.59	3.00	16440.2	16425	7.31	4.57	16425*	16425		
egl-s3-A	159	10291.2	10284	10276.1	10221	10243.2	10221	5.21	2.32	10240.0	10221	7.92	4.39	10220	<u>10201</u>		
egl-s3-B	159	13829.2	13857	13860.7	13694	13714.4	13682	5.25	1.76	13693.5	13682	9.98	6.63	13682	13682		
egl-s3-C	159	17327.9	17316	17277.7	17221	17243.2	17196	5.65	2.38	17191.3	17188	8.93	5.85	17188*	17188		
egl-s4-A	190	12440.4	12348	12406.5	12297	12313.3	12257	6.52	3.10	12287.9	12273	14.72	10.46	12268	<u>12216</u>		
egl-s4-B	190	16410.3	16442	16432.0	16333	16303.3	16265	7.71	4.75	16283.9	16230	18.18	14.21	16321	<u>16214</u>		
egl-s4-C	190	20731.5	20821	20660.5	20563	20637.4	20577	14.76	8.20	20591.4	20500	21.51	17.24	20481	<u>20461</u>		
Gap (%)		0.625%	0.555%	0.562%	0.207%	0.209%	0.088%			0.141%	0.049%						
T(min)		30.00	—	13.31	—			3.45			5.59						
T*(min)		8.39	2.10	—	—			1.47			3.46						
CPU		P-IV 3.6G	Xe 2G	I4 3.0G			Xe 3.07G			Xe 3.07G							

Table EC.6 Results for the CARP – EGL-L instances

Table EC.7 Results for the MCGRP – MGGDB instances

Inst	N_R	E_R	A_R	BLMV14	DHDI14	ILS				UHGS				BKS	
				Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
mggdb-0.25-1	6	3	12	280	280	280.0	280	0.13	0.00	280.0	280	0.19	0.00	280*	280
mggdb-0.25-2	6	4	15	359	349	349.0	349	0.24	0.02	349.0	349	0.24	0.01	349*	349
mggdb-0.25-3	7	3	12	286	278	278.0	278	0.14	0.00	278.0	278	0.20	0.00	278*	278
mggdb-0.25-4	4	3	11	289	289	289.0	289	0.11	0.00	289.0	289	0.15	0.00	289*	289
mggdb-0.25-5	5	4	15	410	394	394.0	394	0.20	0.00	394.0	394	0.21	0.00	394*	394
mggdb-0.25-6	6	3	12	295	292	292.0	292	0.15	0.00	292.0	292	0.19	0.00	292*	292
mggdb-0.25-7	5	3	12	302	290	290.0	290	0.12	0.00	290.0	290	0.17	0.00	290*	290
mggdb-0.25-8	11	8	26	351	336	336.0	336	0.56	0.06	336.0	336	0.57	0.02	336	336
mggdb-0.25-9	9	9	29	316	309	309.0	309	0.62	0.06	309.0	309	0.63	0.04	309	309
mggdb-0.25-10	4	4	14	265	265	265.0	265	0.18	0.00	265.0	265	0.19	0.00	265*	265
mggdb-0.25-11	8	8	25	369	356	356.0	356	0.55	0.00	356.0	356	0.70	0.00	356*	356
mggdb-0.25-12	6	3	13	465	459	459.0	459	0.14	0.00	459.0	459	0.20	0.00	459*	459
mggdb-0.25-13	6	5	15	392	388	388.8	388	0.25	0.08	388.0	388	0.27	0.02	388*	388
mggdb-0.25-14	5	3	12	107	107	107.0	107	0.12	0.00	107.0	107	0.15	0.00	107*	107
mggdb-0.25-15	5	3	12	55	55	55.0	55	0.09	0.00	55.0	55	0.12	0.00	55*	55
mggdb-0.25-16	5	5	15	98	98	98.0	98	0.22	0.00	98.0	98	0.23	0.00	98*	98
mggdb-0.25-17	5	5	15	71	71	71.0	71	0.13	0.00	71.0	71	0.15	0.00	71*	71
mggdb-0.25-18	6	6	20	144	144	144.0	144	0.33	0.00	144.0	144	0.42	0.00	144*	144
mggdb-0.25-19	3	1	6	53	53	53.0	53	0.04	0.00	53.0	53	0.08	0.00	53*	53
mggdb-0.25-20	5	3	12	117	116	116.0	116	0.14	0.00	116.0	116	0.20	0.00	116*	116
mggdb-0.25-21	7	6	18	146	146	146.0	146	0.33	0.00	146.0	146	0.41	0.00	146*	146
mggdb-0.25-22	6	8	24	168	160	160.0	160	0.54	0.02	160.0	160	0.61	0.01	160*	160
mggdb-0.25-23	8	9	31	186	181	181.0	181	0.72	0.04	181.0	181	0.77	0.05	181*	181
mggdb-0.30-1	7	3	11	276	273	273.0	273	0.13	0.00	273.0	273	0.18	0.00	273*	273
mggdb-0.30-2	6	4	14	314	301	301.0	301	0.15	0.00	301.0	301	0.19	0.00	301*	301
mggdb-0.30-3	5	3	11	278	270	270.0	270	0.12	0.00	270.0	270	0.16	0.00	270*	270
mggdb-0.30-4	6	2	10	260	260	260.0	260	0.10	0.00	260.0	260	0.15	0.00	260*	260
mggdb-0.30-5	7	4	14	399	388	388.0	388	0.18	0.00	388.0	388	0.23	0.00	388*	388
mggdb-0.30-6	8	3	11	276	276	276.0	276	0.13	0.00	276.0	276	0.19	0.00	276*	276
mggdb-0.30-7	6	3	11	277	273	273.0	273	0.10	0.00	273.0	273	0.16	0.00	273*	273
mggdb-0.30-8	15	7	24	338	331	331.0	331	0.50	0.02	331.0	331	0.60	0.02	331	331
mggdb-0.30-9	11	8	27	284	281	281.0	281	0.63	0.06	281.0	281	0.62	0.06	281*	281
mggdb-0.30-10	5	4	13	242	242	242.0	242	0.16	0.00	242.0	242	0.21	0.00	242*	242
mggdb-0.30-11	13	7	23	399	387	387.0	387	0.62	0.01	387.0	387	0.73	0.01	387*	387
mggdb-0.30-12	6	3	12	472	467	467.0	467	0.13	0.00	467.0	467	0.18	0.00	467*	467
mggdb-0.30-13	6	4	14	483	483	483.0	483	0.20	0.00	483.0	483	0.24	0.00	483*	483
mggdb-0.30-14	3	3	11	101	101	101.0	101	0.10	0.00	101.0	101	0.15	0.00	101*	101
mggdb-0.30-15	5	3	11	44	44	44.0	44	0.09	0.00	44.0	44	0.11	0.00	44*	44
mggdb-0.30-16	6	4	14	107	105	105.0	105	0.24	0.01	105.0	105	0.24	0.01	105*	105
mggdb-0.30-17	4	4	14	67	65	65.0	65	0.11	0.00	65.0	65	0.14	0.00	65*	65
mggdb-0.30-18	6	6	18	144	144	144.0	144	0.25	0.00	144.0	144	0.27	0.00	144*	144
mggdb-0.30-19	3	1	6	51	51	51.0	51	0.04	0.00	51.0	51	0.07	0.00	51*	51
mggdb-0.30-20	4	3	11	97	94	94.0	94	0.11	0.00	94.0	94	0.17	0.00	94*	94
mggdb-0.30-21	6	5	17	122	121	121.0	121	0.26	0.00	121.0	121	0.32	0.00	121*	121
mggdb-0.30-22	7	7	23	156	153	153.0	153	0.44	0.00	153.0	153	0.53	0.00	153*	153
mggdb-0.30-23	9	9	29	171	167	167.0	167	0.73	0.05	167.0	167	0.88	0.25	167*	167

Table EC.8 Results for the MCGRP – MGGDB instances (continued)

Inst	N_R	E_R	A_R	BLMV14	DHDI14	ILS				UHGS				BKS	
				Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
mggdb-0.35-1	7	3	11	252	252	252.0	252	0.12	0.00	252.0	252	0.17	0.00	252*	252
mggdb-0.35-2	6	3	13	284	284	284.0	284	0.13	0.00	284.0	284	0.18	0.00	284*	284
mggdb-0.35-3	6	3	11	243	243	243.0	243	0.11	0.00	243.0	243	0.17	0.00	243*	243
mggdb-0.35-4	6	2	9	242	242	242.0	242	0.10	0.00	242.0	242	0.14	0.00	242*	242
mggdb-0.35-5	7	3	13	317	309	309.0	309	0.15	0.00	309.0	309	0.21	0.00	309*	309
mggdb-0.35-6	7	3	11	262	262	262.0	262	0.12	0.00	262.0	262	0.18	0.00	262*	262
mggdb-0.35-7	8	3	11	272	272	272.0	272	0.12	0.00	272.0	272	0.18	0.00	272*	272
mggdb-0.35-8	9	7	22	321	316	316.0	316	0.51	0.16	316.0	316	0.49	0.04	316	316
mggdb-0.35-9	13	7	25	274	266	266.0	266	0.55	0.02	266.0	266	0.58	0.03	266	266
mggdb-0.35-10	9	3	12	268	268	268.0	268	0.20	0.00	268.0	268	0.27	0.00	268*	268
mggdb-0.35-11	12	7	22	313	303	303.0	303	0.57	0.01	303.0	303	0.67	0.01	303*	303
mggdb-0.35-12	6	3	11	461	461	461.0	461	0.11	0.00	461.0	461	0.15	0.00	461*	461
mggdb-0.35-13	7	4	13	435	417	417.0	417	0.19	0.00	417.0	417	0.23	0.01	417*	417
mggdb-0.35-14	5	3	10	85	84	84.0	84	0.10	0.00	84.0	84	0.16	0.00	84*	84
mggdb-0.35-15	5	3	10	44	44	44.0	44	0.08	0.00	44.0	44	0.11	0.00	44*	44
mggdb-0.35-16	5	4	13	75	75	75.0	75	0.14	0.00	75.0	75	0.23	0.00	75*	75
mggdb-0.35-17	6	4	13	62	62	62.0	62	0.12	0.00	62.0	62	0.15	0.00	62*	62
mggdb-0.35-18	8	5	17	137	135	135.0	135	0.32	0.00	135.0	135	0.34	0.00	135*	135
mggdb-0.35-19	3	1	5	51	51	51.0	51	0.03	0.00	51.0	51	0.06	0.00	51*	51
mggdb-0.35-20	6	3	11	96	96	96.0	96	0.14	0.00	96.0	96	0.18	0.00	96*	96
mggdb-0.35-21	7	5	16	122	120	120.0	120	0.24	0.00	120.0	120	0.32	0.00	120*	120
mggdb-0.35-22	8	7	21	143	139	139.0	139	0.38	0.00	139.0	139	0.49	0.00	139*	139
mggdb-0.35-23	9	8	27	185	179	179.0	179	0.59	0.02	179.0	179	0.66	0.05	179*	179
mggdb-0.40-1	6	3	10	279	279	279.0	279	0.10	0.00	279.0	279	0.16	0.00	279*	279
mggdb-0.40-2	7	3	12	320	308	308.0	308	0.16	0.00	308.0	308	0.20	0.00	308*	308
mggdb-0.40-3	7	3	10	229	225	225.0	225	0.12	0.00	225.0	225	0.17	0.00	225*	225
mggdb-0.40-4	6	2	9	238	238	238.0	238	0.10	0.00	238.0	238	0.15	0.00	238*	238
mggdb-0.40-5	7	3	12	346	344	344.0	344	0.15	0.00	344.0	344	0.19	0.00	344*	344
mggdb-0.40-6	6	3	10	281	270	270.0	270	0.11	0.00	270.0	270	0.16	0.00	270*	270
mggdb-0.40-7	6	3	10	283	282	282.0	282	0.11	0.00	282.0	282	0.16	0.00	282*	282
mggdb-0.40-8	13	6	21	340	331	331.0	331	0.42	0.01	331.0	331	0.49	0.01	331	331
mggdb-0.40-9	15	7	23	285	275	275.0	275	0.50	0.01	275.0	275	0.55	0.01	275	275
mggdb-0.40-10	8	3	11	191	191	191.0	191	0.15	0.00	191.0	191	0.18	0.00	191*	191
mggdb-0.40-11	12	6	20	287	283	283.0	283	0.46	0.00	283.0	283	0.58	0.01	283	283
mggdb-0.40-12	6	3	10	412	412	412.0	412	0.11	0.00	412.0	412	0.16	0.00	412*	412
mggdb-0.40-13	7	4	12	405	406	405.0	405	0.17	0.01	405.0	405	0.22	0.00	405*	405
mggdb-0.40-14	6	3	9	62	62	62.0	62	0.10	0.00	62.0	62	0.15	0.00	62*	62
mggdb-0.40-15	6	3	9	37	37	37.0	37	0.08	0.00	37.0	37	0.11	0.00	37*	37
mggdb-0.40-16	5	4	12	84	84	84.0	84	0.16	0.00	84.0	84	0.22	0.00	84*	84
mggdb-0.40-17	5	4	12	65	65	65.0	65	0.12	0.00	65.0	65	0.15	0.00	65*	65
mggdb-0.40-18	6	5	16	122	119	119.0	119	0.24	0.00	119.0	119	0.31	0.00	119*	119
mggdb-0.40-19	4	1	5	38	38	38.0	38	0.04	0.00	38.0	38	0.05	0.00	38*	38
mggdb-0.40-20	6	3	10	94	94	94.0	94	0.13	0.00	94.0	94	0.20	0.00	94*	94
mggdb-0.40-21	9	4	15	106	104	104.0	104	0.23	0.00	104.0	104	0.29	0.01	104*	104
mggdb-0.40-22	8	6	19	132	129	129.0	129	0.33	0.01	129.0	129	0.40	0.00	129*	129
mggdb-0.40-23	10	7	25	165	160	160.3	160	0.63	0.17	160.4	160	0.80	0.24	160*	160

Table EC.9 Results for the MCGRP – MGGDB instances (end)

Inst	N_R	E_R	A_R	BLMV14	DHDI14	ILS				UHGS				BKS	
				Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
mggdb-0.45-1	6	2	9	259	259	259.0	259	0.09	0.00	259.0	259	0.14	0.00	259*	259
mggdb-0.45-2	7	3	11	302	298	298.0	298	0.13	0.00	298.0	298	0.18	0.00	298*	298
mggdb-0.45-3	8	2	9	245	237	237.0	237	0.11	0.00	237.0	237	0.16	0.00	237*	237
mggdb-0.45-4	7	2	8	228	228	228.0	228	0.09	0.00	228.0	228	0.14	0.00	228*	228
mggdb-0.45-5	7	3	11	357	350	350.0	350	0.14	0.00	350.0	350	0.19	0.00	350*	350
mggdb-0.45-6	7	2	9	225	218	218.0	218	0.10	0.00	218.0	218	0.15	0.00	218*	218
mggdb-0.45-7	9	2	9	243	243	243.0	243	0.09	0.00	243.0	243	0.15	0.00	243*	243
mggdb-0.45-8	16	6	19	312	296	296.0	296	0.44	0.01	296.0	296	0.47	0.01	296*	296
mggdb-0.45-9	14	6	21	287	277	277.0	277	0.51	0.04	277.0	277	0.54	0.05	277*	277
mggdb-0.45-10	9	3	10	214	214	214.0	214	0.17	0.00	214.0	214	0.22	0.00	214*	214
mggdb-0.45-11	15	6	18	310	297	297.0	297	0.46	0.00	297.0	297	0.57	0.01	297	297
mggdb-0.45-12	10	2	9	406	393	393.0	393	0.13	0.00	393.0	393	0.19	0.00	393*	393
mggdb-0.45-13	7	3	11	423	423	423.0	423	0.22	0.01	423.0	423	0.20	0.00	423*	423
mggdb-0.45-14	6	2	8	67	66	66.0	66	0.08	0.00	66.0	66	0.13	0.00	66*	66
mggdb-0.45-15	6	2	8	36	34	34.0	34	0.07	0.00	34.0	34	0.09	0.00	34*	34
mggdb-0.45-16	6	3	11	70	70	70.0	70	0.12	0.00	70.0	70	0.17	0.00	70*	70
mggdb-0.45-17	7	3	11	53	53	53.0	53	0.09	0.00	53.0	53	0.12	0.00	53*	53
mggdb-0.45-18	7	4	14	123	123	123.0	123	0.20	0.00	123.0	123	0.26	0.00	123	123
mggdb-0.45-19	3	1	4	48	48	48.0	48	0.03	0.00	48.0	48	0.06	0.00	48*	48
mggdb-0.45-20	5	2	9	78	78	78.0	78	0.11	0.00	78.0	78	0.15	0.00	78*	78
mggdb-0.45-21	7	4	13	128	122	122.0	122	0.20	0.00	122.0	122	0.25	0.00	122*	122
mggdb-0.45-22	9	6	18	139	136	136.0	136	0.32	0.00	136.0	136	0.41	0.00	136*	136
mggdb-0.45-23	9	7	23	147	145	144.6	144	0.52	0.11	144.8	144	0.69	0.19	144*	144
mggdb-0.50-1	8	2	8	214	214	214.0	214	0.09	0.00	214.0	214	0.14	0.00	214*	214
mggdb-0.50-2	6	3	10	281	269	269.0	269	0.11	0.00	269.0	269	0.15	0.00	269*	269
mggdb-0.50-3	9	2	8	218	218	218.0	218	0.10	0.00	218.0	218	0.15	0.00	218*	218
mggdb-0.50-4	6	2	7	219	219	219.0	219	0.07	0.00	219.0	219	0.11	0.00	219*	219
mggdb-0.50-5	7	3	10	292	292	292.0	292	0.11	0.00	292.0	292	0.16	0.00	292*	292
mggdb-0.50-6	7	2	8	276	276	276.0	276	0.09	0.00	276.0	276	0.14	0.00	276*	276
mggdb-0.50-7	9	2	8	274	265	265.0	265	0.11	0.00	265.0	265	0.16	0.00	265*	265
mggdb-0.50-8	15	5	17	310	310	310.0	310	0.35	0.01	310.0	310	0.39	0.01	310	310
mggdb-0.50-9	16	6	19	270	265	265.0	265	0.42	0.00	265.0	265	0.48	0.01	265	265
mggdb-0.50-10	7	3	9	194	194	194.0	194	0.12	0.00	194.0	194	0.17	0.00	194*	194
mggdb-0.50-11	16	5	17	278	275	275.0	275	0.53	0.02	275.0	275	0.53	0.02	275	275
mggdb-0.50-12	8	2	9	445	445	445.0	445	0.11	0.00	445.0	445	0.16	0.00	445*	445
mggdb-0.50-13	8	3	10	259	261	259.0	259	0.16	0.00	259.0	259	0.20	0.00	259*	259
mggdb-0.50-14	6	2	8	76	75	75.0	75	0.09	0.00	75.0	75	0.14	0.00	75*	75
mggdb-0.50-15	5	2	8	37	37	37.0	37	0.06	0.00	37.0	37	0.09	0.00	37*	37
mggdb-0.50-16	6	3	10	66	66	66.0	66	0.11	0.00	66.0	66	0.16	0.00	66*	66
mggdb-0.50-17	7	3	10	53	53	53.0	53	0.11	0.00	53.0	53	0.13	0.00	53*	53
mggdb-0.50-18	8	4	13	122	121	121.0	121	0.21	0.00	121.0	121	0.26	0.00	121	121
mggdb-0.50-19	3	1	4	44	44	44.0	44	0.03	0.00	44.0	44	0.05	0.00	44*	44
mggdb-0.50-20	5	2	8	81	81	81.0	81	0.08	0.00	81.0	81	0.11	0.00	81*	81
mggdb-0.50-21	8	4	12	88	86	86.0	86	0.17	0.00	86.0	86	0.21	0.00	86*	86
mggdb-0.50-22	10	5	16	127	123	123.0	123	0.33	0.02	123.0	123	0.34	0.01	123*	123
mggdb-0.50-23	7	6	21	125	126	125.0	125	0.45	0.06	125.0	125	0.52	0.12	125*	125
Gap (%)				1.342%	0.018%	0.006%	0.000%					0.006%	0.000%		
T(min)				0.31	60.00					0.22					0.27
T*(min)				—	0.86					0.01					0.01
CPU				Xe 3.0G	CPU 3G					Xe 3.07G					Xe 3.07G

Table EC.10 Results for the MCGRP – MGVAL instances

Inst	N_R	E_R	A_R	BLMV14		DHDI14		ILS				UHGS				BKS	
				Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New		
mgval-0.25-1A	13	15	26	177	177	177.0	177	0.57	0.01	177.0	177	0.57	0.01	177*	177		
mgval-0.25-1B	10	9	28	217	217	217.0	217	0.85	0.02	217.0	217	0.72	0.02	217*	217		
mgval-0.25-1C	12	12	27	335	279	279.9	279	0.76	0.26	279.0	279	0.84	0.19	279	279		
mgval-0.25-2A	7	12	21	259	259	259.0	259	0.76	0.01	259.0	259	0.98	0.01	259*	259		
mgval-0.25-2B	9	9	30	336	336	336.0	336	0.72	0.00	336.0	336	0.92	0.00	336*	336		
mgval-0.25-2C	12	10	26	528	480	480.0	480	0.56	0.02	480.0	480	0.66	0.04	480	480		
mgval-0.25-3A	9	11	24	89	89	89.0	89	0.55	0.00	89.0	89	0.55	0.01	89*	89		
mgval-0.25-3B	8	12	21	125	125	125.0	125	0.57	0.00	125.0	125	0.78	0.00	125*	125		
mgval-0.25-3C	10	13	18	161	153	153.0	153	0.42	0.00	153.0	153	0.50	0.01	153*	153		
mgval-0.25-4A	19	19	51	514	514	514.0	514	1.85	0.05	514.0	514	2.40	0.05	514*	514		
mgval-0.25-4B	20	14	62	541	537	537.0	537	1.74	0.02	537.0	537	2.36	0.07	537*	537		
mgval-0.25-4C	24	15	61	549	525	525.0	525	1.90	0.04	525.0	525	2.38	0.07	525*	525		
mgval-0.25-4D	19	15	62	724	683	675.0	675	1.81	0.25	675.0	675	2.17	0.57	683*	675		
mgval-0.25-5A	21	16	55	485	485	485.0	485	1.54	0.03	485.0	485	1.81	0.06	485*	485		
mgval-0.25-5B	18	26	42	500	493	493.0	493	1.65	0.05	493.0	493	2.04	0.08	493*	493		
mgval-0.25-5C	21	12	60	599	584	584.0	584	1.60	0.03	584.0	584	2.11	0.07	584*	584		
mgval-0.25-5D	17	21	47	681	644	644.0	644	1.39	0.39	643.8	642	1.80	0.33	644*	642		
mgval-0.25-6A	16	16	35	274	274	274.0	274	0.93	0.02	274.0	274	0.91	0.03	274*	274		
mgval-0.25-6B	14	16	33	263	263	263.0	263	0.96	0.02	263.0	263	1.12	0.03	263*	263		
mgval-0.25-6C	16	17	33	337	324	324.0	324	0.76	0.03	324.0	324	0.87	0.03	324	324		
mgval-0.25-7A	20	27	37	297	297	297.0	297	1.33	0.05	297.0	297	1.31	0.06	297*	297		
mgval-0.25-7B	18	18	49	355	355	355.0	355	1.63	0.04	355.0	355	1.66	0.04	355*	355		
mgval-0.25-7C	18	21	46	407	378	378.0	378	1.31	0.09	378.0	378	1.40	0.05	378	378		
mgval-0.25-8A	16	15	57	510	510	510.0	510	2.07	0.07	510.0	510	1.90	0.07	510*	510		
mgval-0.25-8B	16	20	48	423	423	423.0	423	1.48	0.03	423.0	423	1.97	0.04	423*	423		
mgval-0.25-8C	16	21	41	591	545	544.9	544	1.30	0.17	544.7	544	1.61	0.32	545	544		
mgval-0.25-9A	23	24	75	371	371	371.0	371	2.14	0.09	371.0	371	2.14	0.12	371*	371		
mgval-0.25-9B	22	33	57	363	358	358.0	358	2.43	0.06	358.0	358	2.72	0.11	358*	358		
mgval-0.25-9C	26	31	62	369	365	364.1	364	3.22	0.52	364.0	364	3.25	0.40	365	364		
mgval-0.25-9D	24	28	69	478	429	426.7	425	2.52	0.93	426.9	425	3.57	1.33	429	424		
mgval-0.25-10A	26	24	79	492	492	492.0	492	3.26	0.12	492.0	492	3.58	0.29	492*	492		
mgval-0.25-10B	24	24	75	528	528	528.0	528	3.18	0.12	528.0	528	3.47	0.23	528*	528		
mgval-0.25-10C	23	27	75	501	483	483.0	483	3.06	0.34	483.0	483	3.49	0.29	483	483		
mgval-0.25-10D	23	31	65	616	567	566.7	566	2.24	0.95	567.4	566	3.20	0.92	567*	566		
mgval-0.30-1A	15	14	24	170	170	170.0	170	0.55	0.01	170.0	170	0.52	0.01	170*	170		
mgval-0.30-1B	12	9	26	194	194	194.0	194	0.80	0.02	194.0	194	0.89	0.02	194*	194		
mgval-0.30-1C	12	11	25	280	270	270.0	270	0.61	0.09	270.0	270	0.61	0.03	270	270		
mgval-0.30-2A	12	11	19	233	233	233.0	233	0.75	0.00	233.0	233	0.98	0.00	233*	233		
mgval-0.30-2B	13	8	28	347	347	347.0	347	0.93	0.06	347.0	347	1.06	0.11	347*	347		
mgval-0.30-2C	12	9	24	542	495	495.0	495	0.60	0.10	495.0	495	0.61	0.07	495	495		
mgval-0.30-3A	13	10	23	105	105	105.0	105	0.60	0.00	105.0	105	0.61	0.00	105*	105		
mgval-0.30-3B	10	11	20	115	115	115.0	115	0.53	0.00	115.0	115	0.72	0.00	115*	115		
mgval-0.30-3C	12	12	17	156	153	153.0	153	0.44	0.01	153.0	153	0.55	0.04	153	153		
mgval-0.30-4A	21	18	48	477	477	477.0	477	2.06	0.10	477.0	477	2.29	0.06	477*	477		
mgval-0.30-4B	27	13	58	537	533	533.0	533	2.29	0.29	533.0	533	2.71	0.37	533	533		
mgval-0.30-4C	27	14	57	513	500	499.2	498	1.88	0.26	499.2	498	2.50	0.32	500	498		
mgval-0.30-4D	22	14	58	718	653	653.0	653	1.43	0.44	653.0	653	1.85	0.26	653	653		
mgval-0.30-5A	20	15	51	445	445	445.0	445	1.64	0.02	445.0	445	1.83	0.05	445*	445		
mgval-0.30-5B	20	24	39	492	490	490.0	490	2.04	0.19	490.0	490	2.13	0.24	490	490		
mgval-0.30-5C	20	11	56	568	553	553.0	553	1.50	0.03	553.0	553	1.88	0.07	553	551		
mgval-0.30-5D	22	20	44	675	621	618.3	618	1.65	0.75	616.9	616	2.55	1.09	621	616		
mgval-0.30-6A	19	15	32	252	252	252.0	252	1.06	0.01	252.0	252	1.04	0.01	252*	252		
mgval-0.30-6B	19	15	30	268	262	262.0	262	1.30	0.24	262.0	262	1.18	0.05	262*	262		
mgval-0.30-6C	17	16	31	339	320	317.6	317	0.95	0.28	317.0	317	1.07	0.20	320	317		
mgval-0.30-7A	17	25	35	324	324	324.0	324	1.20	0.03	324.0	324	1.14	0.04	324*	324		
mgval-0.30-7B	19	17	46	344	344	344.0	344	1.64	0.03	344.0	344	1.73	0.05	344*	344		
mgval-0.30-7C	23	19	43	380	354	354.0	354	1.09	0.02	354.0	354	1.24	0.04	354	354		
mgval-0.30-8A	21	14	53	431	431	431.0	431	1.81	0.03	431.0	431	2.00	0.05	431*	431		
mgval-0.30-8B	21	18	44	408	400	400.0	400	1.81	0.05	400.0	400	1.78	0.07	400*	400		
mgval-0.30-8C	18	19	38	570	522	522.0	522	1.13	0.22	522.0	522	1.26	0.11	522	522		
mgval-0.30-9A	26	22	70	357	357	357.0	357	2.26	0.10	357.0	357	2.16	0.11	357*	357		</

Table EC.11 Results for the MCGRP – MGVAL instances (continued)

Inst	N_R	E_R	A_R	BLMV14		DHDI14		ILS			UHGS			BKS	
				Single		Single		Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*
mgval-0.35-1A	12	13	22	158	158	158.0	158	0.47	0.00	158.0	158	0.46	0.00	158*	158
mgval-0.35-1B	16	8	24	192	192	192.0	192	0.81	0.02	192.0	192	0.70	0.02	192*	192
mgval-0.35-1C	14	11	23	284	290	283.5	283	0.66	0.30	283.0	283	0.80	0.20	284	<u>283</u>
mgval-0.35-2A	12	10	18	286	286	286.0	286	0.52	0.01	286.0	286	0.49	0.00	286*	286
mgval-0.35-2B	13	7	26	326	326	326.0	326	0.74	0.01	326.0	326	0.88	0.01	326*	326
mgval-0.35-2C	14	9	22	523	485	485.0	485	0.52	0.06	485.0	485	0.55	0.03	485	485
mgval-0.35-3A	13	9	21	84	84	84.0	84	0.59	0.00	84.0	84	0.61	0.01	84*	84
mgval-0.35-3B	13	10	18	113	113	113.0	113	0.53	0.00	113.0	113	0.74	0.00	113*	113
mgval-0.35-3C	13	11	16	159	150	150.0	150	0.37	0.00	150.0	150	0.47	0.01	150*	150
mgval-0.35-4A	24	16	44	430	430	430.0	430	1.66	0.02	430.0	430	2.18	0.03	430*	430
mgval-0.35-4B	25	12	53	531	531	531.0	531	1.59	0.02	531.0	531	2.11	0.06	531	531
mgval-0.35-4C	27	13	53	553	516	516.0	516	2.24	0.17	516.0	516	2.34	0.32	516*	516
mgval-0.35-4D	30	13	53	661	643	643.0	643	1.54	0.43	643.0	643	1.96	0.31	643	643
mgval-0.35-5A	20	14	48	454	454	454.4	454	1.78	0.49	454.3	454	2.77	0.71	454*	454
mgval-0.35-5B	23	22	36	468	467	467.4	467	2.09	0.51	467.7	467	2.39	0.57	467	467
mgval-0.35-5C	19	11	52	595	586	586.0	586	1.79	0.60	586.1	586	2.31	0.62	586	586
mgval-0.35-5D	22	18	40	648	578	576.5	576	1.66	0.69	576.0	576	1.93	0.67	578	<u>576</u>
mgval-0.35-6A	20	14	30	248	248	248.0	248	1.08	0.01	248.0	248	1.22	0.02	248*	248
mgval-0.35-6B	20	14	28	250	250	250.0	250	0.95	0.01	250.0	250	1.10	0.02	250*	250
mgval-0.35-6C	17	14	29	326	312	312.0	312	0.76	0.07	312.0	312	0.76	0.05	312	312
mgval-0.35-7A	23	23	32	264	264	264.0	264	1.02	0.03	264.0	264	1.01	0.03	264*	264
mgval-0.35-7B	21	16	42	325	325	325.0	325	1.44	0.02	325.0	325	1.61	0.03	325*	325
mgval-0.35-7C	24	18	40	351	336	336.0	336	1.26	0.10	336.0	336	1.31	0.10	336*	336
mgval-0.35-8A	22	13	49	415	415	415.0	415	1.64	0.11	415.0	415	1.51	0.06	415*	415
mgval-0.35-8B	20	17	41	385	385	385.0	385	1.44	0.03	385.0	385	1.77	0.03	385*	385
mgval-0.35-8C	22	18	35	547	494	490.7	489	1.18	0.45	489.0	489	1.34	0.25	494	<u>489</u>
mgval-0.35-9A	31	20	65	324	324	324.0	324	2.29	0.08	324.0	324	2.21	0.11	324*	324
mgval-0.35-9B	29	28	49	332	331	331.0	331	2.93	0.27	331.0	331	2.63	0.17	331*	331
mgval-0.35-9C	35	27	53	338	328	328.0	328	2.38	0.36	328.0	328	3.17	0.55	328*	328
mgval-0.35-9D	31	24	60	473	430	427.2	426	2.46	0.82	429.0	426	3.22	1.14	430	<u>426</u>
mgval-0.35-10A	34	20	68	475	475	475.0	475	3.43	0.12	475.0	475	3.47	0.15	475*	475
mgval-0.35-10B	32	21	65	463	461	461.0	461	2.54	0.07	461.0	461	3.06	0.11	461*	461
mgval-0.35-10C	34	23	65	448	430	430.0	430	3.01	0.38	430.0	430	3.59	0.72	430	430
mgval-0.35-10D	31	27	56	566	523	523.0	523	2.13	0.24	523.4	523	2.79	0.70	523	523
mgval-0.40-1A	15	12	21	165	165	165.0	165	0.53	0.01	165.0	165	0.48	0.01	165*	165
mgval-0.40-1B	14	7	22	196	196	196.0	196	0.68	0.01	196.0	196	0.73	0.01	196*	196
mgval-0.40-1C	15	10	21	272	263	263.0	263	0.67	0.15	263.2	263	0.78	0.23	263	263
mgval-0.40-2A	13	9	16	222	222	222.0	222	0.56	0.00	222.0	222	0.79	0.00	222*	222
mgval-0.40-2B	18	7	24	311	311	311.0	311	0.67	0.00	311.0	311	0.89	0.00	311*	311
mgval-0.40-2C	14	8	21	485	469	469.0	469	0.44	0.01	469.0	469	0.54	0.01	469	469
mgval-0.40-3A	13	9	19	86	86	86.0	86	0.61	0.00	86.0	86	0.56	0.00	86*	86
mgval-0.40-3B	14	9	17	110	110	110.0	110	0.51	0.01	110.0	110	0.64	0.01	110*	110
mgval-0.40-3C	13	10	15	157	148	148.0	148	0.35	0.01	148.0	148	0.43	0.01	148	148
mgval-0.40-4A	26	15	41	400	400	400.0	400	1.80	0.05	400.0	400	2.18	0.07	400*	400
mgval-0.40-4B	29	11	49	423	423	423.0	423	1.51	0.03	423.0	423	2.02	0.05	423	423
mgval-0.40-4C	28	12	49	487	462	462.0	462	1.62	0.05	462.0	462	1.94	0.07	462	462
mgval-0.40-4D	27	12	49	669	622	620.0	620	1.79	0.75	622.4	620	2.61	1.17	622	<u>620</u>
mgval-0.40-5A	25	13	44	426	426	426.0	426	1.45	0.02	426.0	426	1.71	0.04	426*	426
mgval-0.40-5B	23	21	33	428	424	424.0	424	1.43	0.03	424.0	424	1.75	0.05	424	424
mgval-0.40-5C	26	10	48	539	527	524.3	524	1.61	0.52	525.9	524	2.25	0.53	527	<u>524</u>
mgval-0.40-5D	25	17	37	665	608	605.3	604	1.43	0.73	603.8	602	2.68	1.41	608	<u>602</u>
mgval-0.40-6A	20	13	28	224	224	224.0	224	1.11	0.02	224.0	224	0.94	0.03	224*	224
mgval-0.40-6B	19	13	26	211	211	211.0	211	0.82	0.02	211.0	211	0.98	0.03	211*	211
mgval-0.40-6C	22	13	27	316	312	312.0	312	0.68	0.01	312.0	312	0.81	0.01	312	312
mgval-0.40-7A	25	21	30	271	271	271.0	271	0.95	0.02	271.0	271	0.94	0.02	271*	271
mgval-0.40-7B	23	15	39	270	270	270.0	270	1.34	0.03	270.0	270	1.47	0.03	270*	270
mgval-0.40-7C	27	16	37	336	332	330.0	330	1.34	0.37	330.4	330	1.73	0.54	332	330
mgval-0.40-8A	23	12	45	393	393	393.0	393	1.70	0.04	393.0	393	1.83	0.07	393*	393
mgval-0.40-8B	23	16	38	372	371	371.0	371	1.67	0.06	371.0	371	1.75	0.06	371	371
mgval-0.40-8C	23	16	33	573	517	517.0	517	1.09	0.15	517.0	517	1.27	0.15	517	517
mgval-0.40-9A	35	19	60	341	341	341.0	341	1.96	0.06	341.0	341	1.96	0.11	341	341
mgval-0.40-9B	34	26	45	331	327	327.0	327	2.17	0.05	327.0	327	2.56	0.08	327	327
mgval-0.40-9C	30	25	49	301	295	295.0	295	2.34	0.17	295.0	295	2.46	0.29	295	295
mgval-0.40-9D	39	22	55	414	382	382.0	382	2.12	0.64	382.4	382	3.09	1.10	382	

Table EC.12 Results for the MCGRP – MGVAL instances (end)

Inst	N_R	E_R	A_R	BLMV14	DHDI14	ILS			UHGS			BKS			
				Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
mgval-0.45-1A	17	11	19	168	168	168.0	168	0.52	0.01	168.0	168	0.47	0.01	168*	168
mgval-0.45-1B	14	7	20	166	166	166.0	166	0.65	0.02	166.0	166	0.73	0.02	166*	166
mgval-0.45-1C	16	9	19	313	258	258.0	258	0.54	0.04	258.0	258	0.52	0.02	258	258
mgval-0.45-2A	15	8	15	251	251	251.0	251	0.56	0.00	251.0	251	0.58	0.00	251*	251
mgval-0.45-2B	18	6	22	314	314	314.0	314	0.83	0.03	314.0	314	0.91	0.05	314*	314
mgval-0.45-2C	18	7	19	496	462	462.0	462	0.50	0.05	462.0	462	0.51	0.03	462	462
mgval-0.45-3A	15	8	18	82	82	82.0	82	0.60	0.00	82.0	82	0.82	0.00	82*	82
mgval-0.45-3B	16	8	15	91	91	91.0	91	0.50	0.01	91.0	91	0.64	0.01	91*	91
mgval-0.45-3C	16	9	13	143	143	143.0	143	0.33	0.00	143.0	143	0.44	0.00	143	143
mgval-0.45-4A	29	14	37	381	381	381.0	381	2.06	0.10	381.0	381	2.16	0.09	381*	381
mgval-0.45-4B	36	10	45	481	471	471.0	471	1.69	0.04	471.0	471	2.12	0.06	471	471
mgval-0.45-4C	29	11	45	508	480	480.0	480	1.68	0.07	480.0	480	1.89	0.16	480	480
mgval-0.45-4D	31	11	45	626	577	577.0	577	1.46	0.10	577.0	577	1.47	0.08	577	577
mgval-0.45-5A	26	12	40	391	392	391.7	391	1.70	0.32	391.4	391	2.45	0.58	391	391
mgval-0.45-5B	26	19	30	416	416	415.8	414	1.43	0.14	415.8	414	1.62	0.05	416	414
mgval-0.45-5C	26	9	44	502	492	492.0	492	1.63	0.07	492.0	492	1.77	0.19	492	492
mgval-0.45-5D	27	15	34	612	552	552.0	552	1.28	0.29	552.0	552	1.34	0.12	552	552
mgval-0.45-6A	23	12	25	213	213	213.0	213	0.79	0.01	213.0	213	0.94	0.01	213*	213
mgval-0.45-6B	22	12	24	210	210	210.0	210	0.80	0.01	210.0	210	1.00	0.01	210*	210
mgval-0.45-6C	22	12	24	308	296	296.0	296	0.66	0.01	296.0	296	0.70	0.02	296	296
mgval-0.45-7A	27	19	27	261	261	261.0	261	1.19	0.04	261.0	261	1.08	0.03	261*	261
mgval-0.45-7B	28	13	36	294	294	294.0	294	1.40	0.05	294.0	294	1.62	0.05	294	294
mgval-0.45-7C	27	15	34	347	336	336.0	336	1.15	0.10	336.0	336	1.20	0.15	336	336
mgval-0.45-8A	24	11	41	370	370	370.0	370	1.22	0.02	370.0	370	1.38	0.05	370	370
mgval-0.45-8B	23	14	35	376	360	360.0	360	1.63	0.10	360.0	360	1.64	0.08	360	360
mgval-0.45-8C	20	15	30	535	498	498.4	496	1.05	0.59	497.3	496	1.45	0.53	498	496
mgval-0.45-9A	37	17	55	306	306	306.0	306	2.53	0.07	306.0	306	2.45	0.10	306	306
mgval-0.45-9B	35	24	41	323	323	323.0	323	2.28	0.11	323.0	323	2.43	0.11	323	323
mgval-0.45-9C	34	23	45	306	291	291.0	291	2.08	0.17	291.0	291	2.24	0.13	291	291
mgval-0.45-9D	37	20	51	403	387	386.8	386	1.86	0.40	386.9	386	2.15	0.30	387	386
mgval-0.45-10A	40	17	58	388	388	388.0	388	2.55	0.05	388.0	388	3.09	0.11	388	388
mgval-0.45-10B	35	18	55	399	399	399.0	399	2.10	0.04	399.0	399	2.63	0.08	399	399
mgval-0.45-10C	37	19	55	418	403	402.4	401	2.33	0.41	402.8	401	2.79	0.23	403	401
mgval-0.45-10D	35	23	47	504	487	487.5	486	1.89	0.92	487.6	487	2.70	0.89	487*	486
mgval-0.50-1A	16	10	17	145	145	145.0	145	0.60	0.00	145.0	145	0.65	0.00	145*	145
mgval-0.50-1B	17	6	19	170	170	170.0	170	0.70	0.01	170.0	170	0.81	0.01	170*	170
mgval-0.50-1C	14	8	18	261	255	265.0	265	0.56	0.10	265.0	265	0.52	0.05	261	261
mgval-0.50-2A	16	8	14	248	248	248.0	248	0.63	0.00	248.0	248	0.84	0.00	248*	248
mgval-0.50-2B	18	6	20	284	284	284.0	284	0.66	0.01	284.0	284	0.80	0.00	284*	284
mgval-0.50-2C	16	7	17	488	464	464.0	464	0.41	0.01	464.0	464	0.45	0.02	464	464
mgval-0.50-3A	17	7	16	75	75	75.0	75	0.64	0.00	75.0	75	0.79	0.01	75*	75
mgval-0.50-3B	15	8	14	107	107	107.0	107	0.39	0.00	107.0	107	0.57	0.00	107*	107
mgval-0.50-3C	15	9	12	139	137	137.0	137	0.31	0.00	137.0	137	0.39	0.00	137*	137
mgval-0.50-4A	31	13	34	350	350	350.0	350	1.62	0.03	350.0	350	1.85	0.04	350*	350
mgval-0.50-4B	32	9	41	419	413	413.0	413	1.83	0.10	413.0	413	1.85	0.07	413	413
mgval-0.50-4C	32	10	41	512	488	487.7	487	1.46	0.32	487.7	487	2.04	0.33	488	487
mgval-0.50-4D	32	10	41	613	580	582.8	580	1.45	0.65	580.0	580	2.26	0.94	580	580
mgval-0.50-5A	27	11	37	367	367	367.0	367	1.48	0.03	367.0	367	1.69	0.05	367*	367
mgval-0.50-5B	28	17	28	378	378	378.0	378	1.36	0.03	378.0	378	1.60	0.05	378	378
mgval-0.50-5C	26	8	40	476	457	457.0	457	1.59	0.13	457.6	457	1.83	0.38	457	457
mgval-0.50-5D	27	14	31	570	541	541.8	539	1.13	0.39	539.8	539	1.80	0.68	541	539
mgval-0.50-6A	19	11	23	210	210	210.0	210	0.82	0.01	210.0	210	0.98	0.01	210*	210
mgval-0.50-6B	23	11	22	210	210	210.0	210	0.86	0.02	210.0	210	0.91	0.01	210*	210
mgval-0.50-6C	22	11	22	306	293	293.0	293	0.64	0.03	293.0	293	0.69	0.03	293	293
mgval-0.50-7A	31	18	25	248	248	248.0	248	1.40	0.03	248.0	248	1.21	0.04	248*	248
mgval-0.50-7B	26	12	33	276	276	276.0	276	1.05	0.02	276.0	276	1.19	0.04	276*	276
mgval-0.50-7C	29	14	31	333	320	320.0	320	1.03	0.06	320.0	320	1.07	0.05	320	320
mgval-0.50-8A	26	10	38	388	388	388.0	388	1.66	0.05	388.0	388	1.72	0.06	388	388
mgval-0.50-8B	23	13	32	356	350	350.0	350	1.32	0.04	350.0	350	1.46	0.04	350	350
mgval-0.50-8C	22	14	27	535	501	501.0	501	0.79	0.03	501.0	501	0.98	0.04	501	501
mgval-0.50-9A	39	16	50	306	306	306.0	306	2.30	0.08	306.0	306	2.03	0.12	306*	306
mgval-0.50-9B	37	22	38	278	278	278.0	278	2.04	0.07	278.0	278	2.23	0.08	278	278
mgval-0.50-9C	38	21	41	303	292	292.0	292	1.96	0.27	292.0	292	2.23	0.14	292	292
mgval-0.50-9D	38	19	46	408	358	358.0	358	1.87	0.54	358.0	358	2.26	0.51	358	358
mgval-0.50-10A	40	16	5												

Table EC.13 Results for the MCGRP – CBMix instances

Inst	N_R	E_R	A_R	HKSG12		BLMV14		DHDI14		ILS				UHGS				BKS	
				Best-2	Single	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New			
CBMix1	11	0	37	2589	2587	2585	2585.4	2585	0.72	0.06	2570.6	2569	1.05	0.34	2585	<u>2569</u>			
CBMix2	36	0	149	12222	12241	11749	11708.5	11643	8.76	4.74	11663.7	11610	16.17	12.19	11749	<u>11610</u>			
CBMix3	16	8	55	3643	3643	3614	3612.6	3604	1.40	0.40	3614.7	3612	1.99	0.69	3614	<u>3590</u>			
CBMix4	10	75	13	7802	7583	7483	7459.5	7431	2.65	1.15	7437.0	7429	3.80	2.15	7483	<u>7429</u>			
CBMix5	23	4	38	4531	4531	4459	4459.0	4459	0.91	0.20	4454.4	4436	1.27	0.26	4459	<u>4436</u>			
CBMix6	40	4	64	7087	6968	6969	6846.9	6813	2.01	1.01	6816.9	6813	3.46	1.38	6968	<u>6813</u>			
CBMix7	54	8	106	9607	9859	9428	9396.6	9341	5.39	2.79	9400.2	9303	8.66	4.88	9428	<u>9260</u>			
CBMix8	63	6	108	10669	10658	10338	10292.7	10257	6.81	3.24	10270.1	10162	15.42	11.62	10338	<u>10148</u>			
CBMix9	6	39	5	4130	4060	3991	3981.4	3965	0.80	0.24	3973.0	3965	1.35	0.62	3991	<u>3965</u>			
CBMix10	4	94	9	7794	7755	7525	7486.7	7463	2.96	2.06	7462.0	7462	3.33	1.46	7525	<u>7462</u>			
CBMix11	65	6	11	4525	4561	4484	4480.5	4466	1.47	0.36	4470.9	4456	2.61	1.22	4484	<u>4456</u>			
CBMix12	1	0	52	3235	3138	3138	3138.0	3138	0.71	0.02	3138.0	3138	0.88	0.03	3138	3138			
CBMix13	79	2	60	9135	9110	9037	8985.1	8947	4.20	1.94	8970.3	8937	6.76	3.97	9037	<u>8934</u>			
CBMix14	93	0	0	8579	8671	8473	8442.4	8420	2.25	1.12	8448.2	8438	2.64	1.08	8473	<u>8413</u>			
CBMix15	0	91	0	8371	8359	8221	8206.3	8179	1.70	0.71	8177.6	8164	2.46	1.06	8221	<u>8164</u>			
CBMix16	36	0	133	9022	8933	8742	8627.7	8605	4.68	2.23	8607.3	8605	7.46	3.98	8742	<u>8605</u>			
CBMix17	16	16	31	4097	4037	4034	4034.0	4034	0.68	0.03	4034.0	4034	0.81	0.02	4034	4034			
CBMix18	39	0	88	7133	7254	7052	7011.6	6979	2.94	1.73	6997.8	6979	4.21	1.89	7052	<u>6962</u>			
CBMix19	61	9	142	16692	16554	16155	16112.1	16072	12.01	6.93	15982.7	15895	17.58	12.88	16155	<u>15895</u>			
CBMix20	38	2	33	4859	4885	4738	4758.8	4738	1.35	0.70	4752.8	4738	2.10	0.93	4738	4738			
CBMix21	55	68	57	18809	18509	17875	17815.7	17739	6.89	3.67	17749.0	17638	12.07	8.50	17875	<u>17597</u>			
CBMix22	7	10	25	1941	1941	1941	1941.0	1941	0.53	0.00	1941.0	1941	0.70	0.01	1941	1941			
CBMix23	3	2	15	780	780	780	780.0	780	0.12	0.00	780.0	780	0.18	0.00	780	780			
Gap (%)				3.083%	2.705%	0.891%	0.574%	0.287%					0.361%	0.088%					
T(min)				120.00	44.72	60.00					3.13					5.09			
T*(min)				56.92	—	19.60					1.54					3.09			
CPU				CPU 3G	Xe 3.0G	CPU 3G					Xe 3.07G					Xe 3.07G			

Table EC.14 Results for the MCCRP – BHW instances

Inst	N_R	E_R	A_R	HKSG12		DHDI14		ILS				UHGS				BKS	
				Best-2	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New		
BHW1	7	11	11	337	337	337.0	337	0.20	0.00	337.0	337	0.27	0.00	337*	337		
BHW2	4	0	25	470	470	470.0	470	0.20	0.00	470.0	470	0.19	0.00	470*	470		
BHW3	5	8	7	415	415	415.0	415	0.12	0.00	415.0	415	0.16	0.00	415*	415		
BHW4	6	0	44	240	240	240.0	240	0.46	0.00	240.0	240	0.50	0.00	240*	240		
BHW5	30	0	132	506	502	502.0	502	2.86	0.07	502.0	502	3.65	0.10	502*	502		
BHW6	15	37	58	388	388	388.0	388	1.26	0.02	388.0	388	1.35	0.02	388*	388		
BHW7	35	0	194	1094	1070	1066.0	1062	8.34	4.25	1065.0	1060	12.15	6.99	1070	<u>1054</u>		
BHW8	20	0	97	672	668	668.4	668	1.99	0.57	668.4	668	3.35	0.98	668	668		
BHW9	10	26	142	920	875	871.0	867	4.50	1.82	866.0	865	7.06	3.36	875	<u>863</u>		
BHW10	40	0	102	8596	8524	8506.4	8482	3.21	1.54	8483.0	8482	3.54	0.88	8524	<u>8480</u>		
BHW11	20	0	51	5023	4914	4883.0	4883	1.80	0.69	4883.0	4883	1.37	0.16	4914	<u>4883</u>		
BHW12	40	0	75	11042	10887	10871.3	10849	2.53	1.16	10858.9	10849	3.72	1.75	10887	<u>10849</u>		
BHW13	25	0	150	14510	14346	14326.9	14310	5.81	2.80	14318.0	14301	7.92	4.59	14346	<u>14301</u>		
BHW14	25	0	196	25194	24833	24802.8	24730	13.10	7.66	24739.4	24692	18.58	13.98	24833	<u>24668</u>		
BHW15	30	0	98	15509	15354	15361.9	15341	4.01	1.80	15351.6	15331	6.90	4.65	15354	<u>15331</u>		
BHW16	30	0	380	44527	43948	43412.5	43147	43.08	31.30	43293.3	43121	56.82	51.39	43948	<u>43058</u>		
BHW17	50	0	190	26768	26235	26001.9	25898	18.03	13.03	25965.3	25876	24.65	19.23	26235	<u>25850</u>		
BHW18	20	0	174	15833	15170	15203.4	15118	6.92	3.49	15163.0	15098	11.72	7.77	15170	<u>15060</u>		
BHW19	20	0	87	9480	9388	9393.6	9379	2.64	1.50	9382.9	9379	5.37	3.45	9388	<u>9379</u>		
BHW20	50	51	192	16625	16291	16127.2	16038	17.15	8.24	16142.0	16060	29.69	23.16	16291	<u>16019</u>		
Gap (%)				1.976%	0.581%	0.338%	0.126%					0.230%	0.084%				
T(min)				120.00	60.00					6.91					9.95		
T*(min)				60.07	21.44					4.00					7.12		
CPU				CPU 3G	CPU 3G					Xe 3.07G					Xe 3.07G		

Table EC.15 Results for the MCGRP – DI-NEARP instances

Inst	N_R	E_R	A_R	HKSG12		DHDI14		ILS				UHGS				BKS	
				Best-2	Single	Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New		
n240-Q2k	120	120	0	24371	23807	23834.1	23782	20.98	15.04	23799.3	23782	35.45	24.53	23807	<u>23782</u>		
n240-Q4k	120	120	0	18352	18197	18181.0	18181	22.11	10.73	18184.0	18181	22.89	8.84	18197	<u>18181</u>		
n240-Q8k	120	120	0	15937	15884	15865.0	15865	27.10	2.25	15865.0	15865	34.36	14.02	15884	<u>15865</u>		
n240-Q16k	120	120	0	14953	14717	14717.0	14717	24.79	1.28	14717.0	14717	12.35	0.89	14717	<u>14717</u>		
n422-Q2k	302	120	0	18990	18943	18905.9	18902	51.41	27.62	18903.0	18902	45.82	26.89	18943	<u>18902</u>		
n422-Q4k	302	120	0	15987	15869	15851.6	15849	60.11	32.62	15849.0	15849	47.28	17.42	15869	<u>15849</u>		
n422-Q8k	302	120	0	14627	14442	14442.0	14442	59.23	4.29	14442.0	14442	37.22	3.43	14442	<u>14442</u>		
n422-Q16k	302	120	0	14357	14339	14339.0	14339	60.17	7.65	14339.0	14339	40.53	3.58	14339	<u>14339</u>		
n442-Q2k	294	148	0	51656	51052	50442.3	50402	56.26	24.80	50416.2	50386	52.90	32.17	51052	<u>50382</u>		
n442-Q4k	294	148	0	45605	44952	44931.6	44927	49.60	20.02	44931.2	44926	46.74	22.03	44952	<u>44926</u>		
n442-Q8k	294	148	0	44652	43264	43247.0	43247	60.16	18.97	43247.0	43247	49.70	15.10	43264	<u>43247</u>		
n442-Q16k	294	148	0	42797	42683	42681.0	42681	60.19	3.80	42681.0	42681	55.07	3.21	42683	<u>42681</u>		
n477-Q2k	203	274	0	23124	22896	22879.0	22868	47.97	28.36	22876.4	22868	46.54	22.46	22896	<u>22868</u>		
n477-Q4k	203	274	0	20198	20035	19950.0	19950	56.66	7.42	19950.2	19950	46.40	15.05	20035	<u>19950</u>		
n477-Q8k	203	274	0	18561	18490	18490.0	18490	60.16	5.03	18490.0	18490	60.28	33.68	18490	<u>18490</u>		
n477-Q16k	203	274	0	18105	18040	17930.0	17930	60.22	4.39	17930.0	17930	56.81	5.23	18040	<u>17930</u>		
n699-Q2k	335	364	0	59817	58948	58668.1	58595	60.21	34.97	58708.6	58422	60.28	56.13	58948	<u>58422</u>		
n699-Q4k	335	364	0	40473	40124	39723.5	39656	60.21	32.58	39717.0	39637	60.31	47.58	40124	<u>39608</u>		
n699-Q8k	335	364	0	30992	30799	30532.6	30531	60.24	41.47	30536.1	30531	60.40	40.87	30799	<u>30531</u>		
n699-Q16k	335	364	0	27028	26999	26740.5	26718	60.29	32.78	26725.4	26718	60.53	46.47	26999	<u>26703</u>		
n833-Q2k	347	486	0	56877	56102	55818.4	55507	60.30	36.99	55674.1	55556	60.44	57.36	56102	<u>55335</u>		
n833-Q4k	347	486	0	42407	41192	40893.7	40762	60.34	35.30	40747.4	40688	60.53	56.02	41192	<u>40620</u>		
n833-Q8k	347	486	0	35267	34812	34467.5	34374	60.41	45.29	34392.8	34336	60.69	54.92	34812	<u>34275</u>		
n833-Q16k	347	486	0	33013	32567	32348.2	32310	60.47	28.09	32277.2	32242	61.05	55.46	32567	<u>32213</u>		
Gap (%)				1.640%	0.537%	0.159%	0.074%					0.106%	0.041%				
T(min)				120.00	60.00					58.23					53.48		
T*(min)				92.95	36.32					23.62					30.75		
CPU				CPU 3G	CPU 3G					Xe 3.07G					Xe 3.07G		

Table EC.16 Results for the CARP with turn penalties – CMMS instances

Inst	N_R	E_R	A_R	ILS			UHGS (10,000 it)			UHGS (20,000 it)			BKS
				Avg-10	Best-10	T	Avg-10	Best-10	T	Avg-10	Best-10	T	
p01-8	0	80	80	10710.9	10687	9.62	10722.3	10697	19.53	10721.6	10696	34.50	10687
p01-16	0	80	80	12504.6	12452	6.38	12461.4	12434	15.58	12439.4	12414	33.71	12414
p02-11	0	80	140	13631.4	13596	21.27	13636.0	13622	36.86	13639.4	13608	65.69	13596
p02-22	0	80	140	14936.1	14891	18.48	14912.4	14893	27.31	14906.2	14893	61.76	14891
p03-14	0	80	200	17749.9	17725	32.01	17771.7	17743	48.64	17760.2	17733	90.39	17725
p03-28	0	80	200	20236.5	20175	20.99	20183.1	20155	49.60	20145.4	20108	106.49	20108
p04-9	0	100	88	11425.2	11404	9.95	11426.6	11405	18.14	11418.0	11401	40.63	11401
p04-17	0	100	88	12461.3	12438	10.26	12472.6	12442	17.00	12462.4	12446	29.34	12438
p05-12	0	100	154	14689.7	14672	22.34	14708.8	14678	44.52	14692.2	14674	71.84	14672
p05-24	0	100	154	16671.6	16599	18.76	16574.9	16532	37.12	16554.5	16528	66.97	16528
p06-15	0	100	220	18707.3	18674	39.32	18742.1	18723	56.21	18728.9	18688	102.82	18674
p06-30	0	100	220	21597.2	21469	28.14	21551.9	21462	56.21	21499.3	21458	102.08	21458
p07-10	0	120	104	13915.9	13870	10.85	13886.9	13853	27.75	13874.7	13853	36.17	13853
p07-19	0	120	104	15350.1	15255	9.13	15222.3	15166	20.89	15208.5	15181	39.44	15181
p08-14	0	120	182	19086.6	19051	32.11	19101.8	19057	48.34	19093.9	19049	100.92	19049
p08-27	0	120	182	21598.8	21533	32.91	21590.4	21496	45.09	21542.8	21488	93.41	21488
p09-17	0	120	260	22398.8	22339	55.08	22519.3	22374	58.81	22495.5	22373	111.66	22339
p09-34	0	120	260	24841.5	24681	49.19	24810.5	24725	54.95	24720.9	24615	114.01	24615
p10-11	0	140	132	15957.7	15888	21.66	15971.8	15900	38.76	15970.0	15935	75.80	15888
p10-22	0	140	132	17995.8	17943	18.22	17950.8	17912	29.67	17925.7	17753	60.24	17753
p11-16	0	140	231	23153.8	23091	48.22	23223.5	23103	57.88	23193.2	23102	112.58	23091
p11-32	0	140	231	27943.6	27873	26.91	27867.2	27791	57.84	27810.4	27757	110.01	27757
p12-21	0	140	330	27931.7	27860	60.03	28081.2	27961	58.72	27939.3	27813	115.29	27813
p12-41	0	140	330	32521.1	32430	53.05	32520.6	32368	60.10	32413.2	32318	120.00	32318
p13-12	0	160	148	18538.5	18489	20.26	18567.5	18501	36.98	18528.6	18505	60.09	18489
p13-23	0	160	148	20881.8	20806	21.64	20835.8	20623	37.28	20704.3	20563	83.53	20563
p14-17	0	160	259	25875.2	25833	52.82	25954.9	25846	60.13	25922.2	25850	119.52	25833
p14-34	0	160	259	29594.5	29449	45.56	29549.4	29431	56.34	29527.9	29446	117.21	29446
p15-23	0	160	370	32827.5	32720	60.05	32857.0	32786	60.11	32779.9	32626	120.00	32626
p15-45	0	160	370	38039.0	37915	60.03	38134.1	37963	60.04	37897.1	37749	118.40	37749
p16-12	0	180	160	19748.4	19705	26.76	19739.4	19689	51.94	19717.4	19668	98.09	19668
p16-24	0	180	160	23118.4	22999	20.79	22980.6	22841	42.16	22944.6	22825	83.79	22825
p17-18	0	180	280	26295.1	26170	57.19	26405.2	26278	60.00	26377.8	26280	120.02	26170
p17-36	0	180	280	31417.0	31335	58.08	31445.1	31257	54.71	31368.5	31201	118.28	31201
p18-24	0	180	400	31532.1	31221	59.41	31504.9	31295	60.18	31319.7	31175	120.00	31175
p18-48	0	180	400	36709.3	36587	60.05	36772.5	36543	59.55	36528.0	36424	120.00	36424
p19-13	0	200	168	19542.0	19499	26.63	19526.2	19460	42.15	19521.0	19503	66.71	19499
p19-25	0	200	168	22046.6	21963	26.89	21994.9	21724	42.15	21896.7	21717	84.41	21717
p20-19	0	200	294	27896.1	27670	35.59	27710.1	27551	60.08	27608.6	27480	119.60	27480
p20-38	0	200	294	32987.6	32903	59.44	32992.3	32783	60.00	32863.0	32668	112.23	32668
p21-25	0	200	420	34754.4	34590	60.05	34669.1	34451	60.04	34475.4	34278	120.09	34278
p21-50	0	200	420	41919.9	41726	60.06	41947.6	41605	60.00	41683.4	41504	120.07	41504
Gap (%)				0.645%	0.289%		0.600%	0.184%		0.370%	0.035%		
T(min)					34.91				46.41			90.42	
CPU				Xe 3.07G			Xe 3.07G			Xe 3.07G			

Table EC.17 Results for the MCCRP with turn penalties – DI-TP instances

Inst	N_R	E_R	A_R	ILS			UHGS (10,000 it)			UHGS (20,000 it)			BKS
				Avg-10	Best-10	T	Avg-10	Best-10	T	Avg-10	Best-10	T	
n80-Q2k	40	40	0	6941.0	6941	2.69	6941.0	6941	2.40	6941.0	6941	4.80	6941
n80-Q4k	40	40	0	6296.0	6296	3.21	6296.0	6296	2.85	6296.0	6296	5.87	6296
n160-Q2k	80	80	0	16734.4	16688	13.54	16706.0	16688	15.55	16693.3	16682	30.12	16682
n160-Q4k	80	80	0	14289.3	14229	16.93	14265.6	14247	20.71	14248.4	14229	37.24	14229
n240-Q2k	120	120	0	30258.8	30181	41.80	30189.4	30141	50.64	30171.3	30141	83.03	30141
n240-Q4k	120	120	0	23658.9	23643	45.26	23657.4	23643	50.64	23652.1	23643	83.59	23643
n240-Q8k	120	120	0	21061.9	21035	58.80	21064.7	21035	60.39	21064.2	21035	118.87	21035
n240-Q16k	120	120	0	20085.8	19942	60.10	19966.9	19851	53.63	20033.5	19877	112.48	19851
n422-Q2k	302	120	0	25847.8	25776	60.09	25760.4	25718	60.34	25760.8	25719	120.13	25718
n422-Q4k	302	120	0	22239.4	22156	60.12	22133.5	22125	60.03	22120.0	22098	120.10	22098
n422-Q8k	302	120	0	20614.5	20560	60.20	20554.8	20526	60.01	20530.5	20523	120.14	20523
n422-Q16k	302	120	0	20516.6	20432	60.70	20413.4	20376	60.93	20402.1	20376	120.01	20376
n442-Q2k	294	148	0	72638.4	72477	60.07	72483.1	72455	58.56	72446.9	72367	117.42	72367
n442-Q4k	294	148	0	65322.5	65243	60.11	65175.0	65175	59.17	65175.0	65175	118.38	65175
n442-Q8k	294	148	0	63005.6	62902	60.22	62975.9	62902	60.42	62983.3	62902	120.02	62902
n442-Q16k	294	148	0	62177.4	62125	60.61	62188.7	62125	63.25	62200.3	62125	120.01	62125
n477-Q2k	203	274	0	31002.0	30924	60.09	30950.1	30876	60.01	30943.2	30835	120.00	30835
n477-Q4k	203	274	0	26980.1	26903	60.14	26885.7	26820	60.17	26852.8	26820	121.02	26820
n477-Q8k	203	274	0	24990.2	24838	60.21	24983.5	24855	60.01	24867.1	24802	121.31	24802
n477-Q16k	203	274	0	24272.9	24104	60.27	24170.4	24086	65.39	24153.6	24053	120.02	24053
n699-Q2k	335	364	0	77302.9	77155	60.16	77206.4	77120	60.01	77053.0	76804	120.20	76804
n699-Q4k	335	364	0	52944.6	52748	60.22	52746.1	52611	60.01	52750.0	52655	120.29	52611
n699-Q8k	335	364	0	41020.1	40902	60.20	41029.0	40953	60.35	40946.5	40883	120.01	40883
n699-Q16k	335	364	0	36253.8	35991	60.32	36275.0	36045	64.91	36100.1	35955	120.01	35955
n833-Q2k	347	486	0	73094.2	72835	60.34	73241.8	72693	60.27	72700.4	72310	120.02	72310
n833-Q4k	347	486	0	53514.9	53179	60.31	53590.9	53147	60.02	53288.1	52906	120.05	52906
n833-Q8k	347	486	0	45403.7	45035	60.34	45291.2	45051	60.02	44879.5	44640	124.05	44640
n833-Q16k	347	486	0	42958.7	42756	60.44	42716.6	42167	60.02	42479.5	42282	120.02	42167
Gap (%)				0.597%	0.256%		0.414%	0.130%		0.251%	0.018%		
T(min)				51.70			52.53			102.83			
CPU				Xe 3.07G			Xe 3.07G			Xe 3.07G			

Table EC.18 Results for the PCARP – PGDB instances

Table EC.19 Results for the PCARP – PVAL instances

Inst	$ E_R $	n_{SERV}	CLP06						MPY11						UHGS – short						UHGS						BKS							
			Single			Avg-30			Best-30			Avg-10			Best-10			T			Avg-10			Best-10			T		T*		Old		New	
			F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D	F	D				
pval1A	39	105	2	544	2	489.0	2	470	2	467.0	2	466	0.20	0.11	2	466.0	2	466	1.34	0.34	2	470	0.20	0.11	2	470	466	466	466	466				
pval1B	39	105	3	585	3	540.0	3	530	3	510.9	3	506	0.23	0.15	3	507.8	3	506	1.59	0.85	3	530	0.23	0.15	3	530	506	506	506	506				
pval1C	39	105	5	701	4	679.0	4	653	4	635.0	4	628	0.22	0.15	4	627.0	4	624	1.26	0.66	4	653	0.22	0.15	4	653	623	623	623	623				
pval2A	34	94	2	782	2	706.0	2	697	2	687.0	2	685	0.20	0.10	2	685.9	2	685	1.31	0.43	2	697	0.20	0.10	2	697	685	685	685	685				
pval2B	34	94	3	868	2	789.0	2	775	2	766.2	2	757	0.20	0.12	2	758.3	2	757	1.04	0.32	2	775	0.20	0.12	2	775	757	757	757	757				
pval2C	34	94	5	1259	4	1183.0	4	1149	4	1139.9	4	1130	0.18	0.13	4	1125.5	4	1115	1.12	0.66	4	1149	0.18	0.13	4	1149	1115	1115	1115	1115				
pval3A	35	96	2	249	2	226.0	2	222	2	214.7	2	213	0.20	0.11	2	211.9	2	211	1.49	0.67	2	222	0.20	0.11	2	222	211	211	211	211				
pval3B	35	96	3	278	2	264.0	2	255	2	244.6	2	242	0.20	0.13	2	242.0	2	241	1.39	0.78	2	255	0.20	0.13	2	255	241	241	241	241				
pval3C	35	96	4	358	4	347.0	4	336	4	327.8	4	322	0.15	0.09	4	324.2	4	322	1.22	0.73	4	336	0.15	0.09	4	336	321	321	321	321				
pval4A	69	205	3	1330	2	1262.0	2	1228	2	1160.0	2	1154	0.95	0.68	2	1152.6	2	1146	4.81	2.54	2	1228	0.95	0.68	2	1228	1146	1146	1146	1146				
pval4B	69	205	4	1471	3	1315.0	3	1288	3	1227.7	3	1217	0.90	0.67	3	1214.4	3	1208	4.95	3.02	3	1288	0.90	0.67	3	1288	1205	1205	1205	1205				
pval4C	69	205	4	1583	4	1439.0	4	1409	4	1340.2	4	1322	0.75	0.55	4	1323.5	4	1315	4.55	2.82	4	1409	0.75	0.55	4	1409	1315	1315	1315	1315				
pval4D	69	205	8	1988	6	1906.0	6	1858	6	1732.6	6	1740	0.93	0.77	6	1738.2	6	1726	6.21	4.83	6	1858	0.93	0.77	6	1858	1720	1720	1720	1720				
pval5A	65	194	3	1454	2	1353.0	2	1315	2	1241.7	2	1233	0.97	0.73	2	1236.0	2	1232	4.74	2.74	2	1315	0.97	0.73	2	1315	1224	1224	1224	1224				
pval5B	65	194	4	1528	3	1418.0	3	1384	3	1342.6	3	1329	0.95	0.71	3	1330.9	3	1325	4.53	2.62	3	1384	0.95	0.71	3	1384	1317	1317	1317	1317				
pval5C	65	194	4	1655	4	1542.0	4	1522	4	1442.3	4	1429	0.63	0.44	4	1425.3	4	1420	4.74	3.24	4	1522	0.63	0.44	4	1522	1418	1418	1418	1418				
pval5D	65	194	7	2097	6	2033.0	6	1991	6	1892.9	6	1881	0.75	0.60	6	1868.1	6	1858	4.80	3.55	6	1991	0.75	0.60	6	1991	1858	1858	1858	1858				
pval6A	50	150	3	754	2	741.0	2	722	2	704.6	2	702	0.51	0.35	2	702.1	2	698	2.84	1.47	2	722	0.51	0.35	2	722	698	698	698	698				
pval6B	50	150	4	863	3	786.0	3	774	3	740.3	3	734	0.40	0.26	3	733.6	3	732	2.20	1.05	3	774	0.40	0.26	3	774	732	732	732	732				
pval6C	50	150	8	1183	7	1139.0	7	1117	7	1081.6	7	1071	0.41	0.31	7	1069.1	7	1066	2.34	1.43	7	1117	0.41	0.31	7	1117	1066	1066	1066	1066				
pval7A	66	201	3	1040	2	998.0	2	966	2	914.2	2	908	0.94	0.70	2	907.0	2	905	4.83	2.75	2	966	0.94	0.70	2	966	905	905	905	905				
pval7B	66	201	4	1046	3	979.0	3	960	3	922.4	3	916	0.60	0.41	3	915.5	3	911	3.53	1.88	3	960	0.60	0.41	3	960	911	911	911	911				
pval7C	66	201	8	1283	7	1191.0	7	1165	7	1107.6	7	1102	0.55	0.41	7	1094.7	7	1089	3.73	2.45	7	1165	0.55	0.41	7	1165	1089	1089	1089	1089				
pval8A	63	194	3	1343	3	1282.0	2	1292	2	1221.5	2	1196	1.12	0.89	2	1200.8	2	1191	7.63	5.83	2	1292	1.12	0.89	2	1292	1188	1188	1188	1188				
pval8B	63	194	4	1453	3	1330.0	3	1301	3	1239.9	3	1225	0.59	0.39	3	1231.3	3	1224	3.42	1.64	3	1301	0.59	0.39	3	1301	1224	1224	1224	1224				
pval8C	63	194	7	1970	7	1886.0	7	1853	7	1771.1	7	1739	0.69	0.55	7	1737.9	7	1725	4.70	3.51	7	1853	0.69	0.55	7	1853	1717	1717	1717	1717				
pval9A	92	274	3	1083	2	990.0	2	966	2	902.3	2	895	1.83	1.38	2	896.0	2	891	8.97	5.36	2	966	1.83	1.38	2	966	890	890	890	890				
pval9B	92	274	4	1087	3	1014.0	3	990	3	928.8	3	924	1.43	1.02	3	921.0	3	919	8.50	5.03	3	990	1.43	1.02	3	990	915	915	915	915				
pval9C	92	274	4	1195	4	1051.0	4	1031	4	959.7	4	952	1.34	1.01	4	947.9	4	943	7.92	5.18	4	1031	1.34	1.01	4	1031	940	940	940	940				
pval9D	92	274	8	1418	7	1367.0	7	1324	7	1228.1	7	1218	1.37	1.12	7	1215.8	7	1207	9.05	6.93	7	1324	1.37	1.12	7	1324	1203	1203	1203	1203				
pval10A	97	300	3	1478	2	1402.0	2	1385	2	1296.6	2	1290	2.42	1.90	2	1281.3	2	1277	13.83	9.52	2	1385	2.42	1.90	2	1385	1273	1273	1273	1273				
pval10B	97	300	4	1580	3	1416.0	3	1395	3	1321.2	3	1310	1.95	1.46	3	1308.8	3	1303	14.13	10.09	3	1395	1.95	1.46	3	1395	1296	1296	1296	1296				
pval10C	97	300	4	1631	4	1477.0	4	1461	4	1366.5	4	1357	1.67	1.24	4	1358.7	4	1351	9.39	5.89	4	1461	1.67	1.24	4	1461	1351	1351	1351	1351				
pval10D	97	300	9	1964	7	1880.0	7	1837	7	1731.0	7	1720	1.55	1.25	7	1710.1	7	1702	8.82	6.24	7	1837	1.55	1.25	7	1837	1698	1698	1698	1698				

Table EC.20 Results for the MDCARP – GDB instances

Inst	E	d	KY10 Single	UHGS – short				UHGS				BKS	
				Avg-10	Best-10	T	T*	Avg-10	Best-10	T	T*	Old	New
gdb1	22	2	300	300.0	300	0.01	0.00	300.0	300	0.23	0.00	300	300
gdb2	26	2	321	321.0	321	0.01	0.00	321.0	321	0.26	0.00	321	321
gdb3	22	2	263	259.0	259	0.01	0.00	259.0	259	0.22	0.00	263	<u>259</u>
gdb4	19	2	266	266.0	266	0.00	0.00	266.0	266	0.15	0.00	266	266
gdb5	26	2	361	361.0	361	0.01	0.00	361.0	361	0.25	0.00	361	361
gdb6	22	2	291	283.2	282	0.01	0.00	282.0	282	0.24	0.01	291	<u>282</u>
gdb7	22	2	325	325.0	325	0.01	0.00	325.0	325	0.23	0.00	325	325
gdb8	46	2	350	329.7	328	0.03	0.02	328.0	328	0.62	0.02	350	<u>328</u>
gdb9	51	2	309	277.6	277	0.05	0.03	276.0	276	0.96	0.30	309	<u>276</u>
gdb10	25	2	275	275.0	275	0.01	0.00	275.0	275	0.22	0.00	275	275
gdb11	45	2	403	387.4	387	0.03	0.01	387.0	387	0.62	0.01	403	<u>387</u>
gdb12	23	2	440	420.0	420	0.01	0.00	420.0	420	0.22	0.00	440	<u>420</u>
gdb13	28	2	540	529.0	528	0.01	0.00	523.6	522	0.35	0.07	540	<u>522</u>
gdb14	21	2	96	96.0	96	0.01	0.00	96.0	96	0.21	0.00	96	96
gdb15	21	2	56	56.0	56	0.00	0.00	56.0	56	0.17	0.00	56	56
gdb16	28	2	127	125.0	125	0.01	0.00	125.0	125	0.28	0.00	127	<u>125</u>
gdb17	28	2	91	91.0	91	0.01	0.00	91.0	91	0.23	0.00	91	91
gdb18	36	2	158	158.0	158	0.01	0.00	158.0	158	0.40	0.00	158	158
gdb19	11	2	55	55.0	55	0.00	0.00	55.0	55	0.09	0.00	55	55
gdb20	22	2	121	121.0	121	0.01	0.00	121.0	121	0.23	0.00	121	121
gdb21	33	2	158	155.8	154	0.01	0.00	154.0	154	0.43	0.01	158	<u>154</u>
gdb22	44	2	201	196.9	196	0.02	0.01	196.0	196	0.69	0.02	201	<u>196</u>
gdb23	55	2	235	230.0	227	0.03	0.01	225.2	225	1.16	0.36	235	<u>225</u>
Gap (%)			2.041%	0.296%	0.104%			0.017%	0.000%				
T(min)			≈ 0.01			0.01				0.37			
T*(min)							0.01				0.04		
CPU			P-IV 1.4G		Xe 3.07G				Xe 3.07G				

Table EC.21 Results for the MDCARP – VAL instances

Inst	E	d	UHGS				BKS New
			Avg-10	Best-10	T	T*	
1A	39	3	173.0	173	0.51	0.00	173
1B	39	3	173.0	173	0.48	0.00	173
1C	39	3	192.0	192	0.46	0.02	192
2A	34	3	217.0	217	0.40	0.00	217
2B	34	3	217.0	217	0.38	0.00	217
2C	34	3	289.0	289	0.39	0.01	289
3A	35	3	77.0	77	0.41	0.00	77
3B	35	3	77.0	77	0.40	0.00	77
3C	35	3	85.0	85	0.37	0.00	85
4A	69	3	388.0	388	1.17	0.02	388
4B	69	3	388.0	388	1.12	0.02	388
4C	69	3	388.0	388	1.20	0.05	388
4D	69	3	403.0	402	1.29	0.23	402
5A	65	3	415.0	415	1.05	0.01	415
5B	65	3	415.0	415	1.07	0.02	415
5C	65	3	415.0	415	1.11	0.02	415
5D	65	3	431.0	431	1.56	0.45	431
6A	50	3	221.0	221	0.87	0.09	221
6B	50	3	229.0	229	0.80	0.03	229
6C	50	3	303.2	303	0.81	0.11	303
7A	66	3	279.0	279	0.99	0.01	279
7B	66	3	279.0	279	1.01	0.01	279
7C	66	3	308.0	308	1.36	0.26	308
8A	63	3	385.0	385	1.03	0.02	385
8B	63	3	385.0	385	1.03	0.02	385
8C	63	3	404.3	404	1.28	0.27	404
9A	92	3	323.0	323	1.65	0.07	323
9B	92	3	323.0	323	1.73	0.06	323
9C	92	3	323.0	323	1.74	0.06	323
9D	92	3	339.9	339	2.09	0.33	338
10A	97	3	424.0	424	2.10	0.09	424
10B	97	3	424.0	424	2.17	0.13	424
10C	97	3	424.0	424	2.12	0.11	424
10D	97	3	443.9	443	2.40	0.29	442
Gap (%)			0.041%	0.015%			
T(min)					1.13		
T*(min)					0.08		
CPU			Xe 3.07G				

Table EC.22 Results for the MDCARP – EGL instances

Inst	E	d	UHGS				BKS New
			Avg-10	Best-10	T	T*	
egl-e1-A	51	4	2580.0	2580	0.97	0.05	2580
egl-e1-B	51	4	2990.0	2990	0.90	0.02	2990
egl-e1-C	51	4	3614.0	3614	0.99	0.09	3614
egl-e2-A	72	4	3388.0	3388	1.29	0.04	3388
egl-e2-B	72	4	3926.0	3908	2.37	0.90	3908
egl-e2-C	72	4	4845.2	4845	2.01	0.47	4845
egl-e3-A	87	4	3935.8	3935	2.26	0.43	3935
egl-e3-B	87	4	4759.0	4759	2.35	0.52	4759
egl-e3-C	87	4	5937.0	5930	3.35	1.26	5930
egl-e4-A	98	4	4331.9	4324	2.97	1.01	4324
egl-e4-B	98	4	5536.5	5525	3.49	1.33	5525
egl-e4-C	98	4	6697.3	6694	3.87	1.28	6694
egl-s1-A	75	4	3499.6	3491	2.43	0.90	3491
egl-s1-B	75	4	4038.8	4038	1.95	0.36	4038
egl-s1-C	75	4	5063.8	5053	2.47	0.77	5044
egl-s2-A	147	4	6099.1	6097	4.61	0.50	6097
egl-s2-B	147	4	7432.2	7430	8.73	3.64	7430
egl-s2-C	147	4	8810.2	8801	7.46	1.99	8801
egl-s3-A	159	4	6270.6	6253	7.54	2.70	6253
egl-s3-B	159	4	7764.8	7739	12.90	6.80	7736
egl-s3-C	159	4	9231.8	9212	10.19	3.30	9212
egl-s4-A	190	4	7526.5	7493	9.69	2.85	7429
egl-s4-B	190	4	9294.5	9259	14.09	6.55	9227
egl-s4-C	190	4	11243.1	11174	16.86	7.69	11174
Gap (%)			0.225%	0.059%			
T(min)					5.24		
T*(min)						1.89	
CPU				Xe 3.07G			

Table EC.23 Results for the MM-kWRPP – 2 vehicles

Inst	E	BCS10 Single	UHGS				BKS	
			Avg-10	Best-10	T	T*	Old	New
C20 ₁₁₀	63	176	176.0	176	0.32	0.04	176*	176
C20 ₁₅	63	124	124.0	124	0.39	0.05	124*	124
C20 ₁₈	63	156	156.0	156	0.42	0.11	156*	156
C20 ₂₁₀₀	63	1783	1775.8	1775	0.63	0.32	1775*	1775
C20 ₂₂₀₀	63	3325	3318.2	3317	0.51	0.19	3317*	3317
C20 ₂₅₀₀	63	8159	8089.0	8089	0.41	0.14	8089*	8089
C21 ₁₁₀	67	166	166.0	166	0.45	0.10	166*	166
C21 ₁₅	67	93	92.0	92	0.62	0.27	92*	92
C21 ₁₈	67	152	150.0	150	0.45	0.11	150*	150
C21 ₂₁₀₀	67	1671	1653.0	1653	0.71	0.36	1653*	1653
C21 ₂₂₀₀	67	3227	3193.3	3193	0.58	0.24	3193*	3193
C21 ₂₅₀₀	67	8002	7957.0	7946	0.80	0.44	7946*	7946
C22 ₁₁₀	74	289	288.0	288	0.59	0.19	288*	288
C22 ₁₅	74	194	194.0	194	0.73	0.28	194*	194
C22 ₁₈	74	237	236.0	236	0.63	0.14	236*	236
C22 ₂₁₀₀	74	1500	1492.9	1490	0.89	0.38	1487*	1487
C22 ₂₂₀₀	74	2754	2747.0	2747	0.95	0.44	2747*	2747
C22 ₂₅₀₀	74	8601	8599.1	8596	1.13	0.60	8596*	8596
C23 ₁₁₀	78	204	202.4	202	0.90	0.42	202*	202
C23 ₁₅	78	119	119.0	119	0.60	0.12	119*	119
C23 ₁₈	78	185	185.0	185	0.59	0.09	185*	185
C23 ₂₁₀₀	78	1805	1804.0	1804	1.18	0.66	1804*	1804
C23 ₂₂₀₀	78	3473	3423.1	3422	1.18	0.69	3422*	3422
C23 ₂₅₀₀	78	7936	7934.5	7921	1.36	0.84	7914*	7914
C24 ₁₁₀	55	161	161.0	161	0.32	0.06	161*	161
C24 ₁₅	55	114	114.0	114	0.29	0.02	114*	114
C24 ₁₈	55	156	155.0	155	0.36	0.09	155*	155
C24 ₂₁₀₀	55	1319	1313.3	1313	0.58	0.27	1313*	1313
C24 ₂₂₀₀	55	2654	2653.0	2653	0.62	0.32	2653*	2653
C24 ₂₅₀₀	55	6594	6575.0	6575	0.58	0.28	6575*	6575
Gap (%)		0.103%	0.008%	0.002%				
T(min)		0.94		0.18				
T*(min)					0.07			
CPU	I2 2.4G		Xe 3.07G					

Table EC.24 Results for the MM-kWRPP – 3 vehicles

Inst	E	BCS10 Single	UHGS				BKS	
			Avg-10	Best-10	T	T*	Old	New
C20 ₁₁₀	63	125	125.0	125	0.30	0.03	125*	125
C20 ₁₅	63	90	90.0	90	0.36	0.05	90*	90
C20 ₁₈	63	112	112.0	112	0.34	0.04	112*	112
C20 ₂₁₀₀	63	1292	1281.3	1281	0.50	0.16	1287	<u>1281</u>
C20 ₂₂₀₀	63	2402	2360.0	2360	0.52	0.23	2373	<u>2360</u>
C20 ₂₅₀₀	63	5733	5673.0	5673	0.57	0.28	5673*	5673
C21 ₁₁₀	67	124	122.0	122	0.45	0.09	122*	122
C21 ₁₅	67	68	67.0	67	0.41	0.09	67*	67
C21 ₁₈	67	112	112.0	112	0.52	0.10	112*	112
C21 ₂₁₀₀	67	1164	1144.0	1144	0.71	0.37	1146	<u>1144</u>
C21 ₂₂₀₀	67	2238	2211.5	2209	0.84	0.50	2210	<u>2209</u>
C21 ₂₅₀₀	67	5556	5487.0	5487	0.58	0.23	5501	<u>5487</u>
C22 ₁₁₀	74	205	201.0	201	0.60	0.21	201*	201
C22 ₁₅	74	136	136.0	136	0.56	0.11	136*	136
C22 ₁₈	74	166	163.0	163	0.61	0.17	163*	163
C22 ₂₁₀₀	74	1058	1040.9	1040	1.32	0.83	1049	<u>1040</u>
C22 ₂₂₀₀	74	1943	1908.1	1907	1.32	0.83	1909	<u>1907</u>
C22 ₂₅₀₀	74	6054	6016.0	6013	1.18	0.70	6058	<u>6013</u>
C23 ₁₁₀	78	149	146.3	146	0.94	0.42	147	<u>146</u>
C23 ₁₅	78	90	89.0	89	0.66	0.16	89*	89
C23 ₁₈	78	132	131.0	131	0.77	0.15	131*	131
C23 ₂₁₀₀	78	1231	1220.9	1219	1.44	0.90	1221	<u>1219</u>
C23 ₂₂₀₀	78	2371	2286.0	2286	1.10	0.64	2298	<u>2286</u>
C23 ₂₅₀₀	78	5507	5453.1	5435	1.32	0.80	5445	<u>5435</u>
C24 ₁₁₀	55	118	118.0	118	0.28	0.05	118*	118
C24 ₁₅	55	87	87.0	87	0.28	0.03	87*	87
C24 ₁₈	55	120	119.0	119	0.39	0.05	119*	119
C24 ₂₁₀₀	55	964	951.3	951	0.66	0.38	957	<u>951</u>
C24 ₂₂₀₀	55	1940	1928.7	1928	0.52	0.24	1928	1928
C24 ₂₅₀₀	55	4578	4555.6	4555	0.52	0.22	4555*	4555
Gap (%)		0.230%	0.008%	0.000%				
T(min)		0.41		0.18				
T*(min)					0.07			
CPU	I2 2.4G		Xe 3.07G					

Table EC.25 Results for the MM-kWRPP – 4 vehicles

Inst	E	BCS10 Single	UHGS				BKS	
			Avg-10	Best-10	T	T*	Old	New
C20 ₁₁₀	63	108	108.0	108	0.33	0.05	108*	108
C20 ₁₅	63	76	75.0	75	0.37	0.07	75*	75
C20 ₁₈	63	94	94.0	94	0.35	0.06	94*	94
C20 ₂₁₀₀	63	1035	1029.0	1029	0.44	0.15	1029*	1029
C20 ₂₂₀₀	63	1911	1891.0	1891	0.42	0.14	1925	<u>1891</u>
C20 ₂₅₀₀	63	4548	4520.0	4520	0.37	0.09	4533	<u>4520</u>
C21 ₁₁₀	67	100	99.0	99	0.55	0.10	99*	99
C21 ₁₅	67	56	56.0	56	0.41	0.09	56*	56
C21 ₁₈	67	92	91.0	91	0.48	0.15	91*	91
C21 ₂₁₀₀	67	900	888.0	888	0.55	0.16	888	888
C21 ₂₂₀₀	67	1740	1709.7	1709	0.77	0.40	1737	<u>1709</u>
C21 ₂₅₀₀	67	4334	4293.9	4289	0.62	0.28	4348	<u>4289</u>
C22 ₁₁₀	74	167	161.0	161	0.62	0.23	167	<u>161</u>
C22 ₁₅	74	113	110.0	110	0.80	0.24	111	<u>110</u>
C22 ₁₈	74	133	131.4	131	0.73	0.30	132	<u>131</u>
C22 ₂₁₀₀	74	840	824.8	824	1.12	0.65	832	<u>824</u>
C22 ₂₂₀₀	74	1517	1502.5	1501	1.05	0.58	1515	<u>1501</u>
C22 ₂₅₀₀	74	4812	4761.0	4752	0.92	0.45	4797	<u>4752</u>
C23 ₁₁₀	78	123	120.0	120	0.77	0.28	121	<u>120</u>
C23 ₁₅	78	74	73.0	73	0.62	0.14	73	73
C23 ₁₈	78	109	107.0	107	0.66	0.17	109	<u>107</u>
C23 ₂₁₀₀	78	957	939.5	937	1.16	0.69	950	<u>937</u>
C23 ₂₂₀₀	78	1808	1740.8	1733	1.26	0.82	1752	<u>1733</u>
C23 ₂₅₀₀	78	4308	4214.1	4208	1.44	0.95	4264	<u>4208</u>
C24 ₁₁₀	55	99	97.0	97	0.32	0.08	97*	97
C24 ₁₅	55	72	72.0	72	0.31	0.06	72*	72
C24 ₁₈	55	101	101.0	101	0.35	0.08	101*	101
C24 ₂₁₀₀	55	770	767.0	767	0.41	0.11	770	<u>767</u>
C24 ₂₂₀₀	55	1598	1585.6	1584	0.56	0.27	1593	<u>1584</u>
C24 ₂₅₀₀	55	3663	3619.8	3618	0.59	0.31	3634	<u>3618</u>
Gap (%)		0.303%	0.014%	0.000%				
T(min)		0.29		0.18				
T*(min)					0.06			
CPU	I2 2.4G		Xe 3.07G					

Table EC.26 Results for the MM-kWRPP – 5 vehicles

Inst	E	BCS10 Single	UHGS				BKS	
			Avg-10	Best-10	T	T*	Old	New
C20 ₁₁₀	63	95	95.0	95	0.33	0.05	95	95
C20 ₁₅	63	68	68.0	68	0.42	0.07	68*	68
C20 ₁₈	63	88	87.5	87	0.43	0.13	87*	87
C20 ₂₁₀₀	63	905	905.0	905	0.40	0.10	905	905
C20 ₂₂₀₀	63	1676	1649.0	1649	0.57	0.28	1663	<u>1649</u>
C20 ₂₅₀₀	63	3901	3796.2	3793	0.54	0.27	3879	<u>3793</u>
C21 ₁₁₀	67	87	86.0	86	0.42	0.08	86*	86
C21 ₁₅	67	51	50.0	50	0.43	0.10	50*	50
C21 ₁₈	67	80	79.0	79	0.43	0.11	79*	79
C21 ₂₁₀₀	67	755	747.3	747	0.73	0.35	753	747
C21 ₂₂₀₀	67	1525	1469.9	1468	0.64	0.31	1519	<u>1468</u>
C21 ₂₅₀₀	67	3662	3581.3	3577	0.76	0.43	3637	<u>3577</u>
C22 ₁₁₀	74	145	139.1	139	1.05	0.53	141	<u>139</u>
C22 ₁₅	74	97	95.0	95	0.67	0.19	96	<u>95</u>
C22 ₁₈	74	117	113.3	113	0.83	0.34	114	<u>113</u>
C22 ₂₁₀₀	74	707	692.5	692	0.94	0.43	704	<u>692</u>
C22 ₂₂₀₀	74	1282	1257.9	1256	1.25	0.74	1280	<u>1256</u>
C22 ₂₅₀₀	74	4069	4005.9	4001	1.09	0.63	4046	<u>4001</u>
C23 ₁₁₀	78	107	104.0	104	0.83	0.34	105	<u>104</u>
C23 ₁₅	78	65	64.0	64	0.69	0.15	65	<u>64</u>
C23 ₁₈	78	96	93.0	93	0.69	0.25	95	<u>93</u>
C23 ₂₁₀₀	78	788	774.2	771	1.33	0.87	787	<u>771</u>
C23 ₂₂₀₀	78	1459	1409.2	1405	1.45	1.02	1455	<u>1405</u>
C23 ₂₅₀₀	78	3623	3512.5	3505	1.67	1.11	3545	<u>3505</u>
C24 ₁₁₀	55	86	85.0	85	0.32	0.09	85*	85
C24 ₁₅	55	63	63.0	63	0.31	0.07	63*	63
C24 ₁₈	55	90	89.0	89	0.34	0.09	89*	89
C24 ₂₁₀₀	55	673	666.2	666	0.54	0.23	666*	666
C24 ₂₂₀₀	55	1379	1375.6	1374	0.53	0.26	1374*	1374
C24 ₂₅₀₀	55	3091	3091.0	3091	0.39	0.12	3091	3091
Gap (%)		0.392%	0.021%	0.000%				
T(min)		0.24		0.19				
T*(min)					0.07			
CPU	I2 2.4G		Xe 3.07G					

Table EC.27 Results for the MM-kWRPP – 6 vehicles

Inst	E	UHGS				BKS	
		Avg-10	Best-10	T	T*	Old	New
C16 ₁₁₀	34	57.0	57	0.15	0.03	57*	57
C16 ₁₅	34	36.0	36	0.14	0.02	36*	36
C16 ₁₈	34	54.0	54	0.10	0.01	54*	54
C16 ₂₁₀₀	34	365.2	362	0.17	0.05	362*	362
C16 ₂₂₀₀	34	735.0	735	0.17	0.05	735*	735
C16 ₂₅₀₀	34	1584.0	1584	0.24	0.11	1593	<u>1584</u>
C17 ₁₁₀	17	41.0	41	0.04	0.00	41*	41
C17 ₁₅	17	28.0	28	0.04	0.00	28*	28
C17 ₁₈	17	38.0	38	0.03	0.00	38*	38
C17 ₂₁₀₀	17	404.0	404	0.03	0.00	404*	404
C17 ₂₂₀₀	17	523.0	523	0.05	0.01	523*	523
C17 ₂₅₀₀	17	1497.0	1497	0.04	0.01	1497*	1497
C18 ₁₁₀	16	71.0	71	0.04	0.00	71*	71
C18 ₁₅	16	48.0	48	0.03	0.00	48*	48
C18 ₁₈	16	58.0	58	0.03	0.00	58*	58
C18 ₂₁₀₀	16	577.0	577	0.04	0.01	577*	577
C18 ₂₂₀₀	16	1027.0	1027	0.04	0.00	1027*	1027
C18 ₂₅₀₀	16	2584.0	2584	0.04	0.01	2584*	2584
C19 ₁₁₀	29	99.0	99	0.10	0.02	99*	99
C19 ₁₅	29	67.0	67	0.09	0.01	67*	67
C19 ₁₈	29	91.0	91	0.09	0.01	91*	91
C19 ₂₁₀₀	29	604.0	604	0.10	0.02	604*	604
C19 ₂₂₀₀	29	946.0	946	0.12	0.03	975	<u>946</u>
C19 ₂₅₀₀	29	2506.0	2506	0.13	0.04	2506*	2506
C20 ₁₁₀	63	89.0	89	0.31	0.06	89*	89
C20 ₁₅	63	65.0	65	0.37	0.08	65	65
C20 ₁₈	63	84.3	84	0.48	0.19	84*	84
C20 ₂₁₀₀	63	820.0	820	0.50	0.20	829	<u>820</u>
C20 ₂₂₀₀	63	1476.0	1476	0.51	0.21	1487	<u>1476</u>
C20 ₂₅₀₀	63	3324.4	3312	0.58	0.30	3431	<u>3312</u>
C21 ₁₁₀	67	78.0	78	0.50	0.17	79	<u>78</u>
C21 ₁₅	67	46.0	46	0.42	0.10	47	<u>46</u>
C21 ₁₈	67	74.0	74	0.45	0.11	74*	74
C21 ₂₁₀₀	67	660.3	660	0.78	0.44	660*	660
C21 ₂₂₀₀	67	1280.3	1274	0.78	0.44	1299	<u>1274</u>
C21 ₂₅₀₀	67	3181.8	3174	0.78	0.44	3230	<u>3174</u>
C22 ₁₁₀	74	125.0	125	0.66	0.22	128	<u>125</u>
C22 ₁₅	74	85.0	85	0.65	0.21	86	<u>85</u>
C22 ₁₈	74	101.2	101	0.84	0.42	103	<u>101</u>
C22 ₂₁₀₀	74	606.1	606	1.25	0.71	619	<u>606</u>
C22 ₂₂₀₀	74	1104.4	1104	1.01	0.53	1118	<u>1104</u>
C22 ₂₅₀₀	74	3507.7	3500	1.34	0.83	3548	<u>3500</u>
C23 ₁₁₀	78	95.0	95	0.86	0.28	96	<u>95</u>
C23 ₁₅	78	59.0	59	0.62	0.15	60	<u>59</u>
C23 ₁₈	78	85.0	85	0.65	0.17	86	<u>85</u>
C23 ₂₁₀₀	78	660.4	658	1.57	1.05	675	<u>658</u>
C23 ₂₂₀₀	78	1184.2	1179	1.28	0.85	1238	<u>1179</u>
C23 ₂₅₀₀	78	3023.7	3002	1.48	1.00	3071	<u>3002</u>
C24 ₁₁₀	55	79.0	79	0.29	0.06	79*	79
C24 ₁₅	55	58.0	58	0.31	0.06	58*	58
C24 ₁₈	55	84.0	84	0.36	0.09	84*	84
C24 ₂₁₀₀	55	613.7	613	0.66	0.35	613*	613
C24 ₂₂₀₀	55	1233.0	1233	0.54	0.27	1233*	1233
C24 ₂₅₀₀	55	2737.3	2737	0.55	0.27	2743	<u>2737</u>
Gap (%)		0.031%	0.000%				
T(min)						0.19	
T*(min)						0.08	
CPU						Xe 3.07G	