A Priori Route Evaluation for the Lateral Transhipment Problem (ARELTP) with Piecewise Linear Profits

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Title of the talk:

A Priori Route Evaluation for the Lateral Transhipment Problem (ARELTP) with Piecewise Linear Profits and a Lotsizing Application with Requalification Costs.

the presentation covers:

- problem definition
- lot sizing application
- solution approaches (DP, B&B)
- computational experiments and results

ARELTP with PLWP - origin of the problem

- origin of the problem: Single Route Lateral Transhipment Problem (SRLTP)
- SRLTP: redistribution of inventories using one vehicle.
- extension to piecewise linear profits (PWLP).
- ARELTP: evaluation of a-priori routes for this problem

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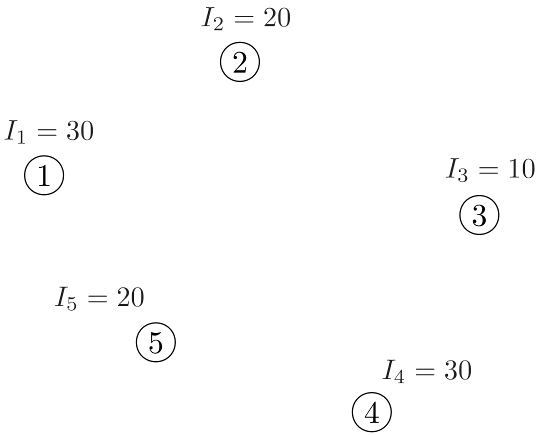
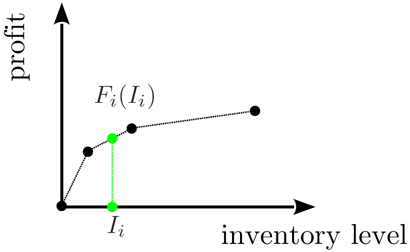
initial inventory levels I_i at the local warehouses.

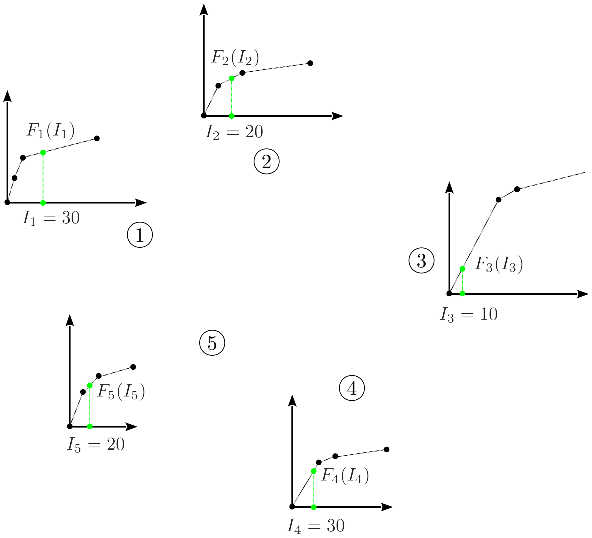


lateral transhipment for a single route - SRLTP

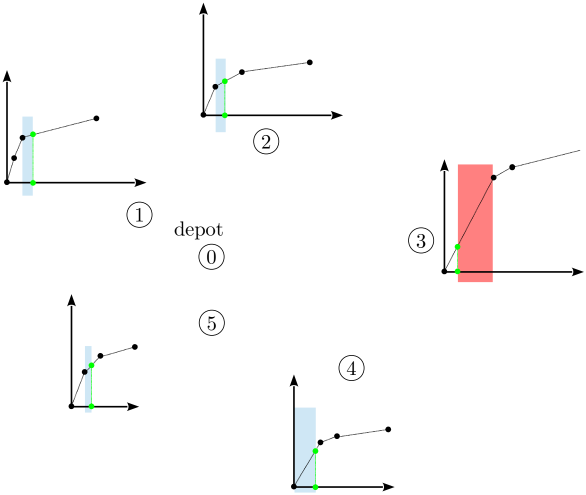
the PWLP function F_i for different inventory levels.



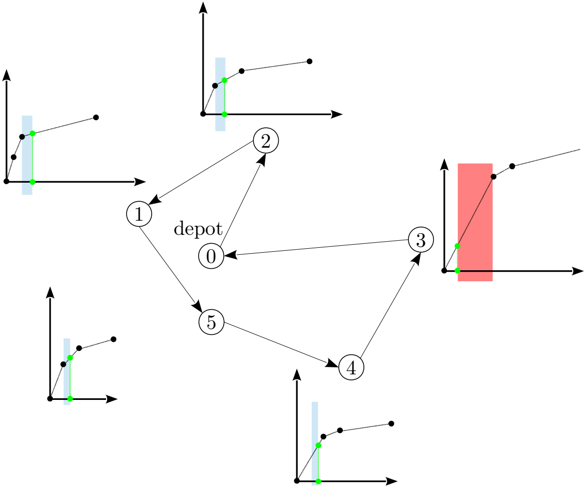
changes of the inventory level y_i and the PWL profit function F_i.



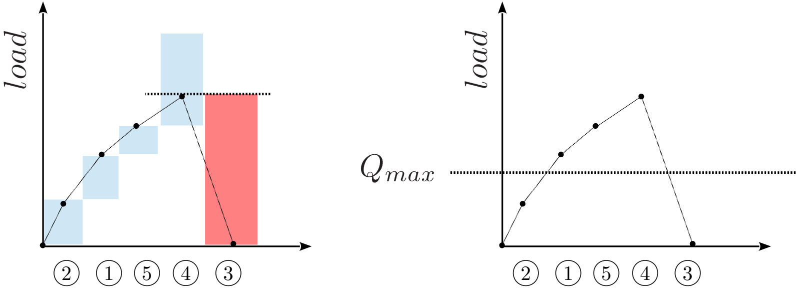
What is a good redistribution?



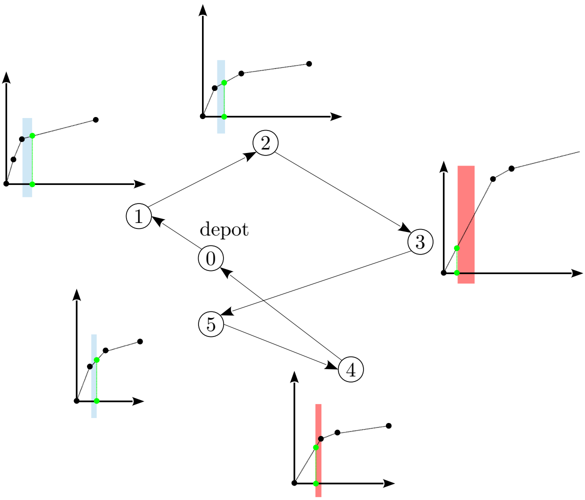
not considered: load capacity, tour length constraint, travel costs.



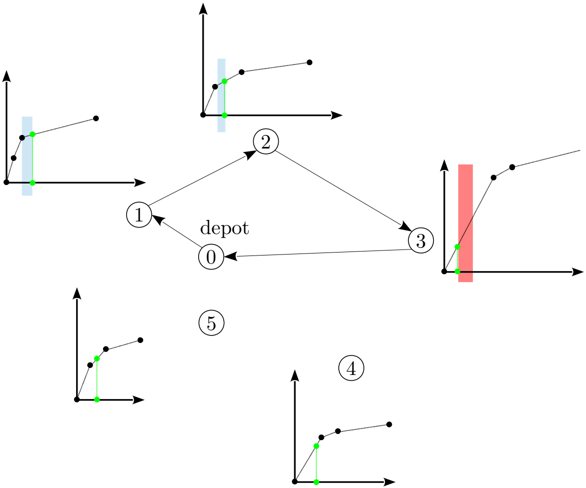
considering the load capacity constraint



considering the load capacity constraint



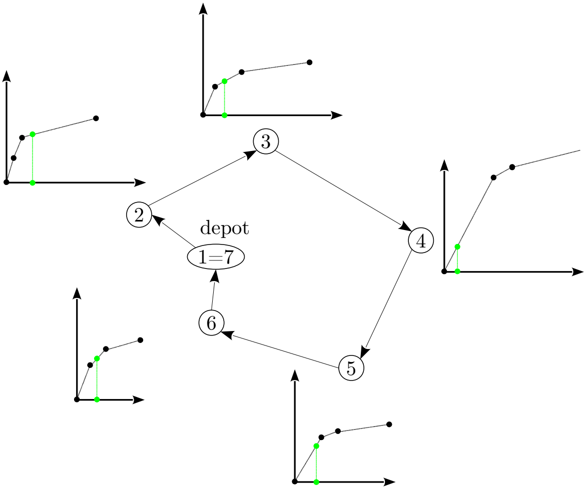
considering the distance constraint



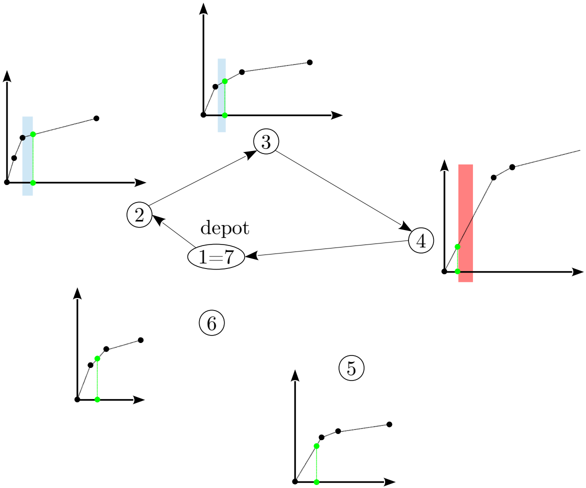
ARELTP: SRLTP for an a-priori route

feasible solution for the a-priori route $1 - 2 - 3 - 4 - 5 - 6 - 7$

- a route that starts in 1 and returns to 7
- indices of the visited customers are increasing



feasible route: $1 - 2 - 3 - 4 - 7$



A-priori route and a-priori route evaluation

motivation to use a-priori routes

- robustness (simple to implement in practice)
- consistency (improve the service quality)

performance issues may also be a motivation to use a-priori routes.

This presentation is about evaluating a single a-priori route for a single scenario (parameter setting) of the SRLTP with PLP.

ARELTP: formulation

decision variables

- arc selection: x_{ij}
- inventory change: y_i (remove y_i)

Remark: load when leaving i is $\sum_{j \leq i} y_j$

parameters

- depots for the truck: $1, n$
- local warehouses: $2, \ldots n-1$:
- revenue change: f_i

$$f_i(y_i) = F_i(I_i - y_i) - F_i(I_i) \quad (a_i \leq y_i \leq b_i \text{ if } i \text{ is visited})$$

- costs: c_{ij}
- time consumption: t_{ij} and upper bound T_{max}
- load limit: Q_{max}

$$\min \sum_{1 \le i < j \le n} c_{ij} x_{ij} - \sum_{1 \le i \le n} f_i(y_i) \tag{1}$$

$$s.t. \quad \sum_{j<i} x_{ji} = \sum_{j>i} x_{ij} \qquad 1 < i < n \tag{2}$$

$$\sum_{j>1} x_{1j} = 1 \tag{3}$$

$$\sum_{j<n} x_{jn} = 1 \tag{4}$$

$$a_i \sum_{j>i} x_{ij} \le y_i \le b_i \sum_{j>i} x_{ij} \qquad 1 \le i \le n \tag{5}$$

$$0 \le \sum_{j \le i} y_j \le Q_{max} \qquad 1 \le i \le n \tag{6}$$

$$x_{ij} \in \{0, 1\} \qquad 1 \le i < j \le n \tag{7}$$

$$y_i \in \mathbb{R} \qquad 1 \le i \le n \tag{8}$$

$$\sum_{1 \le i < j \le n} t_{ij} x_{ij} \le T_{max} \tag{9}$$

ARELTP: MIP formulation

$$\min \sum_{1\le i<j\le n} c_{ij}x_{ij} - \sum_{1\le i\le n} f_i(y_i) \quad (1)$$

$$s.t. \quad \sum_{j<i} x_{ji} = \sum_{j>i} x_{ij} \qquad 1 < i < n \quad (2)$$

$$\sum_{j>1} x_{1j} = 1 \quad (3)$$

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$$\sum_{1\le i<j\le n} t_{ij}x_{ij} \le T_{max} \quad (9)$$

ARELTP: complexity

A polynomially solvable variant of the ARELTP is presented in [Hartl and Romauch(2013)]; simplifications:

- c_{ij} is not considered
- f_i is linear
- T_{max} is not considered

ARELTP is NP hard if one of the following is true if:

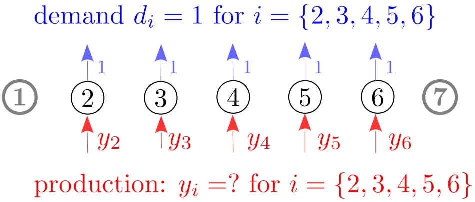
- c_{ij} is considered (linear f_i and $T_{max} = \infty$)
- f_i is piecewise linear ($c_{ij} = 0$ and $T_{max} = \infty$)
- T_{max} is considered (linear f_i and $c_{ij} = 0$)

lot sizing application

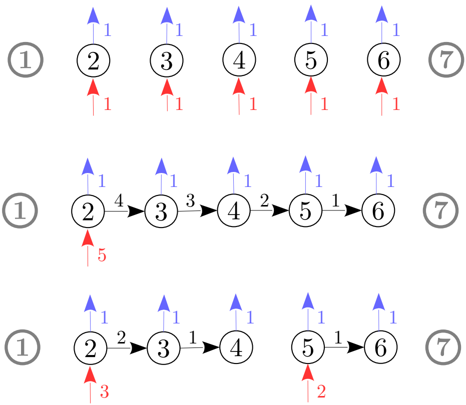
lot sizing and tool qualifications

- frequent use of a tool may lower the setup costs (renewals for tool qualifications).
- pharmaceutical, food and semiconductor industry.

violates the triangle inequality (frequent use of a tool may stretch the duration of a qualification) find the optimal production quantities

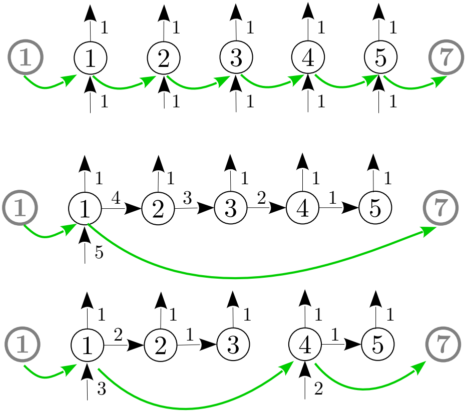


three solutions



lot sizing application - example

three solutions



lot sizing application - parameters

Input data

- periods: $i \in \{1, \ldots n\}$
- d_i: demand
- f_i: production cost for a given quantity
- $[a_i, b_i]$ interval for feasible production quantities - a_i may be positive.
- h_i inventory holding cost per unit (storage between end of period i to start of period $i + 1$)
- c_{ij} setup cost
- t_{ij} setup related resource consumption
- Q_{max} is the maximum inventory level
- T_{max} resource consumption limit

lotsizing application - decision variables

Decision variables

- $x_{ij} = 1$ if i and j are periods with production and there is no production in between.
- y_i is the production quantity

auxiliary varible: $q_i = \sum_{j \leq i}(y_j - d_j)$ is the inventory level after period i

lotsizing application - MIP

$$
\begin{aligned}
\min \quad & \sum_{1 \le i < j \le n} c_{ij} x_{ij} + \sum_{1 \le i \le n} f_i(y_i) + \sum_{1 \le i \le n} h_i q_i \\
s.t. \quad & q_i = \sum_{j \le i} (y_j - d_j) && 1 \le i \le n \\
& \sum_{j < i} x_{ji} = \sum_{j > i} x_{ij} && 1 < i < n \\
& \sum_{j > 1} x_{1j} = 1 \text{ and } \sum_{j < n} x_{jn} = 1 \\
& \sum_{1 \le i < j \le n} t_{ij} x_{ij} \le T_{max} \\
& a_i \sum_{j > i} x_{ij} \le y_i \le b_i \sum_{j > i} x_{ij} && 1 \le i \le n \\
& 0 \le q_i \le Q_{max} && 1 \le i \le n \\
& x_{ij} \in \{0, 1\} && 1 \le i < j \le n \\
& y_i, q_i \ge 0 && 1 \le i \le n
\end{aligned}
$$

DP - case without duration constraint

$V_i(q)$ is the minimum cost for inventory level q considering all customers $j = 1, 2, \ldots, i$ and it is PWL.

recurrence formula

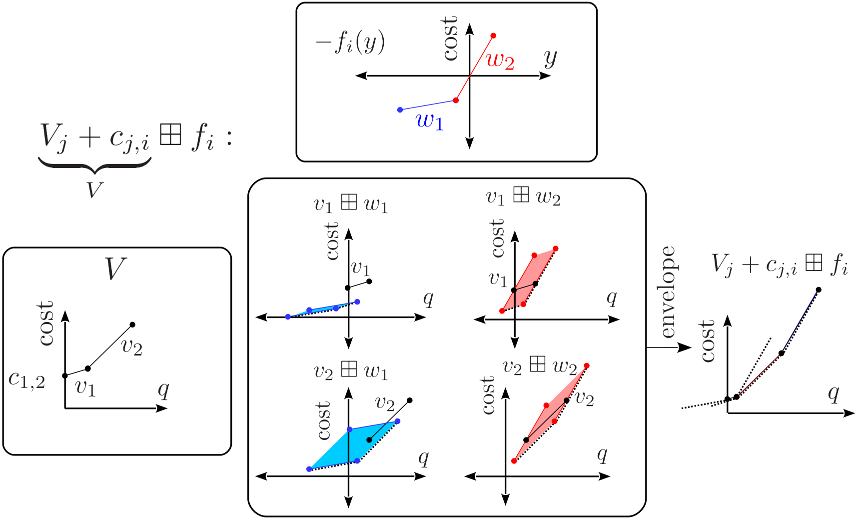
$$V_0 : \{0\} \mapsto \{0\}, \quad V_0(0) = 0 \tag{10}$$

$$V_i = \overbrace{\min_{j<i}(\underbrace{\underbrace{V_j + c_{j,i}}_{\text{shifted value function}} \boxplus f_i}_{\text{superposition}})}^{\text{envelope}} \tag{11}$$

superposition

$$(V \boxplus f)(q) = \min_{\substack{y \in \mathcal{D}(f) \\ q-y \in \mathcal{D}(V)}} \{V(q-y) - f(y)\} \tag{12}$$

$$\underbrace{V_j + c_{j,i}}_{V} \boxplus f_i :$$



DP (case without duration constraint) - complexity

recurrence formula - complexity

$$V_0 : \{0\} \mapsto \{0\}, \quad V_0(0) = 0 \tag{13}$$

$$V_i = \min_{j<i}(V_j + c_{j,i}) \boxplus f_i \tag{14}$$

The complexity for calculating stage i is $\mathcal{O}(\alpha(M_i)\log(i)M_i)$, where M_i is the number of labels of all predecessor value functions.

Dynamic Programming - general case

considering the duration constraint $\sum_{1 \leq i < j \leq n} t_{ij} x_{ij} \leq T_{max}$

$U_{i,t}(q)$ is the minimum cost for inventory level q considering all customers $j = 1, 2, \dots, i$ and a given duration budget t.

recurrence formula

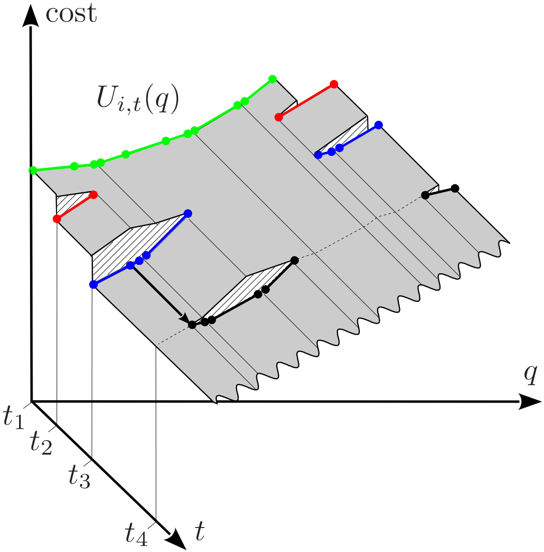
$$U_{0,0} = 0 \quad (15)$$

$$U_{i,t} = \min_{j<i, t'+t_{ji} \leq t} (U_{j,t'} + c_{ji}) \boxplus f_i \quad (16)$$

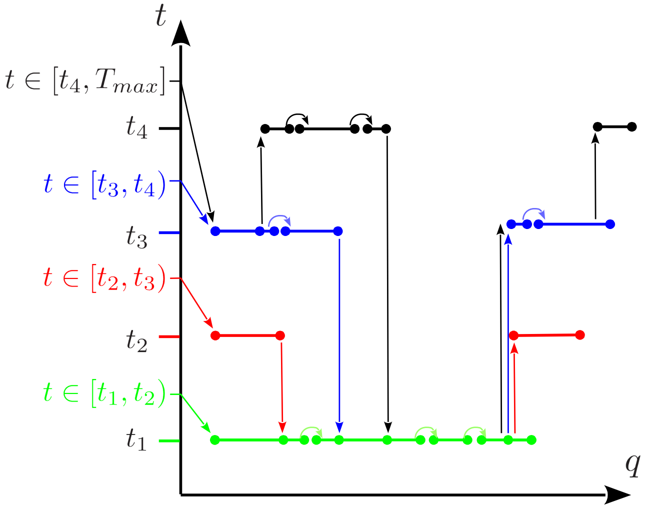
superposition

$$(U \boxplus f)(q) = \min_{\substack{y \in \mathcal{D}(f) \\ q-y \in \mathcal{D}(U)}} \{U(q-y) - f(y)\} \quad (17)$$

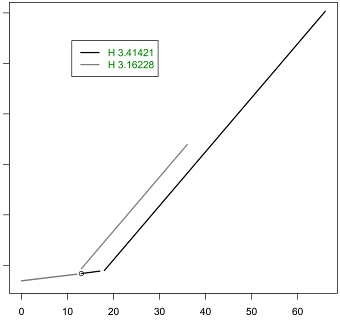
DP - general case - representation of the value function



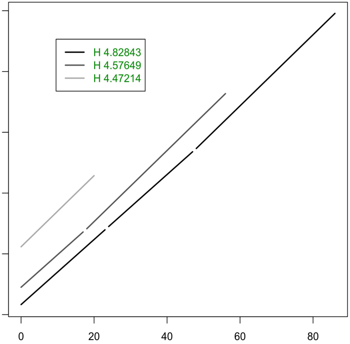
DP - general case - linked lists



DP example - complexity I

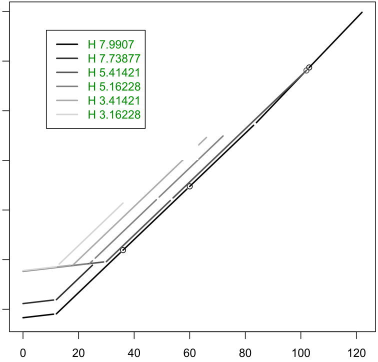


(a) second stage: 5 segments

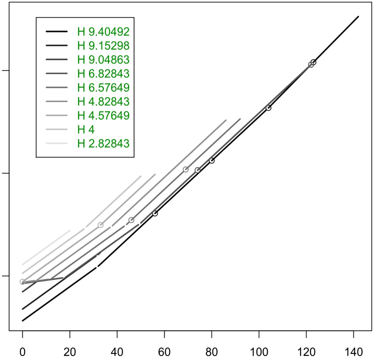


(b) third stage: 6 segments

DP example - complexity II



(c) forth stage: 26 segments



(d) fifth stage: 40 segments

Lagrangian Relaxation

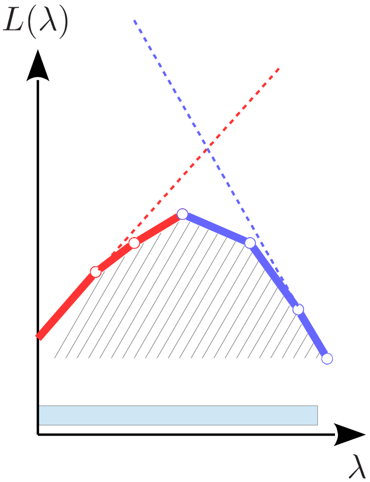
$$L(\lambda) = \min_{x,y:\text{ (2-7)}} \sum_{1 \leq i < j \leq n} c_{ij} x_{ij} - \sum_{1 \leq i \leq n} f_i(y_i) + \lambda \, \overbrace{\Big(\sum_{1 \leq i < j \leq n} t_{ij} x_{ij} - T_{max} \Big)}^{\epsilon(x)}$$

$$L(\lambda^*) = \max_{\lambda \geq 0} L(\lambda)$$

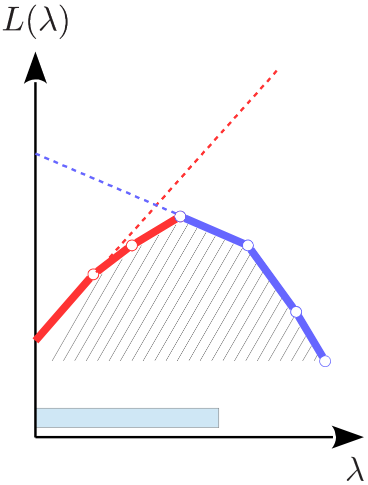
remarks

- $L(\lambda^*)$ is lower bound for the ARELTP
- calculating $L(\lambda)$ is equivalent to ARELTP without duration constraint.
- $L(\lambda)$ is concave.
- $L(\lambda^*)$ is associated to a feasible solution of the ARELTP

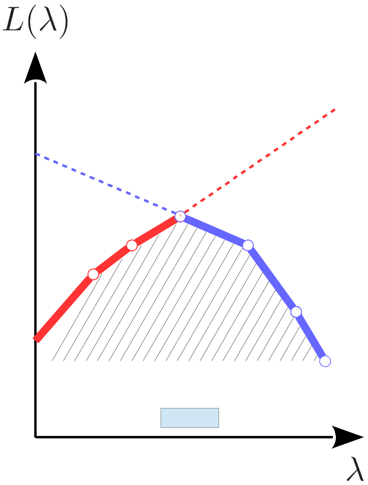
LR - example



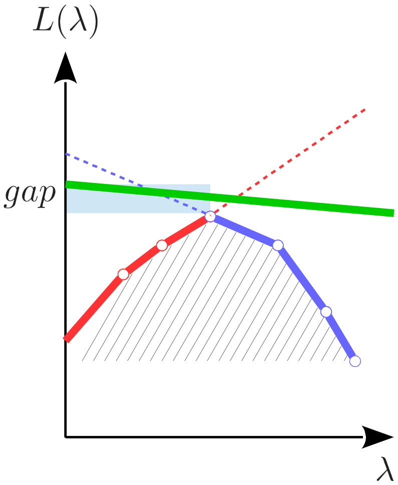
LR - example



LR - example



LR - example



branch & bound

solution branch

A branch is defined by mandatory customers and forbidden customers.

branching

a new branch is generated by additionally excluding and including a customer.

lower bounds

LR provides lower bounds for the branches of the branch & bound tree.

upper bounds

Heuristic that locally optimizes the feasible dual optimal solution.

branch & bound - details

select active branch

select the branch with the largest lower bound

branching

customer is randomly selected from a set where the heuristic solution is locally optimal.

design of experiments

instances

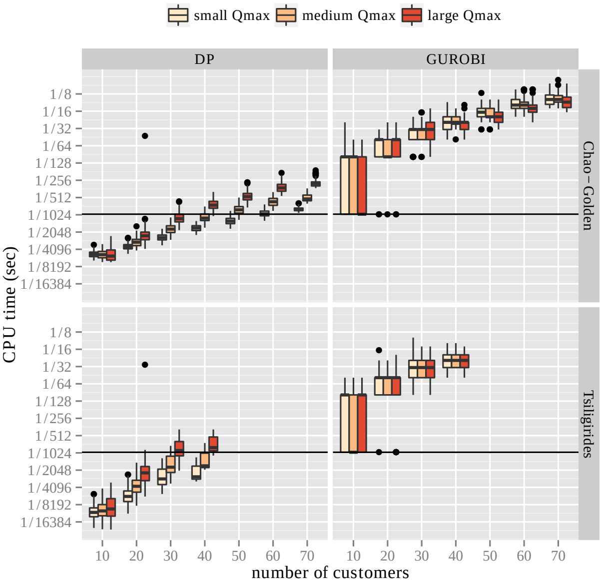
- ARELTP
  - Tisiligirides (size: 32 locations + PWLP 4 steps)
  - Chao-Golden (size: 64/66 locations + PWLP 4 steps)
  - a-priori routes: 20 per instance.
- lot sizing: new benchmark instances

`http://homepage.univie.ac.at/martin.romauch/ARELTP/`

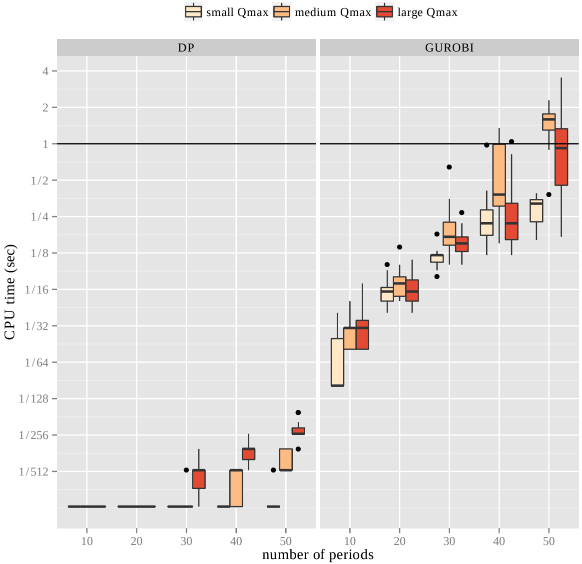
factors

- number customers / periods (n)
- duration limit / maximum resource consumtion (T_{max})
- load capacity / maximum inventory level (Q_{max})

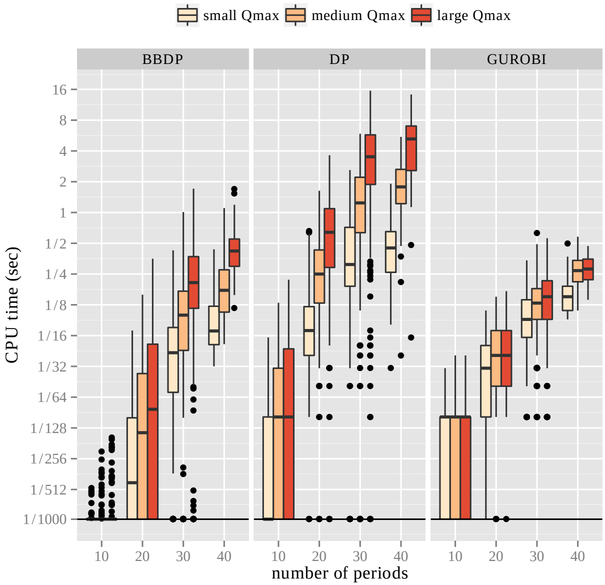
Results: Tsiligirides and Chao-Golden ($T_{max} = \infty$)



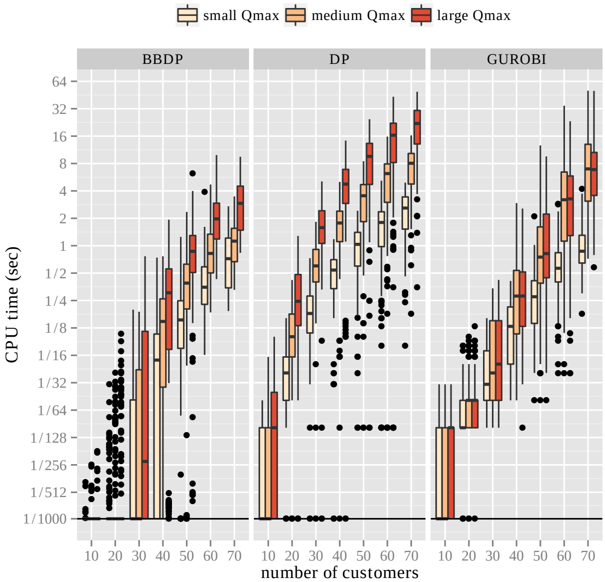
Results: lot sizing ($T_{max} = \infty$)



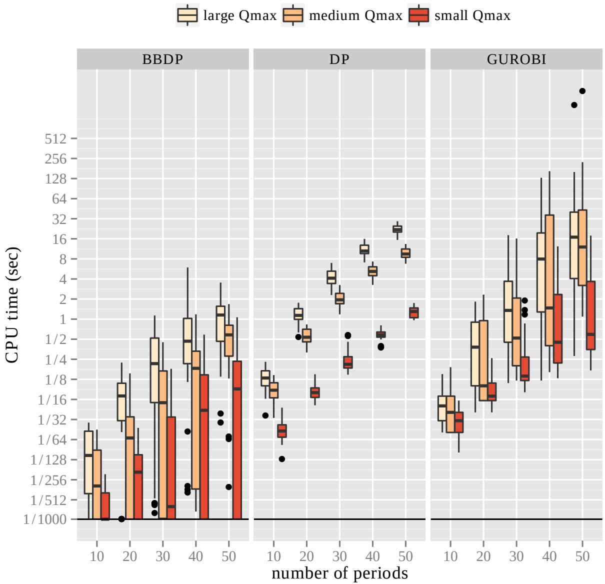
Results: Tsiligirides



Results: Chao-Golden



Results: lot sizing



conclusion and next steps

Conclusion

- With respect to the computational experiments, the presented B&B approach has in average the best performance.
- Very large load capacities Q_{max} are beneficial for Gurobi
- Very large duration limits T_{max} are beneficial for the proposed B&B
- Very good results for the lot sizing instances

next steps

- integration of the ARELTP solver into a framework to solve lateral transhipment for PWLP.
- application: stochastic demands
- extension: more than one route/product, split deliveries ...

Richard F Hartl and Martin Romauch.
The influence of routing on lateral transhipment.
In *Computer Aided Systems Theory-EUROCAST 2013*, pages 267–275. Springer, 2013.