On unified methods for multi-attribute VRPs, route evaluation operators and large neighborhoods

Thibaut VIDAL [1,2]

[1] Massachusetts Institute of Technology, USA
[2] University of Vienna, Austria

Seminar
Wirtschaftsuniversität Wien – WU
December 2TH, 2013

Join Work With

- This work is built on the Unified Hybrid Genetic Search (UHGS), developed during my PhD thesis with
  - Teodor Gabriel Crainic – UQAM, Montréal
  - Michel Gendreau – Polytechnique, Montréal
  - Christian Prins – UTT, Troyes, France

Further collaborations

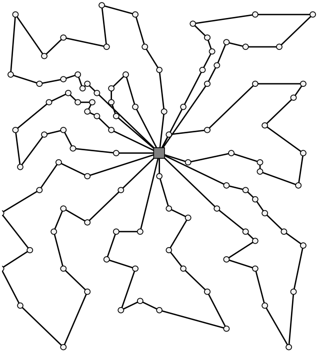
- Recent and current works on specific problem variants:

  - **Prize-collecting VRP**
    - **Nelson Maculan – UFRJ**
    - **Puca Huachi Penna – UFF**
    - **Luis Satoru Ochi – UFF**
  - Heterogeneous VRP
    - Puca Huachi Penna – UFF
    - Luis Satoru Ochi – UFF
  - Clustered VRP
    - Maria Battarra – U. Southampton.
    - Gunes Erdogan – U. Southampton.
    - Anand Subramanian – UFPB
  - **On-line/Stochastic Vehicle Routing**
    - **P. Jaillet – MIT**
    - **and R. Hartl – UniWien**
  - Vehicle Routing and Truck Driver Scheduling Problem
    - A. Goel – Jacobs U.
  - Workover Rig Routing Problem
    - G. Ribeiro, B. Vieira – UFRJ
    - G. Desaulnierss – UdeM
    - J. Desrosiers – UdeM
  - Pollution Routing Problem
    - A. Subramanian – UFPB
    - R. Kramer – UFPB

Contents

Contents

Multi-attribute vehicle routing problems (MAVRPs)

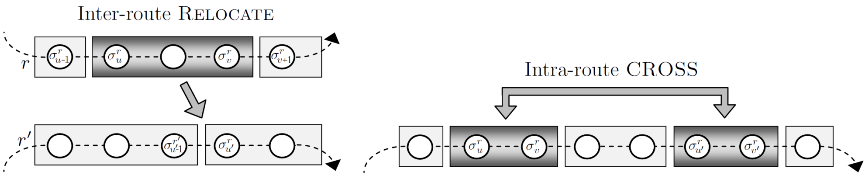


- Vehicle routing problems (VRP)
  - plethora of exact and heuristic methods
- Challenges related to the resolution of VRP with attributes (multi-attribute VRPs, MAVRPs)
  - modeling the specificities of application cases, customers requirements, network and vehicle specificities, operators abilities...
  - Combining several attributes can lead to highly complex rich VRPs.
  - Dramatic increase in the literature dedicated to specific VRP variants.
  - Recent contributions on unified resolution
    - Subramanian et al. (2013) – ILS
    - Vidal et al. (2012, 2013) – Unified hybrid genetic search (UHGS)

Contents

An efficient and unified local search for MAVRPs

- One important structural property of local search:
  - Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
  - The same subsequences appear many times during different moves



  - Data preprocessing on sub-sequences to speed up the search (Savelsbergh, 1985, 1992; Kindervater and Savelsbergh, 1997)

An efficient and unified local search for MAVRPs

- To decrease the computational complexity, we compute auxiliary data on subsequences by induction on concatenation ($\oplus$).
- This is done through four operations:
  - **Initialization**: Initialize the data structures on a single node
  - **Forward Computation**: Append a node at the end of a sequence and compute the data structures
  - **Backward Computation**: Append a node at the beginning of a sequence and compute the data structures
  - **Merge**: Evaluate a move as a concatenation of a bounded number of subsequences using the auxiliary data structures on each subsequence

Route evaluation examples

- Example 1) Distance and capacity constraints

Auxiliary data structures in use:

Partial loads $L(\sigma)$ and distance $D(\sigma)$

Initialization

For a sequence σ_0 with a single visit v_i , $L(\sigma_0) = q_i$ and $D(\sigma_0) = 0$

Forward and Backward

Increment $L(\sigma)$ and $D(\sigma)$

Evaluation

Compute the data by induction on the concatenation operator
$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2)$ $D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$

Route evaluation examples

- Example 2) Objectives based on cumulated arrival time objectives

Auxiliary data structures in use:

Travel time $D(\sigma)$, Cumulated arrival time $C(\sigma)$, Delay Cost $W(\sigma)$ associated to one unit of delay in starting time

Initialization

For a sequence σ_0 with a single visit v_i , $D(\sigma_0) = 0$ and $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ if v_i is a customer, and $W(\sigma_0) = 0$ if v_i is a depot visit.

Forward, Backward and Evaluation

Induction on the concatenation operator:

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2)$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2)$$

Route evaluation examples

- Example 3) Time windows and route duration constraints

Auxiliary data structures in use:

Travel time and service time $T(\sigma)$, earliest feasible completion time $E(\sigma)$, latest feasible starting date $L(\sigma)$, statement of feasibility $F(\sigma)$.

Initialization:

For a sequence σ_0 with a single visit v_i , $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = true$.

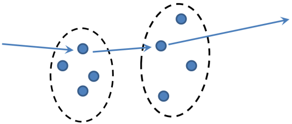
Forward, backward and evaluation:

Induction on the concatenation operator:

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2)$$
$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$
$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$
$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \ \wedge \ F(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$

Route evaluation examples

- Example 4) Generalized VRP : select one customer per cluster
  - Selecting customers for a sequence of clusters is done in route evaluations.



Auxiliary data structures in use:

Shortest path $S(\sigma)[i,j]$ inside sequence σ starting at the location i of the starting group and finishing at location j of the ending group.

Initialization

For a sequence σ_0 with a single visit v_i , $S(\sigma_0)[i,j] = +\infty$ if $i \neq j$, and $S(\sigma)[i,j] = 0$.

Forward, Backward and Evaluation

Induction on the concatenation operator:

$$S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j]$$

$$\forall i \in \{1, \dots, \lambda_{\sigma_1(1)}\}, \forall j \in \{1, \dots, \lambda_{\sigma_2(|\sigma_2|)}\}$$

Unified Hybrid Genetic Search (UHGS)

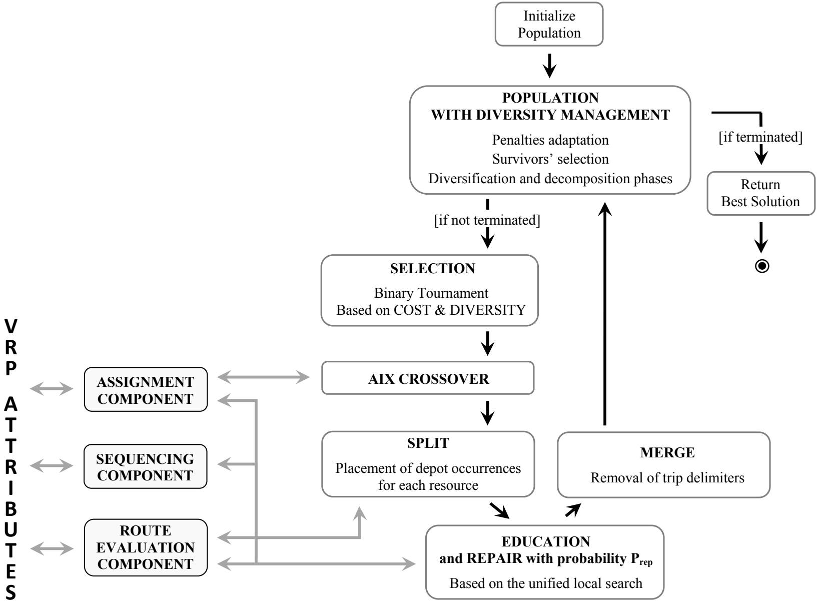
- Generic local-search based on route evaluation operators

Algorithm 1 Unified LS based on route-evaluation operators

Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on sub-sequences using the INIT, FORW and BACK operators.
while some improving moves exist in the neighborhood $\mathcal{N}$ **do**
  for each move μ_i in $\mathcal{N}$ **do**
    for each route r_j^μ produced by the move **do**
      Determine the k sub-sequences $[\sigma_1, \dots, \sigma_k]$ that are concatenated to produce r_j^μ
      if $k = 2$, then NEWCOST(r) = EVAL2(σ_1,σ_2)
      else if $k > 2$, then NEWCOST(r) = EVALN$(\sigma_1,\dots,\sigma_k)$
    if ACCEPTCRITERIA(μ_i) **then** perform the move μ and update the re-optimization data on for each route r_j^μ using the INIT, FORW and BACK operators.

- Can serve as the basis to build any neighborhood-based unified solver based on VNS, Tabu, ILS for vehicle routing variants.
- We even went one step further, and designed a unified hybrid genetic search (UHGS)

Unified Hybrid Genetic Search (UHGS)



Unified Hybrid Genetic Search (UHGS)

- UHGS has been tested on 1099 benchmark instances, for 34 structurally different VRP variants.
  - State-of-the-art results in the literature on all considered problems: VRP with capacity constraints, duration, backhauls, asymmetry, cumulative costs, simultaneous and mix pickup and deliveries, fleet mix, load dependency, multiple periods, depots, generalized deliveries, open routes, time windows, time-dependent travel time and costs, soft and multiple TW, truck driver scheduling regulations, many other problems and their combinations...
  - First method which addresses efficiently more than 6 problems, equals or outperforms all available methods from the literature (more than 204 methods).

A Unified Hybrid Genetic Search (UHGS) for MAVRPs

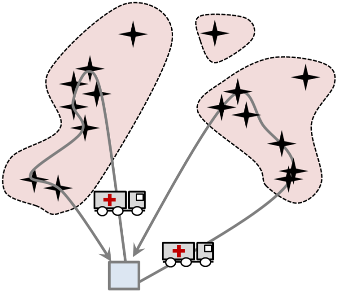
- Now moving towards different problems and richer neighborhood structures

Contents

Large neighborhoods: team-orienteering problem

- **Team-orienteering problem**:
  - Each customer i is associated with a prize p_i. Not all customers are to be serviced.
  - Each route must have a distance of less than D.
  - The goal is to generate m feasible routes while maximizing the total amount of prizes

- Numerous applications, including:
  - Logistics, third party providers, secondary market (Tricoire et al., 2010; Aras et al., 2011; Aksen et al., 2012)
  - Humanitary relief (Campbell et al., 2008)
  - Robotics, maintenance & military surveillance (Falcon et al., 2012; Mufalli et al., 2012).



Large Neighborhoods: Team-orienteering Problem

- Large amount of literature on TOP heuristics and metaheuristics

Acronym	Authors	Methodology
CGW	Chao et al. (1996)	Tabu Search
TMH	Tang and Miller-Hooks (2005)	Tabu Search
GTF	Archetti et al. (2007)	Tabu Search & VNS
ASe	Ke et al. (2008)	Ant colony optimization
BDM	Bouly et al. (2009)	Memetic Algorithm
GLS	Vansteenwegen et al. (2009)	Guided Local Search
SVNS	Vansteenwegen and Souffriau (2009)	Skewed VNS
SPR	Souffriau et al. (2010)	Path Relinking
DGM	Dang et al. (2011)	Particle Swarm Optimization
MSA	Lin (2013)	Multi-Start Simulated Annealing

Table : Metaheuristics for team-orienteering problems

Large Neighborhoods: Team-orienteering Problem

- Past works on heuristics consider separate neighborhood structures for
  - Customers selection : Insert or Remove
  - Route optimization : Relocate, Swap, 2-opt, 2-opt*
- Yet, difficult to insert a new customer in an existing solution structure to improve the objective
  $\Rightarrow$ motivating the use of complicate concepts such as ejection chains to try to fit new customers.

Large Neighborhoods: Team-orienteering Problem

- **Main Idea : always work on a full solution with all visits**
  - Q : How will customers be selected ?
  - A : Directly during separate route evaluations

- The problem of optimally selecting the customers from a complete solution can be assimilated to a shortest path with maximum profit under distance constraints for each route.

- We propose efficient techniques to solve this problem, combining
  - bi-directional dynamic programming,
  - graph sparsification,
  - and data preprocessing techniques.

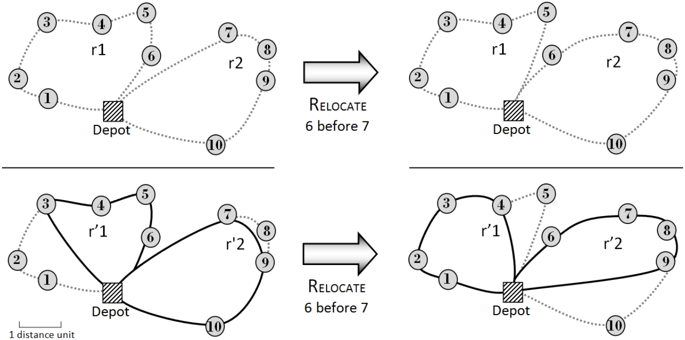
Large Neighborhoods: Team-orienteering Problem

- **Main interest:** Classic VRP neighborhoods on the complete solution representation ⇔ large neighborhoods with an exponential number of implicit insertions and removal of visits.
- SELECT algorithm at each move ⇔ resource-constrained SP

i	$d_{0,i}$	$d_{i-1,i}$	p_i
1	15	–	10
2	25	30	15
3	15	20	15
4	15	20	10
5	20	25	12
6	15	10	15
7	20	15	15
8	25	15	12
9	25	20	15
10	15	35	15

$D_{max} = 100$
$d_{7,9} = 25$
all other distances $= +\infty$

σ	$D(\sigma)$	$P(\sigma)$
(3,4,5,6)	85	52
(7,9,10)	95	45
(1,2,3,4)	100	50
(6,7,8,9)	90	57



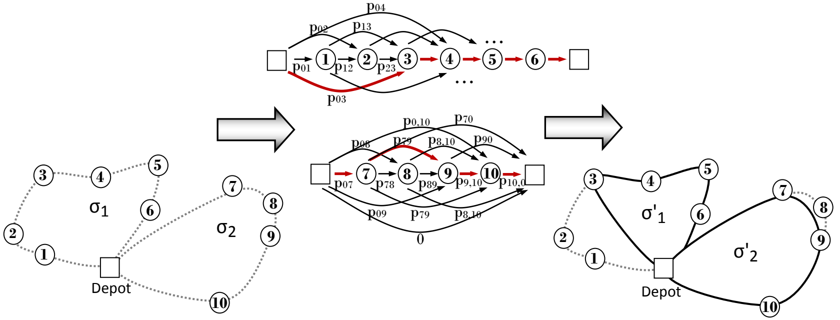
Large Neighborhoods: Team-orienteering Problem

Proposition

Let B be an upper bound on the number of labels per node. Then, the Select *algorithm is pseudo-polynomial, with a complexity of*

$$O(n^2B). \tag{3.1}$$

- In practice the number of labels remains very small, i.e., $B \leq 10$.



Large Neighborhoods: Team-orienteering Problem

- Using a particular hierarchical cost function which considers in priority the Team-Orienteering cost (with only selected customers), and then the VRP cost with all customers.

$$Z' = \max \sum_{\sigma \in \mathcal{R}} Z^{\text{SELECT}}(\sigma) - \omega \sum_{\sigma \in \mathcal{R}} \sum_{i \in \{1,\dots,|\sigma|-1\}} d_{\sigma(i)\sigma(i+1)}$$

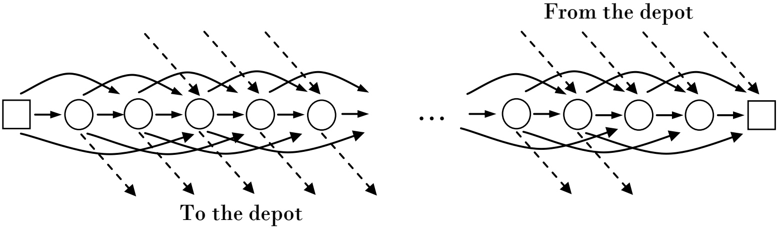
- As a consequence, when the method is unable to improve the primary objective, moves may still be performed to improve the second objective = the positioning of unserviced customers.
- This may lead in turn to a new repartition of customers and new opportunities of improvement of the main objective.

Large Neighborhoods: Team-orienteering Problem

- Speed-ups for move evaluations – 1. Graph Sparsification
  - For a given *sparsification parameter* $H \in \{1, \ldots, n\}$, only the arcs (i, j), with $(i < j)$ satisfying Equation (3.2) are kept.

$$j < i + H \text{ or } i = 0 \text{ or } j = |\sigma| \tag{3.2}$$

  - H is a sparsification parameter, usually small, e.g. $H = 3$.
  - Thus there are only O(Hn) arcs

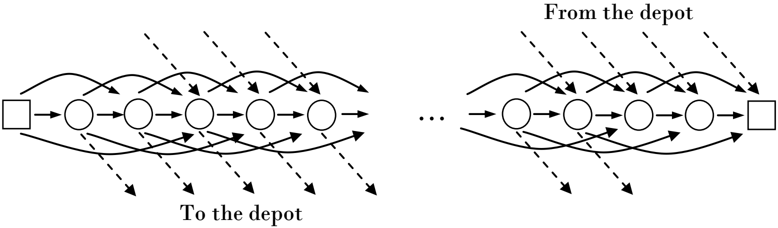


- Speed-ups for move evaluations – 1. Graph Sparsification

Proposition

After sparsification, the number of arcs $|\mathcal{A}'|$ in the new graph becomes $O(nH)$, and the complexity of SELECT*, in terms of number of elementary operations, is*

$$O(nHB). \tag{3.3}$$



Large Neighborhoods: Team-orienteering Problem

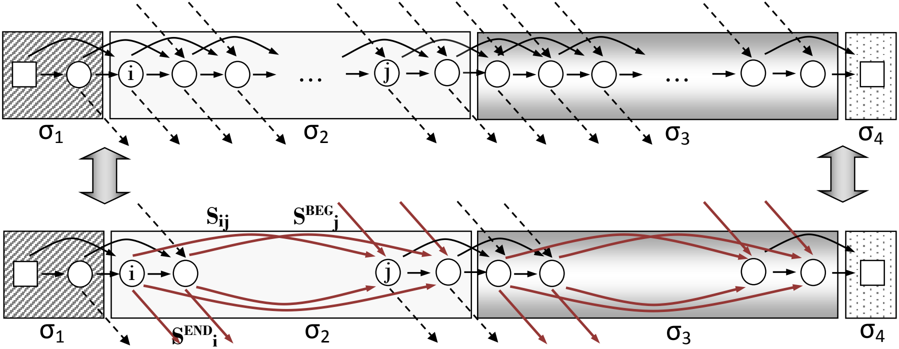
- Speed-ups for move evaluations – 2. Evaluation by Concatenation
- For any sequence σ of successive nodes from the incumbent solution, we propose to pre-process the following information :

Auxiliary data structures in use:

- Set of labels $S_{ij}(\sigma)$ associated to each resource-constrained path (i, j) between any node among the H first of σ, and any node among the H last of σ.
- Set of labels $S_i^{\text{END}}(\sigma)$ associated to each resource-constrained path between any node among the H first nodes of σ and the ending depot.
- Set of labels $S_j^{\text{BEG}}(\sigma)$ associated to each resource-constrained path between the beginning depot and any node among the H last of σ.
- Best profit $Z(\sigma)$ of a inside resource-feasible path in σ, starting from the depot, visiting a subset of customers in σ, and coming back to the depot.

Initialization and Pre-processing:

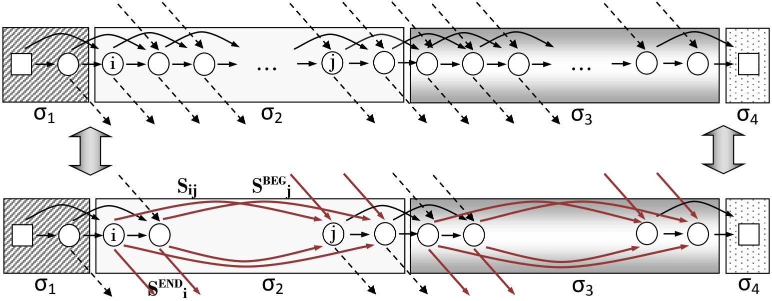
Preprocessing these values for a sequence σ requires $O(n^2HB)$ elementary operations



Large Neighborhoods: Team-orienteering Problem

Initialization and Pre-processing:

Preprocessing these values for a sequence σ requires $O(n^2HB)$ elementary operations



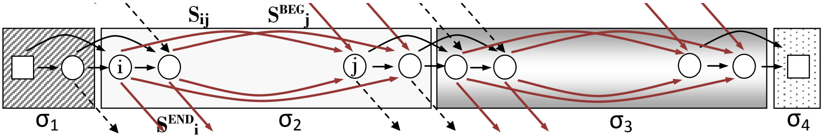
- The resulting *reduced* multi-graph $\mathcal{G}'' = (\mathcal{V}'', \mathcal{A}'')$ is such that $|\mathcal{A}''| = O(MH^2)$ arcs and $|\mathcal{V}''| = O(MH)$ nodes. M is the number of subsequences.

Large Neighborhoods: Team-orienteering Problem

Proposition (**Concatenation – general**)

The optimal profit $Z(\sigma_1 \oplus \cdots \oplus \sigma_M)$ *of* SELECT, *for a recombination of* M *sequences is the maximum between the profit* $\bar{Z}(\sigma_1 \oplus \cdots \oplus \sigma_M)$ *of the resource-constrained shortest path in* $\mathcal{G}''$, *and the maximum profit* $Z(\sigma_i)$ *of an inside resource-feasible path in* σ_i *for* $i \in \{1, \ldots, M\}$. *Furthermore,* $\bar{Z}(\sigma_1 \oplus \cdots \oplus \sigma_M)$ *can be evaluated in*

$$\Phi_{\text{C-M}} = O(MH^2B^2). \tag{3.4}$$

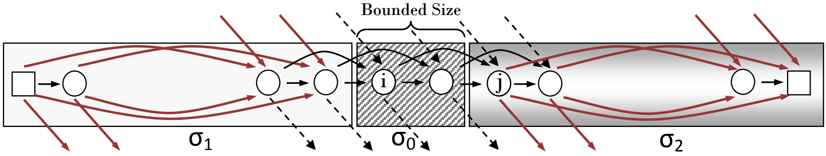


Large Neighborhoods: Team-orienteering Problem

Proposition (Concatenation – 2 or 3 subsequences)

The optimal cost $Z(\sigma_1 \oplus \sigma_0 \oplus \sigma_2)$ of Select*, for a route assimilated to a recombination of three subsequences σ_1, σ_0 and σ_2 such that σ_0 contains a bounded number of customers can be evaluated using bi-directional dynamic programming for a complexity of*

$$\Phi_{\text{C-3}} = O(H^2 B). \tag{3.5}$$



- The same complexity is achieved for a concatenation of two sequences σ_1 and σ_2.

Large Neighborhoods: Team-orienteering Problem

Proof.

- Set of non-dominated labels, for any path going from the beginning depot to one of the last H nodes of σ_1, is $S_k^{\mathrm{BEG}}(\sigma_1)$.
- Propagate these labels on the next $|\sigma_0|$ nodes in $O(HB|\sigma_0|)$ elementary operations.

$\square$

Large Neighborhoods: Team-orienteering Problem

Proof.

- Finally, the best cost $\bar{Z}(\sigma_1 \oplus \sigma_0 \oplus \sigma_2)$ of a path servicing nodes in both $\sigma_1 \oplus \sigma_0$ and σ_2 is equivalent to finding, for each possible arc $(i, j) \in \mathcal{A}''$ between $\sigma_1 \oplus \sigma_0$ and σ_2, the best couple of labels $(s_k^{\mathrm{R}}, s_k^{\mathrm{P}}) \in \bar{S}_i(\sigma_1 \oplus \sigma_0)$ and $(s_l^{\mathrm{R}}, s_l^{\mathrm{P}}) \in S_j^{\mathrm{END}}(\sigma_2)$, which maximizes the total profit while respecting resource constraints :

$$\bar{Z}(\sigma_1 \oplus \sigma_0 \oplus \sigma_2) = \max_{i \in \mathcal{I}} \max_{j \in \{1,\dots,H\}} \left\{ \begin{array}{rl} \max & s_k^{\mathrm{P}} + p_{\sigma'(i)\sigma_2(j)} + s_l^{\mathrm{P}} \\ \text{s.t.} & s_k^{\mathrm{R}} + r_{\sigma'(i)\sigma_2(j)} + s_l^{\mathrm{R}} \leq R \\ & (s_k^{\mathrm{R}}, s_k^{\mathrm{P}}) \in \bar{S}_i(\sigma_1 \oplus \sigma_0) \\ & (s_l^{\mathrm{R}}, s_l^{\mathrm{P}}) \in S_j^{\mathrm{END}}(\sigma_2) \end{array} \right\} \quad (3.6)$$

□

Proof.

$$\bar{Z}(\sigma_1 \oplus \sigma_0 \oplus \sigma_2) = \max_{i \in \mathcal{I}} \max_{j \in \{1,\dots,H\}} \left\{ \begin{array}{rl} \max & s_k^{\mathrm{P}} + p_{\sigma'(i)\sigma_2(j)} + s_l^{\mathrm{P}} \\ \text{s.t.} & s_k^{\mathrm{R}} + r_{\sigma'(i)\sigma_2(j)} + s_l^{\mathrm{R}} \leq R \\ & (s_k^{\mathrm{R}}, s_k^{\mathrm{P}}) \in \bar{S}_i(\sigma_1 \oplus \sigma_0) \\ & (s_l^{\mathrm{R}}, s_l^{\mathrm{P}}) \in S_j^{\mathrm{END}}(\sigma_2) \end{array} \right\} \quad (3.7)$$

For any pair (i, j), the maximum cost is found by sweeping the labels of $\bar{S}_i(\sigma_1 \oplus \sigma_0)$ by increasing resource consumption and, in the meantime, the labels of $S_j^{\mathrm{END}}(\sigma_2)$ by decreasing resource consumption. □

Large Neighborhoods: Team-orienteering Problem

- Finally, using pre-processing and the previous propositions
  - For a neighborhood of size $\Theta(n^2)$, the computational complexity of the preprocessing phase remains is $O(n^2HB)$.
  - Complexity of the evaluation of a full inter-route neighborhood is $\Theta(n^2H^2B)$ as demonstrated in Proposition 4.
  - Intra-route moves are less numerous, an average of $\Theta(\frac{n^2}{m})$ such moves to consider where m is number of routes.
  - The neighborhood evaluation complexity (c.f. Proposition 3) becomes $\Theta(\frac{n^2}{m}H^2B^2) = \Theta(\frac{B}{m}n^2H^2B)$.
- $\Rightarrow$ Move evaluations in amortized $O(H^2B)$ instead of $O(n^2B)$.

Contents

Experimental settings

- Experimental analysis of three heuristics and metaheuristics based on our large-neighborhood concepts
  - A simple local search (LS), restarted 100 times.
  - An Iterated Local Search (ILS), based on Prins (2009)
  - Unified Hybrid Genetic Search (UHGS) of Vidal et al. (2013)
- Benchmark instances:
  - Chao et al. (1996) for the TOP : 7 groups of instances. Groups 4-7 are the largest with 64 to 102 customers.
  - Bolduc et al. (2008) for a variant called VRP with private fleet and common carrier. These instances are derived from the CVRP instances of Christofides et al. (1979) and Golden et al. (1998).
- Tests conducted on a single Xeon 3.0GHz processor.
- Method performance evaluated relatively to Gap to Best Known Solutions BKS and CPU time.

Computational Experiments

Table : Summary of results on TOP benchmark instances

	CGW	TMH	GTF	SVF	ASe	SVNS	SPR	MSA	UHGS	ILS	LI
Best Gap 4	4.36%	1.99%	0.48%	0.06%	0.30%	1.46%	0.11%	0.06%	**0.01%**	0.05%	0.09%
Best Gap 5	1.36%	1.38%	0.01%	0.03%	0.04%	0.61%	0.05%	0.01%	**0.00%**	0.01%	0.01%
Best Gap 6	0.37%	0.79%	0.04%	0.00%	0.00%	0.52%	0.00%	0.00%	**0.00%**	0.00%	0.00%
Best Gap 7	2.68%	1.15%	0.29%	0.06%	0.00%	1.31%	0.04%	0.03%	**0.00%**	0.00%	0.07%
Avg Time 4	796.70	105.30	22.50	11.40	32.00	36.70	367.40	81.00	298.57	301.54	76.72
Avg Time 5	71.30	69.50	34.20	3.50	15.10	11.20	119.90	6.60	222.92	193.97	11.31
Avg Time 6	45.70	66.30	8.70	4.30	14.10	9.00	89.60	1.40	184.60	138.25	6.86
Avg Time 7	432.60	160.00	10.30	12.10	24.60	27.30	272.80	32.20	306.35	309.62	50.22

- Equaled or improved 380 of the 387 best known solutions.
- 4 new BKS, quite surprising since the problems have been studied by dozens of previous papers

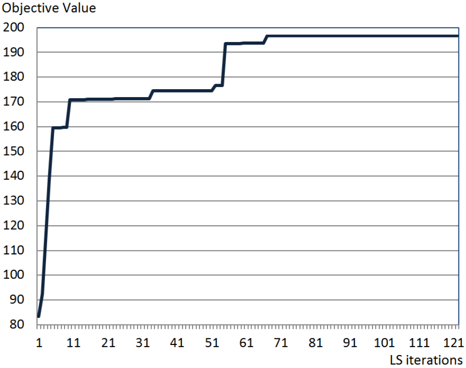
Computational Experiments

Table : Highlight of the results on some of the most difficult problems

Inst	CGW	TMH	GTF	SVF	ASe	SVNS	SPR	MSA	UHGS	ILS	LI	BKS
p4.2.o	1147	1175	1192	**1218**	1215	1195	**1218**	1217	**1218**	**1218**	**1218**	1218
p4.2.p	1199	1208	1239	1241	**1242**	1237	**1242**	1241	1241	1241	1241	1242
p4.2.q	1242	1255	1255	1263	1263	1239	1263	1259	**1267**	1265	1265	1265
p4.2.r	1199	1277	1283	1285	1288	1279	1286	**1290**	1286	1281	1285	1290
p4.2.s	1286	1294	1299	1301	**1304**	1295	1296	1300	1302	1297	1301	1304
p4.2.t	1299	**1306**	**1306**	**1306**	**1306**	1305	**1306**	**1306**	**1306**	**1306**	**1306**	1306
p4.3.o	1078	1151	1157	**1172**	1170	1136	1170	1170	**1172**	**1172**	1170	1172
p4.3.p	1115	1218	1221	**1222**	1221	1200	1220	**1222**	**1222**	**1222**	**1222**	1222
p4.3.q	1222	1249	1241	1245	1252	1236	**1253**	1251	**1253**	**1253**	1251	1253
p4.3.r	1225	1265	1269	1273	1267	1250	1272	1265	**1273**	1272	1269	1272
p4.3.s	1239	1282	1294	**1295**	1293	1280	1287	1293	**1295**	**1295**	**1295**	1295
p4.3.t	1285	1288	1304	1304	**1305**	1299	1299	1299	**1305**	**1305**	1299	1305
p4.4.o	995	1014	1057	**1061**	1036	1030	1057	**1061**	**1061**	**1061**	**1061**	1061
p4.4.p	996	1056	1120	1120	1111	1120	1122	**1124**	**1124**	**1124**	**1124**	1124
p4.4.q	1084	1124	1157	**1161**	1145	1149	1160	**1161**	**1161**	**1161**	1157	1161
p4.4.r	1155	1165	1211	1207	1200	1193	1213	**1216**	**1216**	**1216**	1211	1216
p4.4.s	1230	1243	1256	1260	1249	1213	1250	1256	**1260**	**1260**	**1260**	1259
p4.4.t	1253	1255	**1285**	**1285**	1281	1281	1280	**1285**	**1285**	**1285**	**1285**	1285
Best Gap	4.36%	1.99%	0.48%	0.06%	0.30%	1.46%	0.11%	0.06%	0.01%	0.05%	0.09%	
Avg Time	796.70	105.30	22.50	457.90	32.00	36.70	367.40	81.00	298.57	301.54	76.72	

Computational Experiments

- Objective function during a LS run on a TOP instance (p4.2.a)
  - Many moves only improve the secondary objective, and thus lead to a "plateau" effect
  - Eventually opening the way to new primary objective improvements
  - As a consequence the final local optimum is much "deeper"



- **VRP with Private Fleet and Common Carrier**:
  - Classic VRP formulation with
  - a fixed cost per vehicle, and
  - any customer can be assigned to an external carrier for a price p_i'.
- Can be addressed with a similar technique as for the TOP. The shortest paths subproblems involve a capacity constraint, and seek to minimize the distance.
- The fixed vehicle cost is included in the arc definition.

Computational Experiments

Table : Results on the VRP with Private Fleet and Common Carrier. Instances of Bolduc et al. (2008). Comparison to the current state-of-the-art methods for the problem : Bolduc et al. (2007) – B07, Côté et al. (2009) – CP09, Potvin and Naud (2011) – PN11.

Inst	**n**	**B07**	**CP09**	**PN11**	**UHGS**		**ILS**		**LI**		BKS
		1 Run	Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	
p01	50	1132.91	**1119.47**	**1119.47**	**1119.47**	**1119.47**	**1119.47**	**1119.47**	1124.35	1121.33	1119.47
p02	75	1835.76	1816.07	**1814.52**	1815.55	**1814.52**	1816.23	**1814.52**	1874.98	1844.19	1814.52
p03	100	1959.65	1930.28	1930.66	1923.62	**1919.05**	1925.26	1920.90	1954.52	1941.93	1924.99
p04	150	2545.72	2526.41	2525.17	2514.22	**2505.39**	2514.90	2505.72	2567.10	2553.34	2515.50
p05	199	3172.22	3112.25	3117.10	3097.66	3090.19	3104.11	**3088.03**	3176.73	3151.00	3097.99
p06	50	1208.33	**1207.47**	**1207.47**	**1207.47**	**1207.47**	**1207.47**	**1207.47**	1214.20	1208.33	1207.47
p07	75	**2006.52**	2010.96	**2006.52**	**2006.52**	**2006.52**	2011.24	**2006.52**	2071.89	2036.49	2006.52
p08	100	2082.75	2063.06	2056.59	2056.01	**2052.05**	2060.35	**2052.05**	2084.94	2078.32	2055.64
p09	150	2443.94	2433.86	2435.97	2425.79	**2421.11**	2430.91	2426.30	2491.72	2457.89	2429.19
p10	199	3464.90	3402.72	3401.83	3392.80	**3382.37**	3395.75	3389.04	3472.85	3457.11	3393.41
p11	120	2333.03	2336.59	2332.36	2331.29	**2330.94**	2335.81	**2330.94**	2382.60	2336.78	2330.94
p12	100	1953.55	1961.49	**1952.86**	1953.00	**1952.86**	**1952.86**	**1952.86**	1968.46	1953.55	1952.86
p13	120	2864.21	2863.96	2860.89	2858.99	**2858.83**	2858.93	**2858.83**	2985.81	2882.02	2859.12
p14	100	2224.63	2220.23	2216.97	**2213.02**	**2213.02**	**2213.02**	**2213.02**	2219.94	2215.49	2214.14
Gap		0.90%	0.25%	0.15%	-0.02%	-0.13%	0.07%	-0.10%	1.98%	0.91%	
T(min)		2.97	0.83	20.89	11.77		11.82		0.56		
CPU		Xe3.6G	Opt2.4G	Xe3.6G	Xe3.0G		Xe3.0G		Xe3.0G		

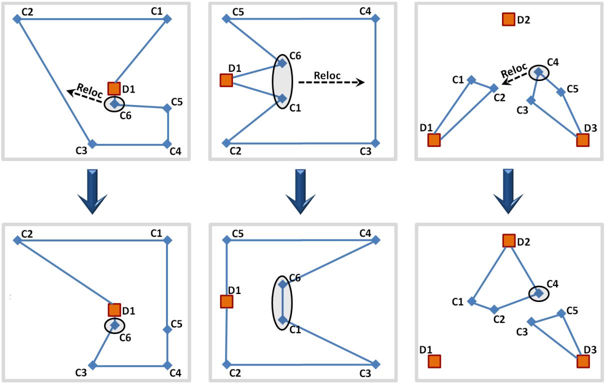
Table : Results on the VRPPFCC. Larger Instances of Bolduc et al. (2008)

Inst	**n**	**B07**	**UHGS**		**ILS**		**LI**		BKS
		1 Run	Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	
pr01	240	14388.58	14143.31	**14136.50**	14147.28	14138.05	14272.52	14228.87	14160.77
pr02	320	19505.00	19148.36	**19126.14**	19152.31	19128.40	19368.25	19325.88	19234.03
pr03	400	24978.17	24463.74	**24411.31**	24478.39	24420.70	24918.23	24772.38	24646.79
pr04	480	34957.98	34269.23	**34202.22**	34396.56	34229.07	34837.96	34663.49	34607.12
pr05	200	14683.03	14268.78	**14223.63**	14309.40	14229.50	14620.50	14488.16	14249.82
pr06	280	22260.19	21435.19	**21365.85**	21506.34	21449.18	21790.29	21677.79	21703.54
pr07	360	23963.36	23367.00	**23298.76**	23417.52	23355.13	23680.03	23568.02	23549.53
pr08	440	30496.18	29780.11	29716.47	29730.26	**29637.38**	30137.73	30059.52	30173.53
pr09	255	1341.17	1330.82	**1326.41**	1334.56	1329.50	1386.97	1364.63	1336.91
pr10	323	1612.09	1599.75	**1594.81**	1607.33	1596.00	1665.23	1653.14	1598.76
pr11	399	2198.45	2189.58	**2182.40**	2207.90	2188.31	2261.97	2232.73	2179.71
pr12	483	2521.79	2520.77	**2516.38**	2541.43	2530.49	2616.88	2593.51	2503.71
pr13	252	2286.91	2260.79	**2255.73**	2268.64	2260.88	2326.98	2311.70	2268.32
pr14	320	2750.75	2685.71	**2681.22**	2702.08	2690.71	2771.63	2752.22	2704.01
pr15	396	3216.99	3151.75	**3142.10**	3173.78	3163.26	3266.62	3244.25	3171.20
pr16	480	3693.62	3633.81	**3622.14**	3652.70	3642.11	3783.29	3752.50	3654.20
pr17	240	1701.58	1668.96	**1666.31**	1671.65	1666.96	1708.43	1703.05	1677.22
pr18	300	2765.92	2732.53	**2729.61**	2738.93	2735.66	2784.13	2776.25	2742.72
pr19	360	3576.92	3500.41	**3495.15**	3511.82	3502.37	3569.20	3554.84	3528.00
pr20	420	4378.13	4317.48	**4311.63**	4333.64	4325.06	4407.83	4391.69	4352.95
Gap		1.98%	-0.69%	-0.02%	-0.13%	-0.48%	2.02%	1.37%	
T(min)		27.59	115.10		98.36		5.59		
CPU		Xe3.6G	Xe3.0G		Xe3.0G		Xe3.0G		

Contents

Implicit rotations and depot management

- **Main Idea:** For the VRP and MDVRP, implicitly and optimally decide the first visit in the route (optimal rotation), the customer-to-depot or customer-to-vehicle type assignment in presence of unlimited fleet.



Implicit rotations and depot management

- **What is computed:** $C(\sigma)$ the distance to service the clients, and $\hat{C}(\sigma)$ the minimum distance supplement to reach a depot during σ.
- Define the distance supplement $\hat{c}_{ij} = \min_{o \in \{1,\dots,d\}}\{c_{io} + c_{oj} - c_{ij}\}$ as the additional distance required to visit the closest depot between customers v_i and v_j rather than driving directly.
- The values $C(\sigma)$ and $\hat{C}(\sigma)$ can be computed by induction on the concatenation operation as follows:

$$C(\sigma \oplus \sigma') = C(\sigma) + c_{\sigma_{|\sigma|}\sigma'_1} + C(\sigma')$$
$$\hat{C}(\sigma \oplus \sigma') = \min\{\hat{C}(\sigma), \hat{c}_{\sigma_{|\sigma|}\sigma'_1}, \hat{C}(\sigma')\}$$
$$Q(\sigma \oplus \sigma') = Q(\sigma) + Q(\sigma')$$

- And the cost $Z(\sigma)$ with the best depot choice is

$$Z(\sigma) = C(\sigma) + c_{\sigma_{|\sigma|}\sigma_1} + \min\{\hat{C}(\sigma), \hat{c}_{\sigma_{|\sigma|}\sigma_1}\} \tag{5.1}$$

Implicit rotations and depot management

- Moves are still evaluated in amortized $O(1)$, and the compound neighborhood is searched in $O(n^2)$, but the size of the neighborhood explored is much higher (all possible depot positions and choices), the size of $O(dn^3)$.
- Impact on solution quality on the MDVRP instances of Cordeau et al. (1997):

			Standard		**Implicit D & R**	
Method	Problem	Set	Avg Gap	T(min)	Avg Gap	T(min)
ILS	MDVRP	CGL 1	0.378%	8.46	0.268%	7.02
ILS	MDVRP	CGL 2	0.778%	11.02	0.133%	7.98
UHGS	MDVRP	CGL 1	0.056%	8.20	0.023%	10.09
UHGS	MDVRP	CGL 2	0.037%	9.25	0.025%	11.45

- Average CPU time is comparable (8min vs 10min), but much better solution quality.

Contents

Conclusions and Perspectives

- A simple LS outperforms all current metaheuristics in the TOP literature, demonstrating the large contribution of the new large neighborhoods on key vehicle routing variants.
- Results of very high quality for the three metaheuristics, extending UHGS and completing the range of problems efficiently addressed.
- Many combinatorial optimization problems are based on a network and a selection of nodes (VRP, steiner trees, generalized VRP, covering tours or trees, among others). The proposed concepts can be generalized to some of these problems and may find interesting applications in various fields.
- The implicit rotations and depot management is widely applicable to many vehicle routing and scheduling settings.

Thank you

THANK YOU FOR YOUR ATTENTION !

For further reading I

Aksen, D., O. Kaya, F. S. Salman, Y. Akça. 2012. Selective and periodic inventory routing problem for waste vegetable oil collection. *Optimization Letters* **6**(6) 1063–1080.

Aras, Necati, Deniz Aksen, Mehmet TuÄ§rul Tekin. 2011. Selective multi-depot vehicle routing problem with pricing. *Transportation Research Part C: Emerging Technologies* **19**(5) 866–884.

Archetti, C., A. Hertz, M.G. Speranza. 2007. Metaheuristics for the team orienteering problem. *Journal of Heuristics* **13**(1) 49–76.

Bolduc, M-C, J Renaud, F Boctor, G Laporte. 2008. A perturbation metaheuristic for the vehicle routing problem with private fleet and common carriers. *Journal of the Operational Research Society* **59**(6) 776–787.

Bolduc, Marie-Claude, Jacques Renaud, Fayez Boctor. 2007. A heuristic for the routing and carrier selection problem. *European Journal of Operational Research* **183**(2) 926–932.

Bouly, H., D.-C. Dang, A. Moukrim. 2009. A memetic algorithm for the team orienteering problem. *4OR* **8**(1) 49–70.

Campbell, A.M., D. Vandenbussche, W. Hermann. 2008. Routing for Relief Efforts. *Transportation Science* **42**(2) 127–145.

Chao, I., B. Golden, E.A. Wasil. 1996. The team orienteering problem. *European Journal of Operational Research* **88**(3) 464–474.

For further reading II

Christofides, N., A. Mingozzi, P. Toth. 1979. The vehicle routing problem. N. Christofides, A. Mingozzi, P. Toth, C. Sandi, eds., *Combinatorial Optimization*. Wiley, Chichester, 315–338.

Cordeau, J.-F., M. Gendreau, G. Laporte. 1997. A tabu search heuristic for periodic and multi-depot vehicle routing problems. *Networks* **30**(2) 105–119.

Côté, J.-F., M. Gendreau, J.-Y. Potvin. 2009. Large Neighborhood Search for the Single Vehicle Pickup and Delivery Problem with Multiple Loading Stacks. *CIRRELT Working Paper 2009-47* .

Dang, D.-C., R. Guibadj, A. Moukrim. 2011. A PSO-Based Memetic Algorithm for the Team Orienteering Problem. *Applications of Evolutionary Computation*, *LNCS*, vol. 6625. Springer Berlin Heidelberg, 471–480.

Falcon, R., X. Li, A. Nayak, I. Stojmenovic. 2012. A Harmony-Seeking Firefly Swarm to the Periodic Replacement of Damaged Sensors by a Team of Mobile Robots. *ICC'12*. 4914–4918.

Golden, B.L., E.A. Wasil, J.P. Kelly, I. Chao. 1998. The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results. T.G. Crainic, G. Laporte, eds., *Fleet management and Logistics*. Kluwer, Boston, 33–56.

Ke, L., C. Archetti, Z. Feng. 2008. Ants can solve the team orienteering problem. *Computers & Industrial Engineering* **54**(3) 648–665.

For further reading III

Kindervater, G.A.P., M.W.P. Savelsbergh. 1997. Vehicle routing: Handling edge exchanges. E.H.L. Aarts, J.K. Lenstra, eds., *Local Search in Combinatorial Optimization*. Princeton Univ Pr, 337–360.

Lin, Shih-Wei. 2013. Solving the team orienteering problem using effective multi-start simulated annealing. *Applied Soft Computing* **13**(2) 1064–1073.

Mufalli, Frank, Rajan Batta, Rakesh Nagi. 2012. Simultaneous sensor selection and routing of unmanned aerial vehicles for complex mission plans. *Computers & Operations Research* **39**(11) 2787–2799.

Potvin, J.-Y., M.-A. Naud. 2011. Tabu search with ejection chains for the vehicle routing problem with private fleet and common carrier. *Journal of the Operational Research Society* **62**(2) 326–336.

Prins, C. 2009. A GRASP - evolutionary local search hybrid for the vehicle routing problem. F.B. Pereira, J. Tavares, eds., *Bio-Inspired Algorithms for the Vehicle Routing Problem*. Springer Berlin Heidelberg, 35–53.

Savelsbergh, M.W.P. 1985. Local Search in Routing Problems with Time Windows. *Annals of Operations Research* **4**(1) 285–305.

Savelsbergh, M.W.P. 1992. The Vehicle Routing Problem with Time Windows: Minimizing Route Duration. *ORSA Journal on Computing* **4**(2) 146–154.

Souffriau, W., P. Vansteenwegen, G. Vanden Berghe, D. Van Oudheusden. 2010. A Path Relinking approach for the Team Orienteering Problem. *Computers & Operations Research* **37**(11) 1853–1859.

For further reading IV

Subramanian, A., E. Uchoa, L.S. Ochi. 2013. A hybrid algorithm for a class of vehicle routing problems. *Computers & Operations Research* **40**(10) 2519–2531.

Tang, H, E. Miller-Hooks. 2005. A TABU search heuristic for the team orienteering problem. *Computers & Operations Research* **32**(6) 1379–1407.

Tricoire, F., M. Romauch, K.F. Doerner, R.F. Hartl. 2010. Heuristics for the multi-period orienteering problem with multiple time windows. *Computers & Operations Research* **37**(2) 351–367.

Vansteenwegen, P, W Souffriau. 2009. Metaheuristics for tourist trip planning. K. Sörensen, M. Sevaux, W. Habenicht, M.J. Geiger, eds., *Metaheuristics in the Service Industry*. Lecture Notes in Economics and Mathematical Systems, Springer-Verlag Berlin Heidelberg, 15–31.

Vansteenwegen, P., W. Souffriau, G.V. Berghe, D.V. Oudheusden. 2009. A guided local search metaheuristic for the team orienteering problem. *European Journal of Operational Research* **196**(1) 118–127.

Vidal, T., T.G. Crainic, M. Gendreau, N. Lahrichi, W. Rei. 2012. A hybrid genetic algorithm for multidepot and periodic vehicle routing problems. *Operations Research* **60**(3) 611–624.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2013. A unified solution framework for multi-attribute vehicle routing problems. *European Journal of Operational Research, Articles in Advance* .