Phase Unwrapping and Operations Research

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Joint work with Ian Herszterg and Marcus Poggi

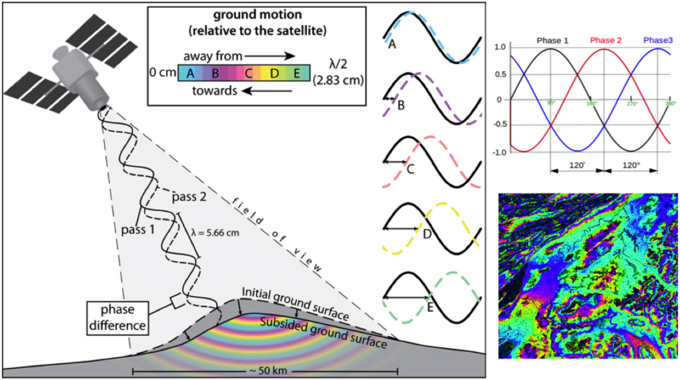


Figure : Radar interferometry

Wrapped phase

- While the phase information can take any real value, it is wrapped to a 2π interval with a $]-\pi, \pi]$ domain by the $\arctan$ operator.

Wrapped phase

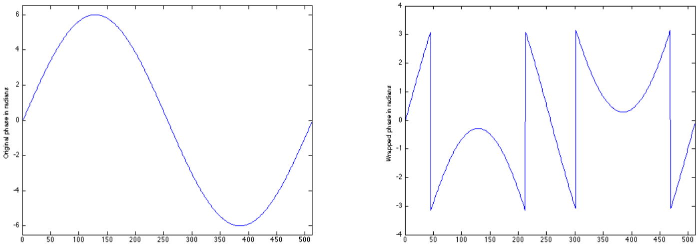


Figure : *Wrapping effect on a 1D continuous phase signal*

Wrapped phase

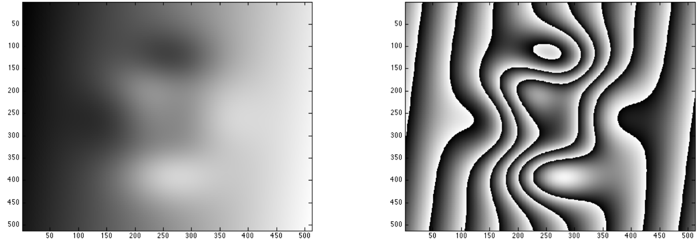


Figure : *Wrapping effect on a 2D phase image*

Phase Unwrapping

- **Phase Unwrapping** = reconstructing the continuous signal by removing the 2π-multiple ambiguity.
- ITOH, K., Analysis of the phase unwrapping algorithm, Applied Optics, v.21, n.14, p. 2470-2470, 1982

Itoh's Unwrapping Method (for discretized phase values):

Input = Wrapped phase values, $\psi(n)$

Output = Unwrapped phase values $\phi(n)$

Initialization: $\phi(1) = \psi(1)$;

For $i \leftarrow 2$ to N

$\Delta_\psi \leftarrow \psi(i) - \psi(i-1)$;
IF $\Delta_\psi \leq -\pi$

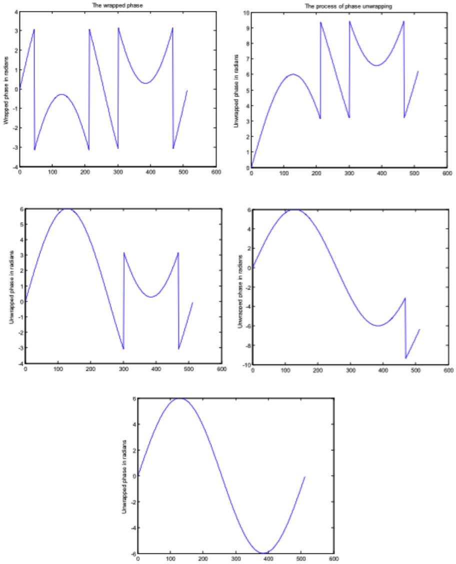
$\Delta_\psi \leftarrow \Delta_\psi + 2\pi$

ELSEIF $\Delta_\psi > \pi$

$\Delta_\psi \leftarrow \Delta_\psi - 2\pi$;

$\phi(i) \leftarrow \phi(i-1) + \Delta_\psi$;

Phase Unwrapping



Phase Unwrapping

Itoh's condition: For unambiguous phase unwrapping, the difference between any two adjacent samples in the continuous phase signal should not exceed a value of π

Phase Unwrapping

- Phase unwrapping problems often comes from complex applications dealing with rich geometries and signal acquisition methods that are highly susceptible to noise.
- Itoh's condition is not fulfilled $\Rightarrow$ Occurrence of *"fake wraps"*.
- Errors are propagated through subsequent samples in the unwrapping process.

Phase Unwrapping

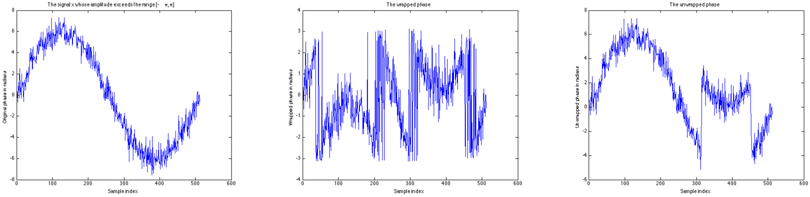


Figure : Unwrapping process over noisy data

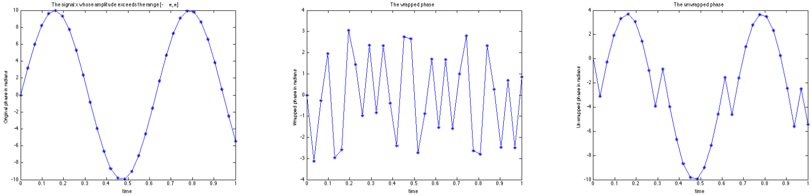


Figure : Unwrapping process over under-sampled data

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Residue theory

- **In higher dimension:** Itoh's algorithm can be applied to any continuous integration path

$\Rightarrow$ Every integration path P can constitute a discrete unwrapping path over any multidimensional space.

$\Rightarrow$ Paths could be selected to avoid damaged regions (noise, under-sampling)

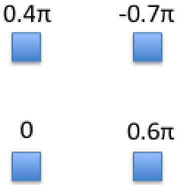
Residue theory

- How to detect singularities in two or more dimensions?

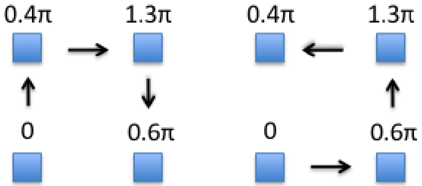
  GHIGLIA, D. C.; MASTIN, G. A.; ROMERO, L. A, Cellular automata method for phase unwrapping, v.4, n.1, p. 267-280, 1987

Residue theory

Wrapped phase example A – No singularity:

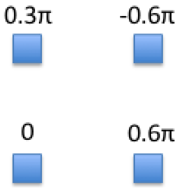


Unwrapped values from Example A:

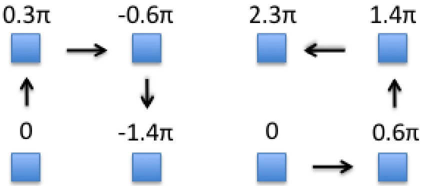


Residue theory

Wrapped phase example B – Singularity:



Unwrapped values from Example B:

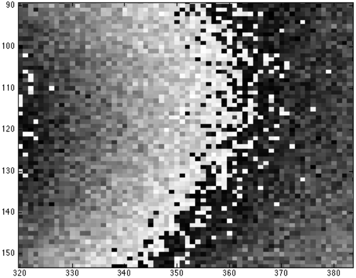


Residue theory

⇒ The location of all singularities can be identified by checking all 2x2 elementary loops (Ghiglia & Pritt, 1998). These specific points are called "residues"

- Residues charges (polarity) are either positive (+1) or negative (-1)
- In the presence of residues, an unambiguous phase unwrapping is possible if, and only if, every integration path encircles none or a *balanced* number of residues (as many positive as negative)

Residue theory



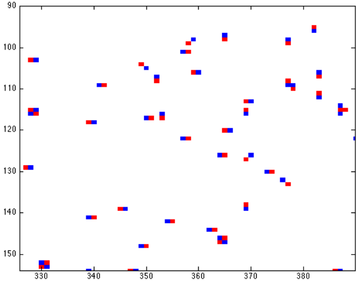


Figure : *Residues detected over a wrapped phase image corrupted by noise*

Residue theory

- Yet, not all residues come from noise
- Phase discontinuities are naturally present in many phase unwrapping applications.
- The topology of residues may suggest structural delimitations in the subject of study.

Residue theory

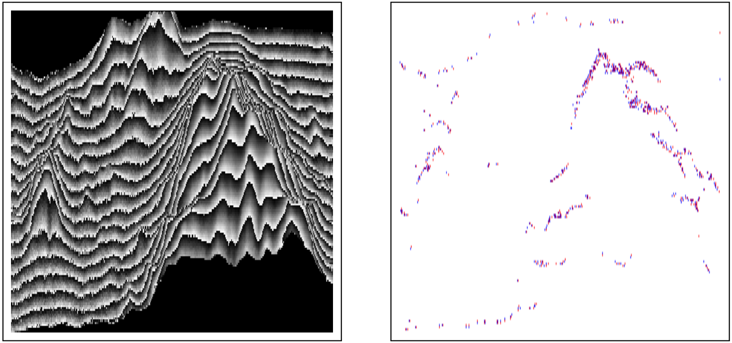
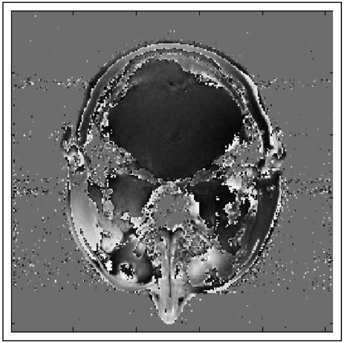


Figure : *Wrapped phase data and residues – high-fidelity InSAR simulator on a steep-relief mountainous region in Colorado*

Residue theory



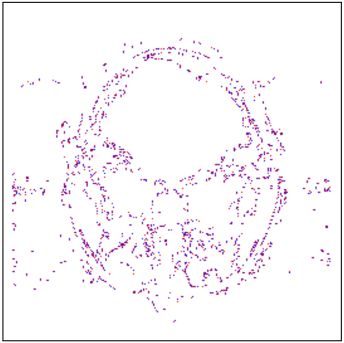


Figure : *Wrapped phase data and residues – head MRI*

Path-following Methods

- **Path-following Methods:**

⇒ Apply the path unwrapping method, but the solution is unique if and only if no integration path can encircle an unbalanced number of residues

⇒ For this purpose, create artificial barriers called *branch-cuts* to solve the path-dependency problem.

⇒ Branch-cuts can introduce a $\pm 2\pi$ discontinuity between samples in opposite sides of the barriers.

Path-following Methods

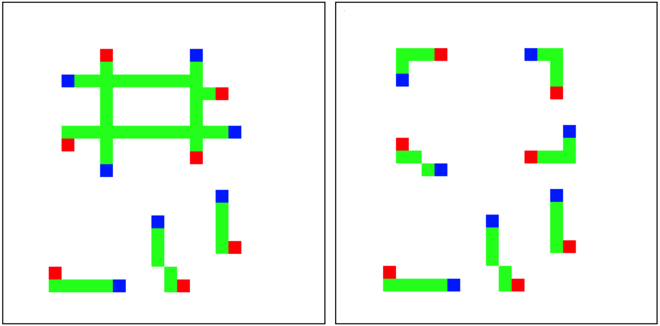


Figure : *Example of residues (blue, red) and possible branch-cuts configurations (green).*

Path-following Methods

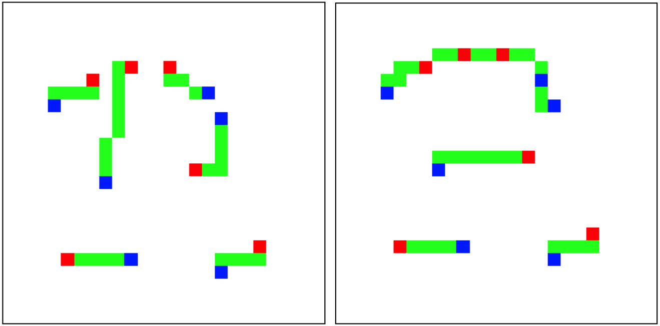


Figure : *Another example of residues (blue, red) and possible branch-cuts configurations (green).*

Path-following Methods

- The placement of branch-cuts fully characterizes the unwrapped solution

$\Rightarrow$ **Matching pairs of residues** is possible (Buckland et al. 1995. *Unwrapping noisy phase maps by use of a minimum-cost-matching algorithm*. Applied Optics, 44(0))

$\Rightarrow$ **Creating trees of residues** is possible, as long as they include a balanced number of positive and negative residues (or are connected to the border of the image).

$\Rightarrow$ **Using Steiner points** is possible

- Minimizing the length of the branch-cuts is a variant of geometrical Steiner problem with additional balance constraints $\Rightarrow$ NP-hard (and quite “tough” in practice for heuristic and exact methods)

Norm minimization

- Finally, from a "norm-minimization" perspective

⇒ Seeking a continuous solution whose gradients are "as close as possible" to those of the wrapped signal (norm minimization)

$$\arg\min_{\Phi} \sum_{m=1}^{M} \sum_{n=1}^{N-1} \left|\Delta^h \phi_{m,n} - \Delta^h \psi_{m,n}\right|^p + \sum_{m=1}^{M} \sum_{n=1}^{N-1} \left|\Delta^v \phi_{m,n} - \Delta^v \psi_{m,n}\right|^p$$

$$\textbf{\textit{s.t.}} \quad \phi_{m,n} = \psi_{m,n} + 2\mathbf{k}_{m,n}\pi \quad \forall (m,n)$$

$$\mathbf{k} \in \mathbb{Z} \quad \forall (m,n)$$

⇒ Remark that the length of the branch cuts is an upper bound of the number of differences of gradient (L^0-norm) between the wrapped signal and the continuous solution.

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Proposed methodology:

- We search for a minimum-cost balanced spanning forest (MCBSF)
- Spanning trees are allowed, as long as they contain a balanced number of residues, or are connected to the border of the image
- *We do not include Steiner points in the solutions.*

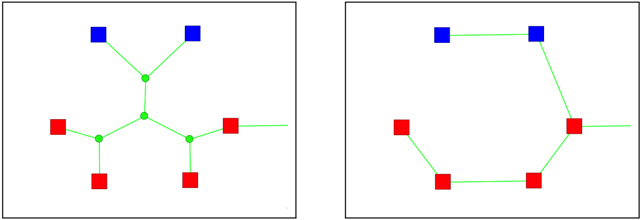


Figure : *Using Steiner-trees to cluster groups of residues.*

Main assumptions

- *We do not include Steiner points in the solutions* ⇒ **Why?**

⇒ Spanning trees better respect the natural boundaries of the image (cliffs, fractures...)

⇒ For most practical purposes, the optimal spanning tree solution is a high-quality approximation of the Steiner solution.

⇒ The model remains NP-hard, but efficient combinatorial optimization methods can be developed.

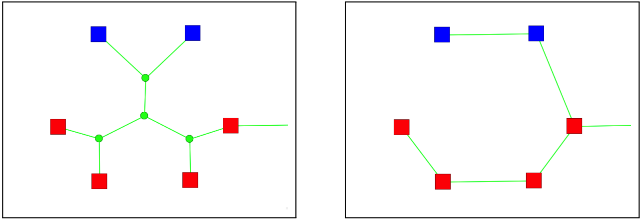


Figure : *Using Steiner-trees or spanning trees to cluster groups of residues.*

Mathematical formulation: Cut-based

Let $G = (V,E)$ be a graph with positive edge costs, where every vertex $v \in V$ has a weight $w_v \in \{-1, 1\}$. Let d_e be the cost (distance) of edge $e \in E$ and x_e be the decision variable indicating whether edge e should be part of the solution.

Mathematical formulation: Cut-based

$$
\begin{aligned}
\min \sum_{e \in E} d_e x_e & \\
\text{s.t.} \sum_{a \in \delta^+(S)} x_a \geq 1 & \qquad \forall S \subset V \text{ such that } \sum_{v \in S} w_v > 0 \\
\sum_{a \in \delta^-(S)} x_a \geq 1 & \qquad \forall S \subset V \text{ such that } \sum_{v \in S} w_v < 0 \\
x_e + x_{e'} \leq 1 & \qquad \forall e = (i,j), e' = (j,i) \in E \\
x_e \in \{0,1\} & \qquad \forall e \in E
\end{aligned}
$$

Mathematical formulation: Set Partitioning

- Set Partitioning formulation (SPF) for the MSFBC:
  - Let J be the set of all balanced subsets V_j of V
  - c_j is the cost of the MST connecting subset V_j

$$\min \sum_{j \in J} c_j x_j$$

$$\text{s.t.} \quad \sum_{j \in J} a_{vj} x_j = 1 \qquad \forall v \in V$$

$$x_j \in \{0,1\} \qquad \forall j \in J$$

$$a_{vj} = \begin{cases} 1 & \text{if } v \in S_j \\ 0 & \text{if } v \notin S_j \end{cases}$$

Solution Methods

- We developed mathematical programming approaches using the cut-based formulation, and metaheuristics
  - Primal Heuristics (metaheuristics)
  - Dual Heuristic + Dual Ascent to discard non-promising arcs
  - Branch-and-cut algorithm

Solving the linear program

- Because of the exponential number of constraints, unrealistic to solve even the linear program ⇒ cuts generation.

```
input  : The instance of the problem
output: The optimal solution set
Initialization: Solve the initial LP considering only the cuts with single
vertices as constraints. Let x be the solution set, lp the current linear
program and balanced a boolean indicating if all trees are balanced.
x ← solve(lp)
balanced ← false;
while not balanced do
    Build G̅ = (V̅, E̅) from the solution set x;
    balanced ← true;
    foreach pair (i,j) of vertices in V̅ do
        {S, maxFlow} ← minCutMaxFlow(G, i, j);
        if S is unbalanced, maxFlow < 1 and S ∉ lp then
            lp ← lp + {S};
            balanced ← false;
        end
    end
    if balanced is false then
        x ← solve(lp);
    end
end
return x;
```

Dual Heuristic

- Dual Heuristics: **Dual Ascent** (over the cut-based, directed, formulation)
  - ⇒ Selects violated cuts and increase their dual variables until one arc becomes saturated
  - ⇒ Based on Wong's Dual Ascent procedure for the Steiner-tree problem with directed cuts formulation (Wong, R.T., 1984. *A dual ascent approach for steiner tree problems on a directed graph*. Mathematical Programming, 28(3), pp.271-287.)
  - ⇒ Selection: Greedy or Random

Dual Ascent + Dual Scaling

- The selection of violated cuts was tested with two different criteria:
  - (**minrc**) by the minimum reduced cost arc in the graph (the cut that contains a minimum reduced cost-edge in its edge set)
  - (**random**) by randomly selecting a non maximal dual variable and saturating at least one of its arcs
- **Dual Scaling**
  - Multiplying the dual solution by a constant factor $0 < \alpha < 1$, and reapplying the dual ascent

Dual Heuristic

input : A dual initial solution π
output: A feasible dual solution π'

Initialization: *Build $G_\pi = (V, E)$ from the saturated arcs in π*
$\pi' \leftarrow \pi$;
while *exists a violated cut $R \in G_\pi$* **do**
    $W \leftarrow$ *selectViolatedCut()*;
    if $\sum_{v \in W} p_v > 0$ **then**
        Augment π'_W until at least one arc in δ^-(W) becomes saturated;
    end
    else if $\sum_{v \in W} p_v < 0$ **then**
        Augment π'_W until at least one arc in δ^+(W) becomes saturated;
    end
    Add the newly saturated arcs in G_π;
end
return π';

Branch-and-cut

- **Branch-and-Cut**
  - Based on the directed formulation
  - Uses primal bounds and dual bound to fix arcs by reduced cost
  - Uses the unbalanced cuts of the dual solution as initial constraints
  - Solves the linear relaxed program at each node
  - Branching: choose the most fractional variable
  - Exploration: depth-first search

Primal Heuristic – ILS

- Iterated Local Search (ILS) metaheuristic
  - Using an **indirect solution representation**: a solution is represented as a partition of the set of vertices into *components* $P_1, \ldots, P_k$ such that $\bigcup P_i = V$
  - The cost $c(P_i)$ of any component P_i can be efficiently derived by solving a minimum-cost spanning tree problem.
  - Any unbalanced component P_i is not considered infeasible, but must be connected to a dummy node that represents the border of the image.

- Iterated Local Search (ILS) metaheuristic
  - **Initial Solution** obtained by computing a minimum-cost spanning tree over V and disconnecting edges that are longer than a threshold d_{MAX}.
  - **Local Search** based on a variety of neighborhoods.
  - **Large neighborhood search** using mathematical programming over a set partitioning formulation
  - **Simple perturbation procedure**

Primal Heuristic – ILS

Algorithm 1 Hybrid Iterated Local Search

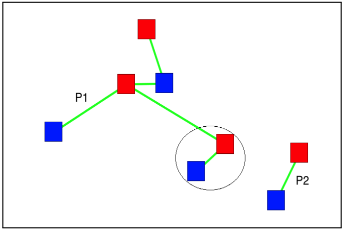
$S \leftarrow$ GenerateInitialSolution;
$S^* \leftarrow S$; $It_{\text{SHAK}} \leftarrow 0$;
while $It_{\text{SHAK}} < It_{\text{MAX}}$ **do**
    $S \leftarrow$ LocalSearch(S);
    if $\exists\, k \in \mathbb{N}^+$ s.t. $It_{\text{SHAK}} = k \times It_{\text{SP}}$ **then**
        $S \leftarrow$ SetPartitioning();
    end if
    if $c(S) < c(S^*)$ **then**
        $S^* \leftarrow S$;
        $It_{\text{SHAK}} \leftarrow 0$;
    end if
    $S \leftarrow$ Perturb(S) or Perturb(S^*) with equal probability;
end while
return S^*;

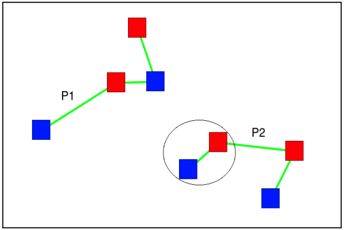
Primal Heuristic – ILS

- Iterated Local Search (ILS) metaheuristic
    - **Local Search** based on a variety of neighborhoods.
    - Enumerating all component pairs (T_i, T_j) in random order to test the associated moves.
    - Each move evaluation requires to build the new spanning trees for the modified components
    - First-improvement policy

ILS – Neighbourhoods

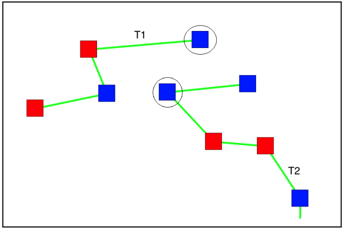
- Relocate - single vertices +/- or pairs (+,-)
  - Relocates one or more vertices v_i from T_1 to T_2, independently of its polarity, or any pair of opposite signed vertices v_i and v_j from T_1 to T_2.

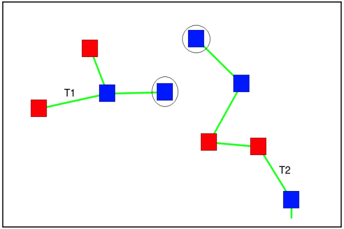




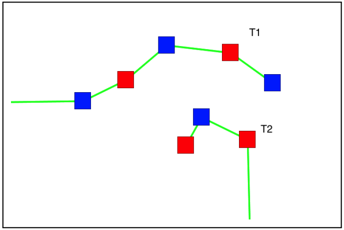
ILS – Neighbourhoods

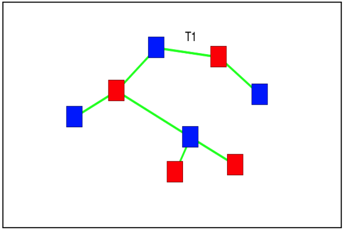
- Swap - single vertices +/- or pairs (+,-)
  - Swaps one or more vertices v_i from T_1 with v_j from T_2, both with same polarity, or a pair of vertices v_i and v_j from T_1 and a pair v_k and v_l from T_2 with opposite signed polarities.





- Merge
  - Merges two components T_1 and T_2, into a single component

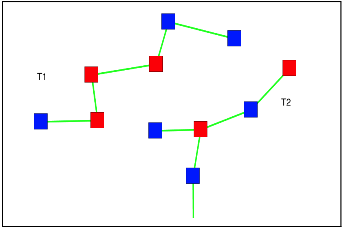


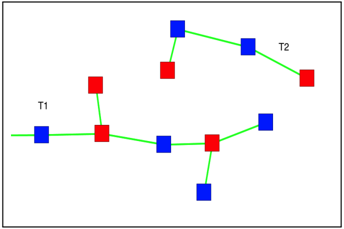


- Break
  - Breaks the longest edge in a given tree T_2, generating two new components

ILS – Neighbourhoods

- Break1-Insert1
  - Merges two given trees, T_1 and T_2, into a single component, compute the spanning tree and disconnect the longest edge, forming two new components.





ILS – Local Search

- **Speed-up** procedures:
  - **Memory structures** to avoid testing again moves that are known to be non-improving.
  - **Pruning**: avoids moves on trees that are very distant from each other by computing a maximum distance radius for each vertex.

ILS – Perturbation

- **Perturbation procedure** is applied to escape from local minima.
- Applied with equal probability to either S or S^*
- From the spanning-tree representation of the solution, with T, components, the perturbation removes $k \in \{1, \lceil 0.15T \rceil\}$ edges, creating disjoint components which are randomly recombined to resume the search with T components.

ILS – Set Partitioning

- Regularly solving the set partitioning formulation, using a pool of columns collected from local minimums of the ILS.
- The size of the pool is limited to 2000 columns.
- Executed every It_{SP} iterations.

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Overview

- Computational experiments designed to address two main objectives:
  - Validade and investigate the performance of the proposed methods.
  - Evaluate the performance of the MSFBC approach in the two-dimensional phase unwrapping domain, when compared to other path-following methods.
- Instances designed to test and identify the limitations and the scalability factor of the proposed methods.

Benchmark Instances

- Generated by randomly spreading p positive and n negative vertices on a $4p \times 4n$ Euclidean space.
- Complete graph: edge costs defined by the 2D Euclidean distance between vertices
- Every vertex is also connected to its closest border point
- 21 sets of 5 instances each: 8 to 1024 nodes
- We have collected the best solutions ever found during the heuristics and exact methods in order to evaluate the quality of each proposed algorithm.

Experiments – Hybrid ILS

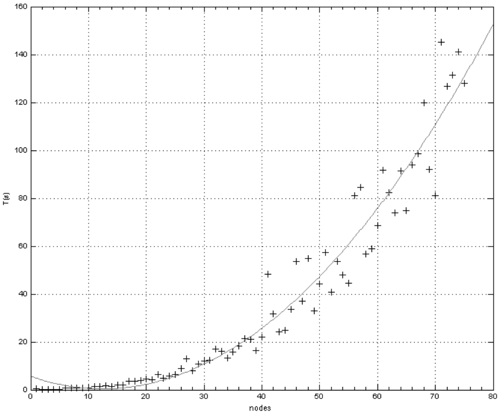
- Executed for 10 times with two termination criterion per run, whichever came first:
  - 100 iterations ($It_{MAX} = 100$) without improving the best solution found
  - A time bound of 3600 seconds
- The set covering routine is executed at every $(1/3)It_{MAX}$ iterations, with a time bound of 300 seconds
- The maximum distance radius for every vertex v is limited to 25% of the shortest distances between v and the set of vertices V-$\{v\}$

Experiments – Hybrid ILS

Group	$\|V\|$	GAP$_B$(%)	OPT	GAP$_{AVG}$(%)	Avg_T
PUC-8	8	0.00	**5/5**	2.49	0.27
PUC-12	12	0.00	**5/5**	0.00	0.77
PUC-16	16	0.00	**5/5**	0.52	1.60
PUC-20	20	0.00	**5/5**	0.21	3.60
PUC-24	24	0.00	**5/5**	0.64	5.46
PUC-28	28	0.00	**5/5**	1.06	10.50
PUC-32	32	0.00	**5/5**	0.76	14.88
PUC-36	36	0.00	**5/5**	0.94	19.91
PUC-40	40	0.00	**5/5**	0.63	32.49
PUC-44	44	0.14	**4/5**	1.44	44.50
PUC-48	48	0.00	**5/5**	0.73	48.91
PUC-52	52	0.00	**5/5**	1.33	70.03
PUC-56	56	0.00	**5/5**	1.35	82.76
PUC-60	60	0.00	**5/5**	1.15	97.13
PUC-64	64	0.35	4/5	3.02	134.45
PUC-80	80	0.40	3/5	3.26	304.64
PUC-96	96	0.10	**2/2**	4.81	650.05
PUC-128	128	1.05	**2/2**	5.45	2091.65
PUC-256	256	0.00	0/0	5.78	3600.00
PUC-512	512	0.00	0/0	6.31	3600.00
PUC-1024	1024	4.00	0/0	4.85	3600.00

Experiments – Hybrid ILS

- Growth of the CPU time appears to be cubic as a function of instance size.



Dual Ascent + Dual Scaling

Group	$\text{GAP}_{minrc}(\%)$	$\text{GAP}_{random}(\%)$	$\text{T}_{minrc}(\text{s})$	$\text{T}_{random}(\text{s})$
PUC-8	2.09	0.00	<0.01	<0.01
PUC-12	7.33	0.00	<0.01	<0.01
PUC-16	2.75	3.55	<0.01	<0.01
PUC-20	6.46	1.45	<0.01	<0.01
PUC-24	3.78	3.78	<0.01	<0.01
PUC-28	8.92	3.34	0.01	<0.01
PUC-32	9.19	3.71	0.01	<0.01
PUC-36	14.63	7.61	0.01	<0.01
PUC-40	13.05	2.96	0.02	0.01
PUC-44	13.82	3.74	0.02	0.01
PUC-48	4.78	3.22	0.02	0.01
PUC-52	11.93	4.21	0.03	0.02
PUC-56	10.80	3.29	0.03	0.02
PUC-60	10.32	3.99	0.07	0.03
PUC-64	11.14	4.15	0.08	0.03
PUC-80	15.94	6.87	0.13	0.07
PUC-96	18.69	9.19	0.11	0.12
PUC-128	16.37	10.28	0.36	0.30
PUC-256	34.81	16.48	10.48	2.64
PUC-512	33.87	18.18	53.37	20.02
PUC-1024	40.44	26.15	1312.55	169.29

Branch-and-cut

- Executed with a time bound of 3600 seconds
- 80 out of 105 primal solutions obtained by the ILS method proved to be optimal
- 20 out of 105 instances were not solved to optimality, with an average gap of 17% between the best lower and upper bounds
- As expected, the separation of cuts by the min-cut/max-flow procedure took more than 50% of the running time in many instances

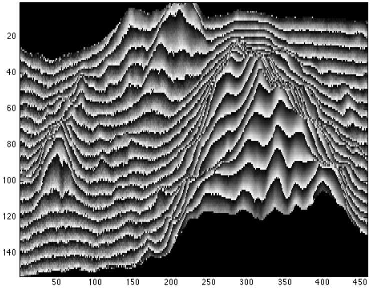
Branch-and-cut

Group	V	Reduction (%)	GAP_{root}(%)	GAP_{LB-UB}(%)	OPT	Avg-T (s)	PtPS(%)
PUC-8	8	85.28	0.00	0.00	**5**	<0.01	4.47
PUC-12	12	92.05	0.00	0.00	**5**	<0.01	0.79
PUC-16	16	91.47	0.07	0.00	**5**	<0.01	9.64
PUC-20	20	93.95	0.42	0.00	**5**	0.01	8.98
PUC-24	24	85.63	0.04	0.00	**5**	0.01	9.05
PUC-28	28	86.95	1.36	0.00	**5**	0.03	11.98
PUC-32	32	85.93	1.54	0.00	**5**	1.13	14.52
PUC-36	36	66.95	2.32	0.00	**5**	2.27	32.37
PUC-40	40	79.10	1.33	0.00	**5**	1.15	52.03
PUC-44	44	84.56	1.75	0.00	**5**	9.43	43.30
PUC-48	48	88.39	0.82	0.00	**5**	0.39	35.11
PUC-52	52	85.93	1.34	0.00	**5**	1.76	32.35
PUC-56	56	74.77	1.17	0.00	**5**	30.61	37.13
PUC-60	60	72.58	1.45	0.00	**5**	4.37	39.82
PUC-64	64	74.62	1.46	0.00	**5**	128.94	55.28
PUC-80	80	48.02	2.05	0.00	**5**	1057.56	38.61
PUC-96	96	48.01	1.96	0.60	3	2182.13	54.17
PUC-128	128	55.32	2.95	2.62	2	2302.03	45.18
PUC-256	256	2.69	9.87	2.13	0	3600.00	34.03
PUC-512	512	0.00	17.47	15.34	0	3600.00	19.04
PUC-1024	1024	0.00	18.62	18.54	0	3600.00	15.56

2DPU: Instances & Metrics

- Methods tested with three well-known benchmark instances and compared against two classic 2DPU algorithms
- Four metrics in order to evaluate and compare the quality of each solution :
  - (**N**) the total number of absolute phase gradients that differ from their wrapped counterparts
  - (**L**) The total length of the branch-cuts
  - (**T**) The number of trees produced by the branch-cuts.
  - (**I**) The number of isolated regions produced by the branch-cuts

Long's Peak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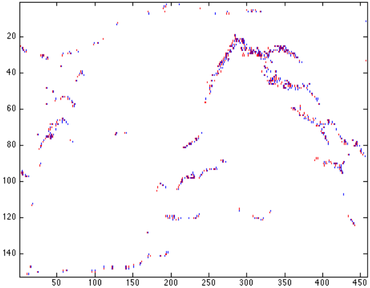
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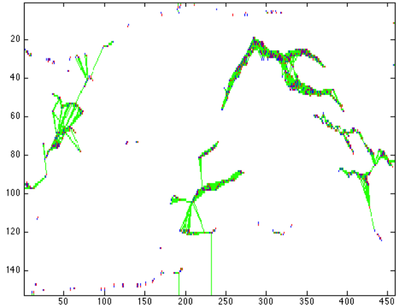
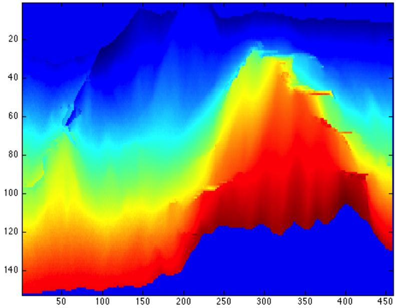
Long's Peak

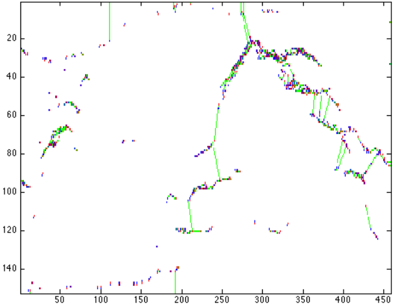
- Radar interferometry example
- 846 residues (422 positives and 426 negatives) distributed over a 152x458-pixel image
- Greatest challenge: Efficiently cluster the sparse group of residues and respect the structural delimitations

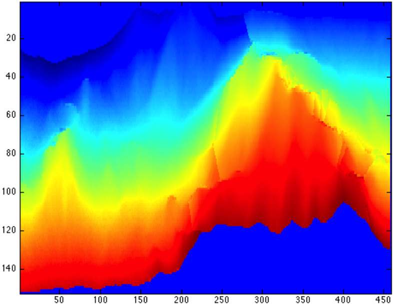
Long's Peak: Goldstein



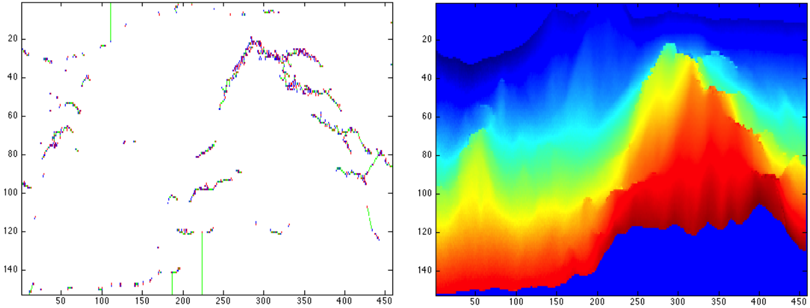


Long's Peak: Minimum-cost matching algorithm





Long's Peak: MSFBC

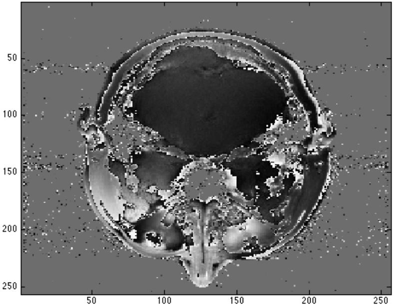


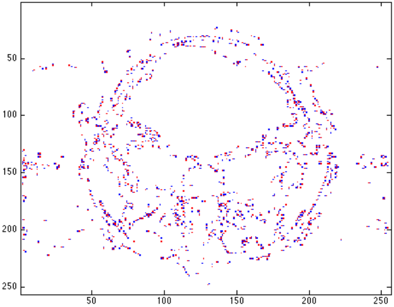
Long's Peak: Summary

Table : Results for Long's Peak data set

Method	N	L	T	I
Goldstein	1437	10647.96	49	110
MCM	1075	1545.38	429	47
MSFBCP	975	1264.31	68	25

Head Magnetic Resonance Image (MRI)

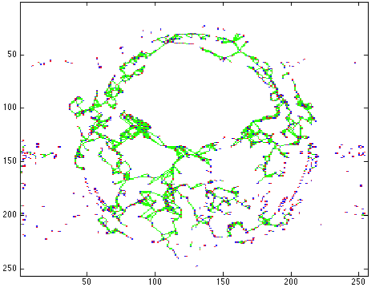


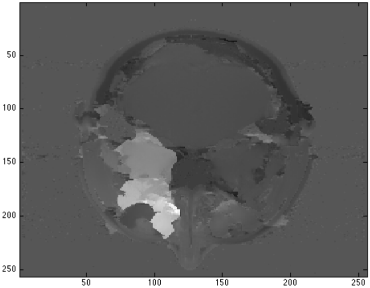


Head Magnetic Resonance Image (MRI)

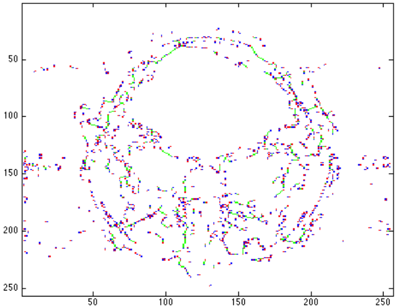
- Magnetic Resonance Image example
- 1926 residues (963 positives and 963 negatives) defined on a 256x256-pixel grid
- Greatest challenge: Considered to pose a difficult problem to the unwrapping procedure since various regions are delimited by residues and appear to be completely isolated from one another.

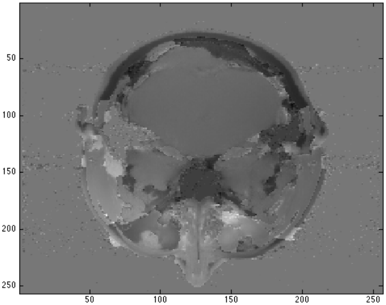
Head MRI: Goldstein



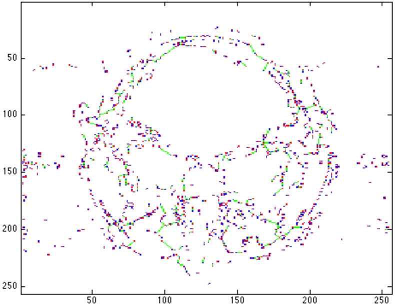


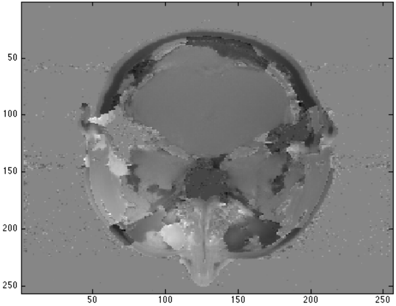
Head MRI: Minimum-cost matching algorithm





Head MRI: MSFBC





Head MRI: Summary

Table : Results for Head MRI data set

Method	N	L	T	I
Goldstein	2570	11696.44	153	257
MCM	1789	1588.72	963	16
MSFBCP	1810	1722.56	57	19

Table of Contents

Contributions

- We have proposed a new model for the 2D Phase Unwrapping problem, along with a new set of mathematical formulations and methods
- We developed efficient methods known from the field of optimization and operations research to address the minimization of the branch-cuts
- The proposed methods constituted a better approximation of the "L^0-norm" problem in the field of phase unwrapping

Discussions and Future work

- Solutions obtained by heuristic methods, with no guarantee on optimality
- In fact, the optimal solution for the MSFBC approach would be theoretically better than any path-following method
- Steiner $\times$ MSFBC?
- Devise a column generation approach and general improvements over the heuristic methods

Thanks!