Benchmark Instances for the Fixed-Route Lateral Transhipment Problem with Piecewise Linear Profits (FRLTP)

Martin Romauch[1], Thibaut Vidal[2], Richard F. Hartl [3]

Abstract
This paper contains MIP representation for the fixed-route lateral transhipment problem with piecewise linear Profits (FRLTP) and a related lot sizing problem, which is called the lotsizing problem with requalification costs (LScRC). You can also find some details on benchmark instances and the corresponding file formats.

[1] *University of Applied Sciences Wiener Neustadt, Johannes-Gutenberg-Strasse 3, 2700 Wiener Neustadt, Austria*
[2] *PUC-Rio - Pontifícia Universidade Católica do Rio de Janeiro, Brazil*
[3] *University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria*

Contents

1. A priori route evaluation for the FRLTP

This section presents a MIP formulation of the FRLTP - it is a subproblem that arises in the context of the lateral transhipment problem for a single route (SRLTP, cf. [1, 2]). The SRLTP considers traveling costs and profits, i.e. each node $i \in \{1,2,\ldots,n\}$ is related to a profit function p_i that is piecewise linear and therefore allows to model expected profits (cf. newsboy problem). The problem is to find inventory changes y_i, such that the revenue minus the costs for the pickup and delivery route is maximized. Suppose that I_0^i is the initial inventory and I_{min}^i, I_{MAX}^i are the bounds on the inventory level, then the total revenue has the following form:

$$\sum_{i\in I} p_i(I_0^i - y_i) \quad \text{for } I_0^i - y_i \in [I_{min}^i, I_{MAX}^i] \tag{1}$$

To define the FRLTP, assume that the route τ is a predefined sequence of locations for the SRLTP that starts in location 1 and ends in location n; and without loss of generality, this route is defined as $\tau_i = i$ $(i = 1,\ldots n)$. Now, suppose that some of the locations in τ are skipped, forming a subtour $\tau^* = (\tau_1, \tau_2, \ldots, \tau_m)$ and the corresponding inventory changes are y_i^*. Then according to (1) and considering the routing costs, the total profit is equal to:

$$\sum_{i=1}^{n} p_i(I_0^i + y_i^*) - \sum_{l=1}^{m-1} c_{\tau_i,\tau_{i+1}}$$

If the total revenue is larger than the initial revenue $\sum_{i=1}^n p_i(I_0^i)$, then rebalancing the inventory is profitable.

In order to simplify the notation, the change in the cost for location i is considered; it is defined as follows:

$$f_i(y_i) = p_i(I_0^i) - p_i(I_0^i - y_i) \quad \text{for } y_i \in [a_i, b_i]$$

The limits for feasible inventory changes a_i and b_i are defined by minimum and maximum inventory levels and the initial inventory level:

$$a_i = I_0^i - I_{MAX}^i$$

$$b_i = I_0^i - I_{min}^i$$

The following formula is used to calculate the corresponding optimal cost change:

$$\sum_{l=1}^{m-1} c_{\tau_{i_l},\tau_{i_{l+1}}} + \sum_{l=1}^{m} f_i(y_{\tau_{i_l}}^*)$$

With the introduced notations, a MIP for the FRLTP with and without duration limit constraint is presented:

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{n} f_i(y_i) \tag{2}$$

$$s.t. \quad \sum_{j=1}^{i-1} x_{ji} = \sum_{j=i+1}^{n} x_{ij} \quad 2 \le i \le n-1 \tag{3}$$

$$\sum_{j=2}^{n} x_{1j} = 1 \tag{4}$$

$$\sum_{j=1}^{n-1} x_{jn} = 1 \tag{5}$$

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} t_{ij} x_{ij} \le T_{MAX} \tag{6}$$

$$a_1 \le y_1 \le b_1 \quad a_n \le y_n \le b_n \tag{7}$$

$$a_i \sum_{j=i+1}^{n} x_{ij} \le y_i \le b_i \sum_{j=i+1}^{n} x_{ij} \quad 1 \le i \le n \tag{8}$$

$$0 \le \sum_{j=1}^{i} y_j \le Q_{MAX} \quad 1 \le i \le n \tag{9}$$

$$x_{ij} \in \{0,1\} \quad 1 \le i < j \le n \tag{10}$$

$$y_i \in \mathbb{R} \quad 1 \le i \le n \tag{11}$$

The objective (2) is equivalent to maximizing the total profit. The arc selection variables x_{ij} are defined for $i < j$, therefore it is sufficient to formulate the flow balance (3) and the constraints for the source (4) and the sink (5) to define a subsequence of τ.

According to (8), changing the inventory level at a location i ($y_i \neq 0$) implies that the location must also be visited. The load of the truck when leaving i is $\sum_{j \le i} y_j$, therefore (9) enforces that Q_{MAX} is the corresponding upper limit. The ARELTP is defined by (2-6) and the limit on duration is formulated in (6). The MIP (2-11) without duration limit constraint (6) is called the FRLTP without duration limit.

In the following a problem related to the FRLTP is presented.

2. Lot sizing with requalification costs (LSwRC)

Various lot sizing models are dealing with product dependent setup times; a wide range of models that cover this aspect can be found in [3] and [4]. In contrast to that, the LScRC considers idle time dependent setups. This model is motivated by applications in food and pharmaceutical industry where the qualification of processes and tools have a given duration or expiration (cf 5.4 (j) in [5] and revalidation in good manufacturing practice [6]). In these applications a frequent use of a tool may stretch the duration of its qualification. This characteristic can also be found in semiconductor industry where the products are permanently inspected; for instance steppers (lithography) that are used very frequently need less activities fpr regular process qualification, because the standard checks of products can be used to certify the quality standard of the equipment.

In the following, a mathematical model that reflects this kind of costs (or resource consumption) is formulated. Suppose that y_i is the production quantity in period $i$then b_i is the corresponding upper bound, the lower bound a_i may also be positive ($y_i \in [a_i, b_i]$); that means that for a given period i the production quantity is either zero ($y_i = 0$) or the lower bound a_i is activated ($a_i \le y_i$).

The production cost for each period i is represented by a piecewise linear functions $f_i(y_i)$, therefore the total production cost is $\sum_i f_i(y_i)$. The inventory level q_i at the end of period i and the unit holding cost h_i for holding one item in period i for one period defines the inventory holding cost $\sum_i h_i q_i$. The inventory level is zero in the beginning ($q_0 = 0$) and the balance equation $q_i = q_{i-1} + (y_i - d_i)$ states that the inventory level q_i at the end of period i is non-negative. In other words, the demand is satisfied at all times, i.e. $q_i = \sum_{j \le i}(y_j - d_j) \ge 0$. In order to model time dependent setup costs, the setup variable x_{ij} for succeeding setups is introduced, i.e. if $y_i > 0$ and $y_j > 0$ and if there is no production between period i and period j then $x_{ij} = 1$, else $x_{ij} = 0$. The corresponding setup costs c_{ij} also cover the qualification costs and may be larger for long idle times when considering expensive re–qualifications and setups.

Furthermore, a resource consumption t_{ij} associated to $x_{ij} = 1$ allows to set restrictions on the total setup related expenditure T_{MAX}. For instance if all setup times (t_{ij}) are equal to one, then T_{MAX} is an upper bound on the number of production periods. Or alternatively, set $t_{ij} = c_{ij}$ to place an upper bound on the total setup cost.

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{n} f_i(y_i) + \sum_{1 \le i \le n} h_i q_i \tag{12}$$

$$s.t. \quad q_i = \sum_{j=1}^{i} (y_j - d_j) \quad 1 \le i \le n \tag{13}$$

$$a_i \sum_{j=i+1}^{n} x_{ij} \le y_i \le b_i \sum_{j=i+1}^{n} x_{ij} \quad 1 \le i \le n \tag{14}$$

$$0 \le q_i \le Q_{MAX} \quad 1 \le i \le n \tag{15}$$

$$x_{ij} \in \{0,1\} \quad 1 \le i < j \le n \tag{16}$$

$$y_i \in \mathbb{R} \quad 1 \le i \le n \tag{17}$$

including $(3-6)$

3. File formats for instances

3.1 FRLTP instances

An instances for the FRLTP is defined by:

- an a priori route τ.

- the nodes $1, 2, \ldots, n$, including the depots 1 and n.
- the cost matrix c_{ij} for $i, j = 1 \ldots n$.
- a distance matrix t_{ij} for $i, j = 1 \ldots n$.
- revenue change functions f_i
- values for Q_{MAX} and T_{MAX}

The file names follow a convention, for instance the file name `set_64_1_15_3078.txt`, tells that $T_{MAX} = 15$ and that the random seed 3078 was used to generate the a-priori route. The parameter Q_{MAX} is not given, since the same instance is used for $Q_{MAX} = 30$, $Q_{MAX} = 60$ and $Q_{MAX} = 120$. In the instance file, the nodes are labeled from 0 to $n-1$ and the order is the following: $0, n-1, 1, 2, 3 \ldots, n-2$ (depots first). The first line in the file represents the a-priori route, which is a list of indices. The first index is always 0 and the last index is always 1. If a listed index is larger than one then it corresponds to a node that is not a depot. In the instance file the a-priori route is followed by the cost matrix c_{ij}. The first line corresponds to costs from node $i = 1$ (index 0) and the second line corresponds to cost $i = n$ (index 1), and row i ($i > 2$) corresponds to index $i - 1$.

The matrix c_{ij} is followed by the matrix (t_{ij}) which is identical ($t_{ij} = c_{ij}$). At the end of the file a representation of the piecewise linear cost change functions f_i can be found. To describe the used format, a definition for f_i is given:

$$f_i(y) = \begin{cases} d_1^{f_i} + k_1^{f_i} y & y \in I_1^{f_i} \\ d_2^{f_i} + k_2^{f_i} y & y \in I_2^{f_i} \\ \ldots \\ d_{l_{MAX}^{f_i}}^{f_i} + k_{l_{MAX}^{f_i}}^{f_i} y & y \in I_{l_{MAX}^{f_i}}^{f_i} \end{cases}$$

where $a_{l-1}^{f_i}$ and $a_l^{f_i}$ are the borders of the interval $I_l^{f_i}$. In Table 1 a tabular representation of the necessary parameters for three piecewise linear functions (with four steps) can be found. Piecewise linear functions are also used for the LSwRC instances and a graphical representation can be found in Figure 1.

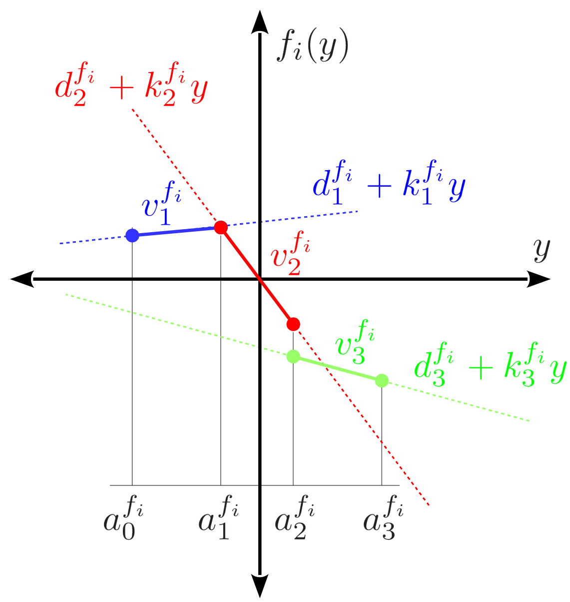

Figure 1. Example: Definition of the segments

Table 1. Format for piecewise linear functions

i	$l-1$	$a_{l-1}^{f_i}$	$a_l^{f_i}$	$d_l^{f_i}$	$k_l^{f_i}$
0	3	-20	20	0	-76
1	3	-20	20	0	-76
2	0	-50	-40	1710	-19
2	1	-40	-30	950	-38
2	2	-30	-20	380	-57
2	3	-20	20	0	-76
3	0	-50	-40	1350	-21
3	1	-40	-30	750	-36
3	2	-30	-20	300	-51
3	3	-20	20	0	-66
4	0	-20	20	0	-11
4	1	20	26	620	-42
4	2	26	32	1426	-73
4	3	32	38	2418	-104

3.2 LSwRC instances

An instances for the ARELTP are defined by:

- a setup cost matrix c_{ij} for $i, j = 1 \ldots n$.
- setup related resource consumption: for the experiments $t_{ij} = c_{ij}$.
- start in period 0 and finish in period n.
- an a priori route τ.
- values for the maximum inventory Q_{MAX} and the maximum setup cost T_{MAX}

- production costs f_i
- holding costs h_i per unit.

The following R-code illustrates how to generate instances for the lot sizing application. In Figure 2 the corresponding piecewise linear functions is displayed. The instance generator is configured with a set of parameters, for instance the ranges for the holding costs, for the demand and for production costs are defined.

```
> source("instances.R")
> parameters$seed = 123
> parameters$periods=4
> parameters

$demand
[1]  50 100

$periods
[1] 4

$HC
[1] 1 2

$PC
[1] 30 90

$PCsubintervals
[1] 3

$PCsubintervallength
[1]  1 60

$recalificationcost
[1] 100

$SC
[1] 1 5

$idlemax
[1] 4

$inventorycapacity
[1] 100

$maxsetupinvest
[1] 21

$seed
[1] 123

$type
[1] "convex"

> instance = generateinstance(parameters)
> instance
$periods
[1] 4

$inventorycapacity
[1] 100

$maxsetupinvest
[1] 21

$demand
[1] 64 90 70 95

$HC
[1] 2 1 2 2

$PCd
     [,1] [,2]  [,3]
[1,]    0  -34 -2284
[2,]    0  -84 -1164
[3,]    0 -216 -1720
[4,]    0 -297 -2026

$PCdemandbounds
     [,1] [,2] [,3] [,4]
[1,]    0   34   75  110
[2,]    0    7   27   90
[3,]    0   54   94  137
[4,]    0   33   91  146

$PCk
     [,1] [,2] [,3]
kvec   56   57   87
kvec   32   44   84
kvec   68   72   88
kvec   38   47   66

$SC
     [,1] [,2] [,3] [,4]
[1,]    0    4    4    1
[2,]    0    0    3    4
[3,]    0    0    0    2
[4,]    0    0    0    0

$SCstart
     [,1] [,2] [,3] [,4]
[1,]    2    1    3    1

$SCend
     [,1] [,2] [,3] [,4]
[1,]    2    3    2    1
```

There is also a function included that exports the instance into a text file. The corresponding text file is also displayed.

```
> exportinstance(instance,"instance.txt");
```

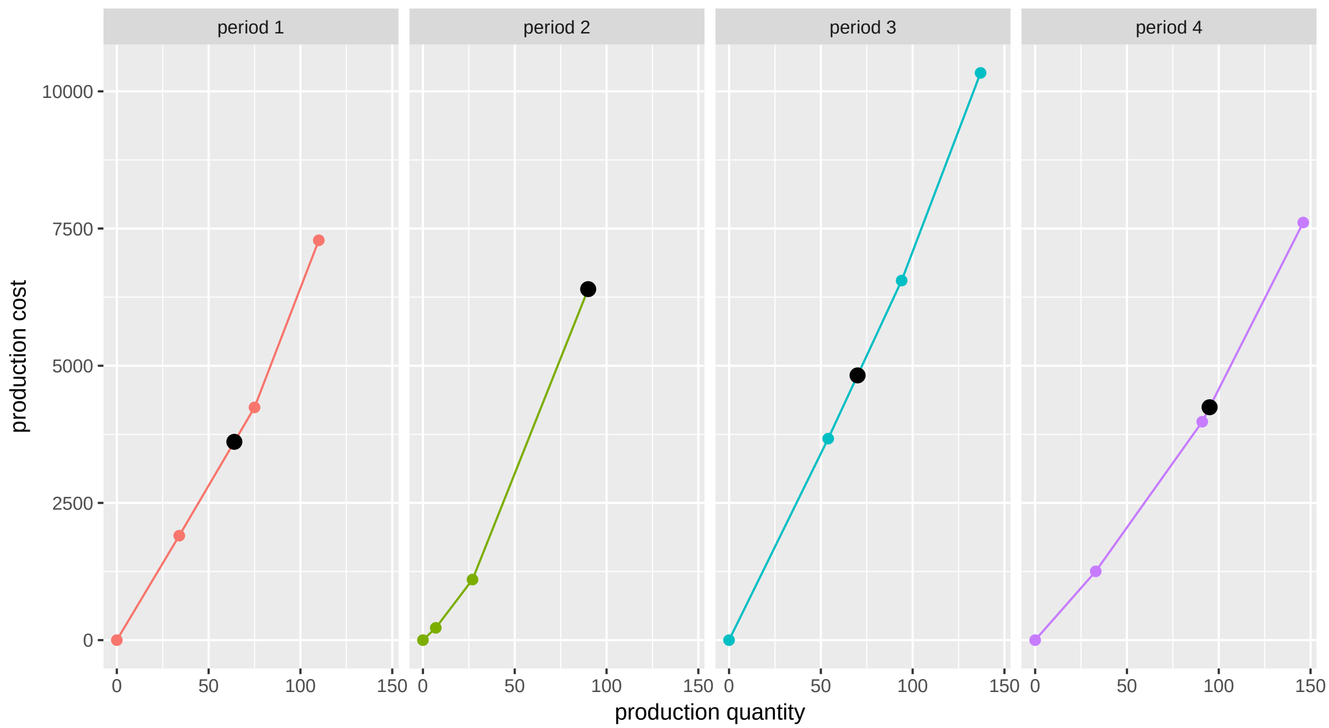

Figure 2. Example: piecewise linear function for the lot sizing instance; the demand is represented by black points.

```
[1] 1
```

```
> cat(readLines("instance.txt"),sep='\n')
```

```
4
100
21
64      90      70      95
2       1       2       2
0       4       4       1
0       0       3       4
0       0       0       2
0       0       0       0
56      57      87
32      44      84
68      72      88
38      47      66
0       -34     -2284
0       -84     -1164
0       -216    -1720
0       -297    -2026
0       34      75      110
0       7       27      90
0       54      94      137
0       33      91      146
2       1       3       1
2       3       2       1
```

References

[1] Richard F Hartl and Martin Romauch. The influence of routing on lateral transhipment. In *Computer Aided Systems Theory-EUROCAST 2013*, pages 267–275. Springer, 2013.

[2] Richard F Hartl and Martin Romauch. Notes on the single route lateral transhipment problem. *Journal of Global Optimization*, pages 1–26, 2015.

[3] Andreas Drexl and Alf Kimms. Lot sizing and scheduling – survey and extensions. *European Journal of operational research*, 99(2):221–235, 1997.

[4] Marc Salomon, Marius M Solomon, Luk N Van Wassenhove, Yvan Dumas, and Stephane Dauzère-Pérès. Solving the discrete lotsizing and scheduling problem with sequence dependent set-up costs and set-up times using the travelling salesman problem with time windows. *European Journal of Operational Research*, 100(3):494–513, 1997.

[5] P. Bedson and M. Sargent. The development and application of guidance on equipment qualification of analytical instruments. *Accreditation and Quality Assurance*, 1(6):265–274, 1996.

[6] Bill Bennett and Graham Cole. *Pharmaceutical production: an engineering guide*. IChemE, 2003.