

Electronic Companion – The Vehicle Routing Problem with Service Level Constraints

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Data	Compact Formulation - No cut				Compact Formulation - All cuts				Branch-and-Price						
	n	m	n_{\min}	LB_0	$T_0(s)$	LB	Tree	Gap	$T(s)$	LB_0	$T_0(s)$	LB	Tree	Gap	$T(s)$
A1-1	31	5	13	563.9	0.46	631.0	950k	0.00	3725.46	584.3	0.61	631.0	117k	0.00	382.51
A2-1	32	6	9	619.2	0.37	660.0	108k	0.00	709.89	626.6	0.17	660.0	23k	0.00	65.77
A3-1	35	5	23	580.8	0.45	629.3	1808k	3.49	7200.00	607.1	0.51	652.0	896k	0.00	4749.94
A4-1	36	6	13	697.3	1.04	750.0	641k	0.00	2626.70	712.9	0.29	750.0	14k	0.00	53.99
A5-1	38	5	12	569.4	0.97	605.6	999k	1.69	7200.00	575.7	0.20	616.0	137k	0.00	821.22
A6-1	43	6	20	642.5	0.21	676.0	248k	0.00	898.46	649.1	0.26	676.0	36k	0.00	106.86
A7-1	44	7	24	855.6	1.41	886.2	610k	5.12	7200.00	884.0	1.98	919.1	410k	1.59	7200.00
A8-1	47	7	21	842.2	1.10	883.6	1213k	4.58	7200.00	872.3	1.22	902.6	841k	2.52	7200.00
A9-1	53	7	41	981.8	2.69	999.8	652k	10.17	7200.00	1015.5	1.77	1032.4	432k	7.24	7200.00
A10-1	59	9	58	1193.8	3.53	1223.6	573k	9.63	7200.00	1238.5	3.19	1260.3	434k	6.92	7200.00
A11-1	61	8	35	989.9	3.65	1015.7	397k	9.40	7200.00	1028.7	3.63	1053.5	244k	6.02	7200.00
A12-1	62	10	27	1007.0	2.02	1028.7	517k	5.89	7200.00	1024.8	2.18	1052.7	407k	3.69	7200.00
A13-1	64	9	22	968.8	1.97	990.3	541k	4.23	7200.00	984.9	1.11	1010.6	503k	2.27	7200.00
A14-1	79	10	24	1260.1	4.25	1282.1	257k	5.17	7200.00	1284.7	4.69	1305.9	150k	3.41	7200.00
B1-1	30	5	14	473.4	1.36	493.8	3293k	2.60	7200.00	477.1	0.30	501.0	1651k	1.18	7200.00
B2-1	34	5	11	649.6	0.07	688.0	2k	0.00	5.14	651.9	0.10	688.0	9k	0.00	14.81
B3-1	38	5	13	415.6	0.80	450.6	1277k	4.13	7200.00	421.6	0.21	454.0	1383k	3.41	7200.00
B4-1	42	6	28	606.1	0.85	642.6	1080k	5.50	7200.00	622.8	0.93	647.6	956k	4.77	7200.00
B5-1	44	5	17	512.5	0.69	547.3	1505k	7.86	7200.00	523.4	0.82	549.0	1393k	7.58	7200.00
B6-1	49	7	29	590.2	1.03	632.1	1157k	3.64	7200.00	602.5	0.63	628.8	1430k	4.15	7200.00
B7-1	50	7	49	903.7	0.75	941.4	2092k	8.78	7200.00	907.6	0.56	933.9	1973k	9.51	7200.00
B8-1	55	7	54	586.5	1.91	614.7	1822k	13.06	7200.00	597.5	1.15	605.2	1318k	14.39	7200.00
B9-1	56	9	25	1198.6	1.32	1220.5	666k	2.83	7200.00	1215.2	1.05	1236.1	827k	1.58	7200.00
B10-1	63	9	38	722.8	1.64	762.1	841k	4.38	7200.00	729.8	2.07	762.5	732k	4.33	7200.00
B11-1	66	10	65	952.3	1.82	971.9	1371k	5.82	7200.00	966.7	1.86	977.4	921k	5.29	7200.00
B12-1	77	10	37	936.4	2.33	958.4	849k	7.04	7200.00	959.8	2.79	972.5	466k	5.68	7200.00
p03-1	100	3	80	569.0	1.93	584.0	310k	0.00	3503.05	570.5	2.87	584.0	426k	0.00	6053.84
p06-1	50	2	22	350.4	0.25	365.0	321	0.00	1.85	350.4	0.49	365.0	201	0.00	1.63
p07-1	75	2	22	491.0	0.84	506.0	13k	0.00	34.97	493.1	1.46	506.0	5k	0.00	21.03
p08-1	100	3	50	513.2	2.21	528.0	325k	0.00	5840.35	513.7	3.00	525.6	366k	0.46	7200.00
p09-1	150	4	63	639.4	7.73	658.1	279k	1.34	7200.00	642.5	17.46	660.2	315k	1.02	7200.00
p10-1	199	6	132	769.2	63.65	778.6	22k	3.28	7200.00	768.3	52.82	778.0	20k	3.35	7200.00
p13-1	120	3	84	516.7	6.89	529.6	211k	16.46	7200.00	522.9	8.92	533.4	116k	15.86	7200.00
p14-1	100	3	34	484.8	3.73	514.9	401k	12.29	7200.00	492.1	5.95	504.4	212k	14.07	7200.00
p15-1	100	4	99	496.3	2.96	526.8	325k	10.86	7200.00	499.4	4.11	514.5	222k	12.94	7200.00
p16-1	199	6	87	763.8	39.44	777.0	21k	2.87	7200.00	767.8	55.50	779.3	23k	2.59	7200.00

Table 1: Results of the exact methods for the VRP-SL instances with 1 group

Data	n	m	n _{MIN}	Compact Formulation - No cut				Compact Formulation - All cuts				Branch-and-Price									
				LB ₀	T ₀ (s)	LB	Tree	Gap	T(s)	LB ₀	T ₀ (s)	LB	Tree	Gap	T(s)	LB ₀	T ₀ (s)	LB	Tree	Gap	T(s)
A1-2R	31	5	27	604.5	0.49	672.8	1464k	2.78	7200.00	616.4	0.47	684.1	781k	1.14	7200.00	672.2	0.09	692.0	71	0	5.55
A2-2R	32	6	17	604.8	0.55	676.0	2329k	0.00	6118.92	626.2	0.54	676.0	964k	0.00	3208.36	665.3	0.15	676.0	178	0	8.37
A3-2R	35	5	21	628.9	0.76	675.9	1360k	2.89	7200.00	653.6	0.69	696.0	325k	0.00	1655.71	683.2	0.13	696.0	1k	0	277.99
A4-2R	36	6	13	689.2	0.67	746.0	465k	0.00	2876.92	708.5	0.31	746.0	31k	0.00	114.96	742.4	0.10	746.0	9	0	0.54
A5-2R	38	5	26	594.6	0.77	627.1	1284k	2.93	7200.00	605.9	0.62	646.0	1366k	0.00	6638.05	632.3	0.21	646.0	3k	0	1607.53
A6-2R	43	6	22	824.5	1.27	869.8	712k	5.04	7200.00	850.9	1.19	879.1	752k	4.03	7200.00	884.0	0.32	912.4	8k	0.39	7200.00
A7-2R	44	7	31	900.1	1.61	922.6	848k	6.81	7200.00	935.6	1.38	956.6	646k	3.38	7200.00	976.9	0.30	990.0	714	0	85.38
A8-2R	47	7	32	819.8	1.47	851.6	846k	7.94	7200.00	846.5	1.30	873.5	515k	5.56	7200.00	899.2	0.22	925.0	9k	0	2547.87
A9-2R	53	7	45	885.9	1.80	913.0	659k	4.80	7200.00	910.5	1.65	936.2	299k	2.38	7200.00	951.0	0.36	959.0	290	0	49.87
A10-2R	59	9	18	1132.1	5.03	1153.7	466k	9.51	7200.00	1168.4	3.52	1193.1	418k	6.43	7200.00	1246.5	0.69	1268.2	10k	0.53	7200.00
A11-2R	61	8	25	979.8	2.96	1006.3	388k	8.93	7200.00	1010.5	3.48	1041.3	233k	5.76	7200.00	1073.7	0.64	1092.2	10k	1.16	7200.00
A12-2R	62	10	33	1048.1	4.34	1076.7	435k	7.97	7200.00	1075.8	3.28	1100.9	285k	5.91	7200.00	1143.2	0.68	1164.2	22k	0.49	7200.00
A13-2R	64	9	45	1034.9	2.40	1060.4	591k	6.90	7200.00	1060.1	2.33	1079.6	526k	5.21	7200.00	1103.3	0.61	1118.7	11k	1.78	7200.00
A14-2R	79	10	60	1300.0	4.34	1315.6	306k	6.96	7200.00	1324.6	4.96	1340.1	155k	5.23	7200.00	1368.6	0.91	1383.5	7k	2.16	7200.00
B1-2R	30	5	23	503.7	0.47	593.4	4079k	2.56	7200.00	509.6	0.35	595.2	2389k	2.26	7200.00	597.1	0.23	606.9	8k	0.35	7200.00
B2-2R	34	5	19	628.7	0.08	658.0	49k	0.00	73.35	631.0	0.12	658.0	26k	0.00	29.74	607.4	0.25	635.0	5k	3.5	7200.00
B3-2R	38	5	19	419.8	0.68	459.0	2178k	4.77	7200.00	431.4	0.52	458.8	1117k	4.82	7200.00	457.5	0.27	473.0	9k	1.88	7200.00
B4-2R	42	6	33	608.8	0.89	647.7	800k	1.57	7200.00	613.8	0.77	648.5	633k	1.44	7200.00	648.9	0.32	658.0	129	0	20.88
B5-2R	44	5	15	509.8	0.91	562.4	1390k	8.40	7200.00	521.6	0.95	554.0	962k	9.76	7200.00	577.3	0.78	591.0	7k	3.74	7200.00
B6-2R	49	7	19	581.6	1.29	633.3	1273k	4.77	7200.00	592.9	1.50	632.4	1166k	4.90	7200.00	634.3	0.78	655.1	10k	1.49	7200.00
B7-2R	50	7	49	828.3	0.24	864.3	2024k	2.78	7200.00	830.8	0.30	865.4	1371k	2.65	7200.00	877.6	0.38	887.3	5k	0.19	7200.00
B8-2R	55	7	28	533.9	2.23	555.9	1111k	10.48	7200.00	541.3	1.14	555.2	1039k	10.59	7200.00	588.5	1.16	598.0	7k	3.71	7200.00
B9-2R	56	9	46	1353.7	1.72	1375.6	1061k	4.67	7200.00	1377.8	1.26	1397.2	827k	3.17	7200.00	1372.5	0.70	1385.9	12k	3.96	7200.00
B10-2R	63	9	30	738.8	1.86	763.7	993k	7.43	7200.00	743.6	1.72	782.4	834k	5.16	7200.00	799.7	0.99	808.9	7k	1.95	7200.00
B11-2R	66	10	23	842.8	1.80	871.0	633k	4.91	7200.00	862.8	2.02	881.6	862k	3.76	7200.00	900.4	0.82	906.2	9k	1.07	7200.00
B12-2R	77	10	40	935.5	3.63	965.2	391k	8.33	7200.00	957.0	3.18	985.9	281k	6.37	7200.00	1025.3	2.15	1033.3	8k	1.87	7200.00
p03-2R	100	3	43	538.9	2.45	551.0	9k	0.00	60.46	540.6	3.61	551.0	65k	0.00	344.21	541.1	86.11	544.5	271	1.18	7200.00
p06-2R	50	2	40	350.4	0.29	367.0	14k	0.00	29.72	350.5	0.36	367.0	1k	0.00	5.42	356.8	4.90	362.7	627	1.18	7200.00
p07-2R	75	3	49	491.0	0.87	506.0	8k	0.00	27.37	492.8	1.51	506.0	6k	0.00	25.34	500.6	18.08	502.3	675	0.72	7200.00
p08-2R	100	3	70	544.0	2.23	556.0	69k	0.00	450.81	544.4	3.49	556.0	15k	0.00	128.46	547.5	71.41	550.5	299	0.99	7200.00
p09-2R	150	4	122	639.4	7.73	659.2	178k	1.17	7200.00	641.4	15.19	660.6	359k	0.96	7200.00	660.6	197.42	662.4	91	0.68	7200.00
p10-2R	199	6	98	769.2	51.83	775.8	17k	3.62	7200.00	769.4	66.60	780.7	19k	3.02	7200.00	792.8	487.17	794.2	53	1.34	7200.00
p13-2R	120	3	50	543.7	6.96	556.7	198k	15.14	7200.00	548.7	9.10	559.8	111k	14.67	7200.00	609.2	1445.75	610.8	27	6.89	7200.00
p14-2R	100	4	65	490.2	2.65	515.1	279k	12.69	7200.00	494.2	3.62	507.2	245k	14.03	7200.00	569.7	105.70	572.4	490	2.98	7200.00
p15-2R	100	4	43	492.1	2.78	539.0	337k	8.64	7200.00	493.4	3.83	508.4	212k	13.82	7200.00	568.8	114.57	571.9	522	3.07	7200.00
p16-2R	199	6	136	763.5	35.13	778.9	28k	2.64	7200.00	768.0	60.64	780.2	28k	2.48	7200.00	790.5	404.15	791.7	39	1.04	7200.00

Table 2: Results of the exact methods for the VRP-SL instances with 2 groups (random distribution)

Data	Compact Formulation - No cut				Compact Formulation - All cuts				Branch-and-Price						
	n	m	n_{\min}	LB_0	$T_0(s)$	LB	Tree	Gap	$T(s)$	LB_0	$T_0(s)$	LB	Tree	Gap	$T(s)$
A1-2C	31	5	15	669.7	0.67	733.6	1972k	3.98	7200.00	689.7	0.53	743.0	1935k	2.75	7200.00
A2-2C	32	6	24	574.0	0.67	624.9	1814k	1.12	7200.00	581.9	0.43	632.0	278k	0.00	946.35
A3-2C	35	5	24	589.0	0.69	645.0	1513k	0.00	4879.83	606.1	0.19	645.0	600k	0.00	1895.55
A4-2C	36	6	12	705.0	0.74	763.6	941k	2.61	7200.00	729.7	0.73	784.0	712k	0.00	4181.35
A5-2C	38	5	17	632.7	0.95	672.2	1037k	2.44	7200.00	652.1	0.79	689.0	194k	0.00	1070.78
A6-2C	43	6	36	648.3	0.90	696.0	828k	0.00	6262.34	655.2	0.73	696.0	91k	0.00	678.72
A7-2C	44	7	26	996.2	1.78	1029.9	800k	7.22	7200.00	1040.3	1.99	1064.2	548k	4.13	7200.00
A8-2C	47	7	26	868.8	1.31	902.6	1012k	8.28	7200.00	890.3	1.50	917.1	679k	6.79	7200.00
A9-2C	53	7	16	1011.8	3.15	1030.7	645k	8.95	7200.00	1050.8	2.45	1072.5	499k	5.25	7200.00
A10-2C	59	9	45	1024.1	3.13	1044.8	564k	6.47	7200.00	1052.2	3.01	1074.6	310k	3.80	7200.00
A11-2C	61	8	29	962.7	4.79	986.9	356k	8.03	7200.00	998.9	3.61	1018.4	236k	5.09	7200.00
A12-2C	62	10	33	974.3	2.62	1001.9	464k	5.12	7200.00	995.8	1.98	1020.9	409k	3.32	7200.00
A13-2C	64	9	52	1004.0	2.38	1030.4	569k	6.92	7200.00	1023.2	2.20	1041.1	479k	5.95	7200.00
A14-2C	79	10	32	1458.8	6.20	1486.1	250k	8.26	7200.00	1501.9	6.06	1524.1	199k	5.92	7200.00
B1-2C	30	5	24	497.2	0.46	521.0	2318k	0.00	4832.25	500.1	0.50	521.0	60k	0.00	172.73
B2-2C	34	5	11	746.0	1.38	787.0	2987k	2.84	7200.00	744.4	0.29	786.4	2124k	2.92	7200.00
B3-2C	38	5	12	415.1	0.57	455.3	3227k	6.71	7200.00	426.2	0.64	449.9	2086k	7.81	7200.00
B4-2C	42	6	19	628.4	0.85	659.9	1068k	4.91	7200.00	639.0	0.99	674.5	915k	2.81	7200.00
B5-2C	44	5	25	500.5	0.77	543.9	1203k	5.24	7200.00	506.7	0.78	547.8	881k	4.57	7200.00
B6-2C	49	7	32	577.3	1.03	614.9	825k	6.27	7200.00	584.6	1.06	612.3	582k	6.66	7200.00
B7-2C	50	7	21	903.7	0.75	941.4	2107k	8.78	7200.00	924.2	0.81	986.1	734k	4.45	7200.00
B8-2C	55	7	30	534.2	2.16	565.4	965k	5.77	7200.00	547.2	1.37	562.3	919k	6.28	7200.00
B9-2C	56	9	41	1368.6	1.83	1404.4	1293k	6.93	7200.00	1385.8	1.28	1424.8	760k	5.58	7200.00
B10-2C	63	9	37	704.2	1.45	749.1	867k	7.98	7200.00	710.5	2.11	751.9	674k	7.63	7200.00
B11-2C	66	10	23	835.9	2.12	884.1	766k	4.32	7200.00	848.5	1.74	891.6	858k	3.51	7200.00
B12-2C	77	10	39	1016.5	4.04	1036.8	681k	8.81	7200.00	1044.5	4.62	1056.7	504k	7.07	7200.00
p03-2C	100	3	49	508.2	2.85	528.0	315k	0.00	4301.03	507.9	3.36	525.1	376k	0.54	7200.00
p06-2C	50	2	22	380.9	0.30	395.0	543	0.00	3.26	381.2	0.46	395.0	999	0.00	5.80
p07-2C	75	3	31	494.6	0.94	511.0	106k	0.00	348.31	494.8	1.32	511.0	80k	0.00	261.69
p08-2C	100	3	59	564.2	1.64	578.0	52k	0.00	390.47	566.0	2.42	578.0	66k	0.00	773.46
p09-2C	150	5	51	700.8	9.16	716.7	187k	1.69	7200.00	701.8	18.96	713.2	118k	2.16	7200.00
p10-2C	199	6	111	768.9	51.21	773.7	12k	3.89	7200.00	768.4	58.02	779.4	15k	3.18	7200.00
p13-2C	120	3	94	509.4	6.53	520.1	228k	16.92	7200.00	516.3	9.63	527.6	131k	15.73	7200.00
p14-2C	100	4	70	484.8	3.76	514.1	408k	12.56	7200.00	490.6	3.87	503.9	245k	14.31	7200.00
p15-2C	100	3	65	484.8	4.10	513.6	385k	12.50	7200.00	490.3	4.99	500.9	206k	14.67	7200.00
p16-2C	199	6	83	767.3	54.32	773.8	10k	3.76	7200.00	767.9	73.73	780.2	13k	2.95	7200.00

Table 3: Results of the exact methods for the VRP-SL instances with 2 groups (clustered)

Data	Compact Formulation - No cut				Compact Formulation - All cuts				Branch-and-Price												
	n	m	n_{\min}	T_0 (s)	LB	Tree	Gap	T(s)	LB ₀	T_0 (s)	LB	Tree	Gap	T(s)							
A1-5R	31	5	19	660.5	0.50	726.0	1704k	0.00	3841.07	678.2	0.41	726.0	211k	0.00	515.85	711.4	0.12	726.0	99	0	8.08
A2-5R	32	6	23	569.9	0.50	615.0	54k	0.00	147.04	579.3	0.25	615.0	6k	0.00	22.11	597.2	0.12	615.0	140	0	5.71
A3-5R	35	5	24	558.7	0.95	599.0	155k	0.00	462.60	566.7	0.13	599.0	20k	0.00	51.48	589.2	0.14	599.0	84	0	7.28
A4-5R	36	6	19	833.8	0.85	871.8	1325k	5.95	7200.00	860.6	0.62	913.9	1011k	1.42	7200.00	896.5	0.16	927.0	145	0	8.58
A5-5R	38	5	22	617.3	1.07	655.5	1279k	4.02	7200.00	633.2	0.83	674.5	1132k	1.25	7200.00	622.9	0.23	675.4	9k	1.11	7200.00
A6-5R	43	6	30	702.7	0.93	739.2	1075k	3.11	7200.00	712.2	1.04	744.8	550k	2.39	7200.00	742.2	0.25	763.0	426	0	47.40
A7-5R	44	7	29	946.0	1.42	977.8	719k	6.07	7200.00	967.9	1.53	1002.0	547k	3.74	7200.00	1018.2	0.34	1041.0	8k	0	6104.53
A8-5R	47	7	31	801.2	1.37	840.7	902k	7.52	7200.00	825.1	1.47	859.9	630k	5.41	7200.00	875.8	0.32	901.0	16k	0.88	7200.00
A9-5R	53	7	23	1015.6	2.92	1036.9	578k	9.76	7200.00	1049.5	2.33	1068.3	619k	7.02	7200.00	1101.7	0.93	1132.5	6k	1.44	7200.00
A10-5R	59	9	34	1119.4	3.45	1139.2	490k	9.30	7200.00	1144.2	2.95	1178.9	296k	6.14	7200.00	1223.5	0.37	1247.9	23k	0.65	7200.00
A11-5R	61	8	39	1029.2	3.77	1050.3	305k	7.62	7200.00	1065.2	3.06	1088.6	309k	4.26	7200.00	1113.3	0.82	1137.0	5k	0	2945.54
A12-5R	62	10	34	1076.1	4.05	1114.7	439k	7.65	7200.00	1117.2	2.47	1141.3	381k	5.44	7200.00	1186.4	0.64	1200.2	16k	0.57	7200.00
A13-5R	64	9	41	1044.4	2.41	1070.0	455k	6.06	7200.00	1066.1	2.91	1091.1	372k	4.21	7200.00	1114.3	0.72	1132.0	13k	0.61	7200.00
A14-5R	79	10	45	1484.3	5.94	1512.4	253k	7.10	7200.00	1534.5	6.81	1563.8	225k	3.95	7200.00	1592.7	1.85	1610.8	6k	1.06	7200.00
B1-5R	30	5	25	504.5	0.35	598.6	3678k	2.82	7200.00	507.4	0.32	598.4	2872k	2.86	7200.00	561.5	0.16	607.0	10k	1.46	7200.00
B2-5R	34	5	18	736.1	0.72	804.0	1671k	0.00	3731.52	730.8	0.33	766.7	2460k	4.64	7200.00	744.4	0.47	779.6	9k	3.03	7200.00
B3-5R	38	5	24	417.2	0.57	456.9	2242k	10.06	7200.00	429.5	0.59	457.1	1829k	10.01	7200.00	475.9	0.65	485.5	9k	4.44	7200.00
B4-5R	42	6	14	590.1	0.77	633.4	1464k	0.73	7200.00	603.7	0.63	638.0	731k	0.00	2960.06	627.7	0.45	638.0	328	0	56.91
B5-5R	44	5	26	614.5	0.99	664.6	1565k	2.41	7200.00	620.7	0.89	655.2	1304k	3.79	7200.00	671.5	1.02	678.6	5k	0.36	7200.00
B6-5R	49	7	41	594.9	1.35	631.0	1323k	4.97	7200.00	605.8	1.09	640.9	1030k	3.48	7200.00	652.5	0.78	664.0	5k	0	3847.52
B7-5R	50	7	32	835.3	0.64	886.2	1420k	2.72	7200.00	832.1	0.38	873.0	1002k	4.17	7200.00	882.4	0.58	906.2	6k	0.53	7200.00
B8-5R	55	7	39	549.0	1.69	564.0	1055k	9.76	7200.00	558.6	1.29	568.4	1066k	9.05	7200.00	602.7	1.42	609.4	5k	2.5	7200.00
B9-5R	56	9	33	1231.8	1.94	1250.5	773k	5.76	7200.00	1247.2	1.60	1270.4	638k	4.26	7200.00	1287.2	0.66	1309.8	28k	1.3	7200.00
B10-5R	63	9	33	712.8	1.73	757.2	826k	6.17	7200.00	717.1	1.94	764.1	735k	5.31	7200.00	768.6	1.55	791.3	9k	1.95	7200.00
B11-5R	66	10	60	821.6	1.85	859.8	686k	5.10	7200.00	831.7	1.69	871.6	638k	3.80	7200.00	891.9	1.17	899.6	8k	0.7	7200.00
B12-5R	77	10	46	1005.6	4.50	1031.6	548k	8.55	7200.00	1030.8	3.77	1048.0	495k	7.09	7200.00	1088.6	1.98	1094.9	7k	2.93	7200.00
p03-5R	100	3	44	546.9	2.47	557.0	272	0.00	6.67	550.1	3.43	557.0	716	0.00	12.20	550.3	179.11	553.0	109	0.72	7200.00
p06-5R	50	2	32	398.4	0.29	421.0	712	0.00	3.95	399.3	0.43	421.0	767	0.00	4.35	401.3	25.31	413.2	359	1.86	7200.00
p07-5R	75	3	49	541.1	0.74	549.0	20k	0.00	37.72	541.4	1.43	549.0	6k	0.00	17.64	538.8	45.54	545.5	546	0.64	7200.00
p08-5R	100	3	47	516.9	2.08	532.0	60k	0.00	475.99	516.7	3.69	532.0	48k	0.00	426.86	517.9	42.63	521.3	193	2	7200.00
p09-5R	150	5	106	673.9	11.33	689.3	89k	2.09	7200.00	675.0	18.91	691.7	107k	1.75	7200.00	683.1	298.86	686.1	73	2.54	7200.00
p10-5R	199	6	127	768.4	50.95	772.7	14k	4.01	7200.00	769.0	74.53	778.1	14k	3.34	7200.00	792.8	834.54	794.1	45	1.35	7200.00
p13-5R	120	3	62	531.3	6.99	540.2	174k	16.90	7200.00	536.3	11.04	542.8	71k	16.50	7200.00	595.1	847.17	598.2	31	7.97	7200.00
p14-5R	100	4	61	486.7	2.87	510.6	423k	13.02	7200.00	490.3	5.58	505.1	224k	13.95	7200.00	553.4	80.92	563.6	593	3.99	7200.00
p15-5R	100	4	56	487.4	3.42	508.4	423k	13.55	7200.00	491.7	5.95	505.1	199k	14.09	7200.00	553.4	103.23	563.4	523	4.18	7200.00
p16-5R	199	6	146	763.6	34.40	778.8	26k	2.65	7200.00	767.0	49.22	777.6	16k	2.80	7200.00	790.5	763.59	791.6	31	1.04	7200.00

Table 4: Results of the exact methods for the VRP-SL instances with 5 groups (random distribution)

Data	Compact Formulation - No cut				Compact Formulation - All cuts				Branch-and-Price						
	n	m	n_{\min}	LB_0	T_0 (s)	LB	Tree	Gap	T(s)	LB_0	T_0 (s)	LB	Tree	Gap	T(s)
A1-5C	31	5	23	633.7	0.50	712.6	2734k	1.57	7200.00	647.1	0.47	724.0	734k	0.00	1821.30
A2-5C	32	6	15	620.1	0.67	667.4	1519k	5.73	7200.00	636.0	0.45	680.6	1980k	3.87	7200.00
A3-5C	35	5	17	688.8	0.96	735.0	1148k	0.55	7200.00	701.7	0.50	739.0	193k	0.00	681.90
A4-5C	36	6	33	751.8	0.87	805.7	1186k	1.50	7200.00	776.8	0.86	818.0	161k	0.00	862.17
A5-5C	38	5	17	666.8	0.92	713.0	992k	0.00	5765.82	679.7	0.32	713.0	88k	0.00	334.38
A6-5C	43	6	19	798.1	0.93	857.4	764k	4.41	7200.00	832.0	1.20	867.4	640k	3.30	7200.00
A7-5C	44	7	31	929.4	1.96	976.6	997k	6.09	7200.00	956.9	1.43	1004.7	647k	3.40	7200.00
A8-5C	47	7	23	910.0	1.42	939.8	726k	5.74	7200.00	938.2	1.51	976.2	684k	2.08	7200.00
A9-5C	53	7	46	912.8	2.99	936.7	469k	7.54	7200.00	940.6	1.49	966.4	342k	4.60	7200.00
A10-5C	59	10	32	1045.6	3.71	1070.8	468k	8.48	7200.00	1079.8	3.25	1108.8	490k	5.23	7200.00
A11-5C	61	8	37	1024.2	5.39	1052.4	342k	11.12	7200.00	1062.3	4.31	1084.7	219k	8.39	7200.00
A12-5C	62	9	49	1031.4	4.01	1056.5	348k	6.99	7200.00	1062.1	2.77	1084.4	265k	4.54	7200.00
A13-5C	64	9	48	995.5	2.53	1021.9	794k	6.16	7200.00	1016.5	2.29	1041.3	762k	4.38	7200.00
A14-5C	79	10	55	1469.8	6.76	1485.0	233k	7.88	7200.00	1519.0	7.48	1531.5	156k	5.00	7200.00
B1-5C	30	5	22	579.1	0.41	671.0	1457k	0.00	2827.55	635.7	0.45	665.7	1958k	0.79	7200.00
B2-5C	34	5	25	719.2	0.44	770.7	3198k	1.31	7200.00	718.6	0.46	781.0	1020k	0.00	2939.93
B3-5C	38	5	22	447.9	0.70	478.6	2565k	11.05	7200.00	463.1	0.64	483.4	1950k	10.14	7200.00
B4-5C	42	6	16	597.9	0.78	632.1	865k	4.36	7200.00	611.7	0.85	644.8	594k	2.45	7200.00
B5-5C	44	5	31	561.4	0.69	626.9	1267k	5.30	7200.00	571.7	0.68	629.1	657k	4.96	7200.00
B6-5C	49	7	29	599.8	1.23	639.8	1588k	8.99	7200.00	609.5	1.58	636.2	1036k	9.51	7200.00
B7-5C	50	7	26	813.6	0.60	897.5	1311k	2.13	7200.00	823.7	0.84	884.3	1211k	3.57	7200.00
B8-5C	55	7	35	557.9	2.08	574.1	1499k	9.44	7200.00	570.4	1.48	580.3	1241k	8.48	7200.00
B9-5C	56	9	31	1213.8	1.46	1237.0	1055k	8.10	7200.00	1231.8	1.50	1252.0	566k	6.99	7200.00
B10-5C	63	9	32	722.8	1.77	754.4	1101k	7.66	7200.00	730.0	2.02	746.2	677k	8.66	7200.00
B11-5C	66	10	39	869.2	2.12	897.3	790k	8.34	7200.00	883.2	1.92	916.7	655k	6.36	7200.00
B12-5C	77	10	53	998.3	4.18	1017.8	578k	8.14	7200.00	1024.2	4.47	1038.6	501k	6.27	7200.00
p03-5C	100	3	53	541.7	2.77	555.0	15k	0.00	103.67	541.6	4.13	555.0	7k	0.00	60.77
p06-5C	50	2	34	369.8	0.25	387.0	4k	0.00	10.01	370.6	0.44	387.0	2k	0.00	9.12
p07-5C	75	3	63	511.8	0.79	526.0	276k	0.00	1046.23	512.3	1.31	526.0	245k	0.00	1180.62
p08-5C	100	3	50	514.7	2.44	530.0	76k	0.00	500.02	515.8	4.05	530.0	49k	0.00	392.13
p09-5C	150	5	99	684.3	9.90	696.9	187k	1.99	7200.00	686.0	15.02	696.9	183k	1.98	7200.00
p10-5C	199	6	104	793.2	46.89	798.9	15k	3.87	7200.00	794.0	57.02	800.5	13k	3.67	7200.00
p13-5C	120	3	85	514.4	7.22	526.1	168k	16.88	7200.00	520.7	11.47	531.0	136k	16.11	7200.00
p14-5C	100	4	46	488.0	2.96	512.5	269k	12.84	7200.00	491.9	5.11	503.8	209k	14.32	7200.00
p15-5C	100	4	65	487.0	3.72	518.5	268k	11.82	7200.00	492.3	4.73	505.2	219k	14.08	7200.00
p16-5C	199	7	89	776.4	51.98	783.1	9k	3.91	7200.00	777.9	91.69	793.6	19k	2.62	7200.00

Table 5: Results of the exact methods for the VRP-SL instances with 5 groups (clustered)

Inst	n	m	Wor-10	Avg-10	Best-10	Gap_{BKS}	Gap_{LB}	T(s)	BKS	BKLB
A1-1	31	5	631	631.00	631	0.00	0.00	7.49	631	631.0
A2-1	32	6	660	660.00	660	0.00	0.00	7.23	660	660.0
A3-1	35	5	652	652.00	652	0.00	0.00	9.48	652	652.0
A4-1	36	6	750	750.00	750	0.00	0.00	8.87	750	750.0
A5-1	38	5	616	616.00	616	0.00	0.00	8.57	616	616.0
A6-1	43	6	676	676.00	676	0.00	0.00	9.92	676	676.0
A7-1	44	7	934	934.00	934	0.00	0.00	12.84	934	934.0
A8-1	47	7	926	926.00	926	0.00	0.00	11.84	926	926.0
A9-1	53	7	1113	1113.00	1113	0.00	1.44	22.17	1113	1097.2
A10-1	59	9	1354	1354.00	1354	0.00	1.37	21.97	1354	1335.7
A11-1	61	8	1121	1121.00	1121	0.00	1.29	18.51	1121	1106.7
A12-1	62	10	1093	1093.00	1093	0.00	0.00	16.74	1093	1093.0
A13-1	64	9	1034	1034.00	1034	0.00	0.00	13.69	1034	1034.0
A14-1	79	10	1352	1352.00	1352	0.00	0.30	18.68	1352	1347.9
B1-1	30	5	507	507.00	507	0.00	0.00	7.37	507	507.0
B2-1	34	5	688	688.00	688	0.00	0.00	7.47	688	688.0
B3-1	38	5	470	470.00	470	0.00	3.52	7.69	470	454.0
B4-1	42	6	680	680.00	680	0.00	2.27	13.56	680	664.9
B5-1	44	5	594	594.00	594	0.00	3.09	9.84	594	576.2
B6-1	49	7	656	656.00	656	0.00	0.00	14.77	656	656.0
B7-1	50	7	1032	1032.00	1032	0.00	6.04	16.96	1032	973.2
B8-1	55	7	707	707.00	707	0.00	10.31	21.66	707	640.9
B9-1	56	9	1256	1256.00	1256	0.00	0.00	13.75	1256	1256.0
B10-1	63	9	797	797.00	797	0.00	2.06	19.19	797	780.9
B11-1	66	10	1032	1032.00	1032	0.00	2.55	26.67	1032	1006.3
B12-1	77	10	1031	1031.00	1031	0.00	1.14	24.61	1031	1019.4
p03-1	100	3	584	584.00	584	0.00	0.00	31.34	584	584.0
p06-1	50	2	365	365.00	365	0.00	0.00	9.04	365	365.0
p07-1	75	2	506	506.00	506	0.00	0.00	15.98	506	506.0
p08-1	100	3	528	528.00	528	0.00	0.46	21.75	528	525.6
p09-1	150	4	667	667.00	667	0.00	0.69	41.16	667	662.4
p10-1	199	6	805	805.00	805	0.00	1.35	77.11	805	794.3
p13-1	120	3	634	634.00	634	0.00	8.45	40.60	634	584.6
p14-1	100	3	587	587.00	587	0.00	1.31	23.57	587	579.4
p15-1	100	4	591	591.00	591	0.00	1.65	28.38	591	581.4
p16-1	199	6	800	800.00	800	0.00	1.05	77.44	800	791.7

Table 6: HGS results for the VRP-SL instances with 1 group

Inst	n	m	Wor-10	Avg-10	Best-10	Gap _{BKS}	Gap _{LB}	T(s)	BKS	BKLB
A1-2R	31	5	692	692.00	692	0.00	0.00	8.72	692	692.0
A2-2R	32	6	676	676.00	676	0.00	0.00	9.17	676	676.0
A3-2R	35	5	696	696.00	696	0.00	0.00	10.12	696	696.0
A4-2R	36	6	746	746.00	746	0.00	0.00	7.72	746	746.0
A5-2R	38	5	646	646.00	646	0.00	0.00	10.08	646	646.0
A6-2R	43	6	916	916.00	916	0.00	0.39	17.79	916	912.4
A7-2R	44	7	990	990.00	990	0.00	0.00	13.43	990	990.0
A8-2R	47	7	925	925.00	925	0.00	0.00	13.48	925	925.0
A9-2R	53	7	959	959.00	959	0.00	0.00	12.06	959	959.0
A10-2R	59	9	1275	1275.00	1275	0.00	0.54	20.40	1275	1268.2
A11-2R	61	8	1105	1105.00	1105	0.00	1.17	16.64	1105	1092.2
A12-2R	62	10	1170	1170.00	1170	0.00	0.50	17.33	1170	1164.2
A13-2R	64	9	1140	1139.50	1139	0.04	1.86	28.29	1139	1118.7
A14-2R	79	10	1414	1414.00	1414	0.00	2.20	20.48	1414	1383.5
B1-2R	30	5	609	609.00	609	0.00	0.35	10.25	609	606.9
B2-2R	34	5	658	658.00	658	0.00	0.00	6.39	658	658.0
B3-2R	38	5	482	482.00	482	0.00	1.90	8.45	482	473.0
B4-2R	42	6	658	658.00	658	0.00	0.00	11.77	658	658.0
B5-2R	44	5	614	614.00	614	0.00	3.89	12.96	614	591.0
B6-2R	49	7	665	665.00	665	0.00	1.51	14.39	665	655.1
B7-2R	50	7	889	889.00	889	0.00	0.19	12.33	889	887.3
B8-2R	55	7	621	621.00	621	0.00	3.85	17.68	621	598.0
B9-2R	56	9	1443	1443.00	1443	0.00	3.28	17.37	1443	1397.2
B10-2R	63	9	825	825.00	825	0.00	1.99	22.26	825	808.9
B11-2R	66	10	916	916.00	916	0.00	1.08	19.97	916	906.2
B12-2R	77	10	1053	1053.00	1053	0.00	1.91	24.21	1053	1033.3
p03-2R	100	3	551	551.00	551	0.00	0.00	20.30	551	551.0
p06-2R	50	2	367	367.00	367	0.00	0.00	9.50	367	367.0
p07-2R	75	3	506	506.00	506	0.00	0.00	15.86	506	506.0
p08-2R	100	3	556	556.00	556	0.00	0.00	23.92	556	556.0
p09-2R	150	4	667	667.00	667	0.00	0.69	40.23	667	662.4
p10-2R	199	6	805	805.00	805	0.00	1.36	87.22	805	794.2
p13-2R	120	3	656	656.00	656	0.00	7.40	40.81	656	610.8
p14-2R	100	4	590	590.00	590	0.00	3.07	27.86	590	572.4
p15-2R	100	4	590	590.00	590	0.00	3.16	26.09	590	571.9
p16-2R	199	6	801	800.10	800	0.01	1.06	74.66	800	791.7

Table 7: HGS results for the VRP-SL instances with 2 groups (random distribution)

Inst	n	m	Wor-10	Avg-10	Best-10	Gap _{BKS}	Gap _{LB}	T(s)	BKS	BKLB
A1-2C	31	5	764	764.00	764	0.00	0.00	9.81	764	764.0
A2-2C	32	6	632	632.00	632	0.00	0.00	8.53	632	632.0
A3-2C	35	5	645	645.00	645	0.00	0.00	10.04	645	645.0
A4-2C	36	6	784	784.00	784	0.00	0.00	8.64	784	784.0
A5-2C	38	5	689	689.00	689	0.00	0.00	11.86	689	689.0
A6-2C	43	6	696	696.00	696	0.00	0.00	11.64	696	696.0
A7-2C	44	7	1110	1110.00	1110	0.00	0.30	14.89	1110	1106.7
A8-2C	47	7	984	984.00	984	0.00	1.31	13.67	984	971.3
A9-2C	53	7	1132	1132.00	1132	0.00	0.32	19.02	1132	1128.4
A10-2C	59	9	1117	1117.00	1117	0.00	0.00	12.95	1117	1117.0
A11-2C	61	8	1073	1073.00	1073	0.00	0.00	15.96	1073	1073.0
A12-2C	62	10	1057	1056.20	1056	0.02	0.33	17.21	1056	1052.7
A13-2C	64	9	1107	1107.00	1107	0.00	1.82	23.24	1107	1087.2
A14-2C	79	10	1620	1620.00	1620	0.00	1.41	31.96	1620	1597.5
B1-2C	30	5	521	521.00	521	0.00	0.00	8.48	521	521.0
B2-2C	34	5	810	810.00	810	0.00	2.08	9.03	810	793.5
B3-2C	38	5	488	488.00	488	0.00	4.18	10.26	488	468.4
B4-2C	42	6	694	694.00	694	0.00	0.00	14.01	694	694.0
B5-2C	44	5	574	574.00	574	0.00	1.02	8.93	574	568.2
B6-2C	49	7	656	656.00	656	0.00	1.56	12.37	656	645.9
B7-2C	50	7	1032	1032.00	1032	0.00	4.65	17.88	1032	986.1
B8-2C	55	7	600	600.00	600	0.00	0.64	15.61	600	596.2
B9-2C	56	9	1509	1509.00	1509	0.00	2.76	25.46	1509	1468.5
B10-2C	63	9	814	814.00	814	0.00	2.60	18.76	814	793.4
B11-2C	66	10	924	924.00	924	0.00	0.74	20.82	924	917.2
B12-2C	77	10	1141	1139.00	1137	0.18	2.65	36.61	1137	1109.6
p03-2C	100	3	528	528.00	528	0.00	0.55	19.88	528	525.1
p06-2C	50	2	395	395.00	395	0.00	0.00	12.19	395	395.0
p07-2C	75	3	511	511.00	511	0.00	0.00	17.29	511	511.0
p08-2C	100	3	578	578.00	578	0.00	0.00	25.58	578	578.0
p09-2C	150	5	729	729.00	729	0.00	2.20	50.95	729	713.3
p10-2C	199	6	805	805.00	805	0.00	1.39	74.98	805	794.0
p13-2C	120	3	628	626.80	626	0.13	9.29	49.09	626	573.5
p14-2C	100	4	588	588.00	588	0.00	4.35	27.26	588	563.5
p15-2C	100	3	587	587.00	587	0.00	1.28	22.54	587	579.6
p16-2C	199	6	807	804.90	804	0.11	1.53	109.63	804	792.8

Table 8: HGS results for the VRP-SL instances with 2 groups (clustered)

Inst	n	m	Wor-10	Avg-10	Best-10	Gap _{BKS}	Gap _{LB}	T(s)	BKS	BKLB
A1-5R	31	5	726	726.00	726	0.00	0.00	9.67	726	726.0
A2-5R	32	6	615	615.00	615	0.00	0.00	8.34	615	615.0
A3-5R	35	5	599	599.00	599	0.00	0.00	8.27	599	599.0
A4-5R	36	6	927	927.00	927	0.00	0.00	12.51	927	927.0
A5-5R	38	5	683	683.00	683	0.00	1.13	9.93	683	675.4
A6-5R	43	6	763	763.00	763	0.00	0.00	11.23	763	763.0
A7-5R	44	7	1041	1041.00	1041	0.00	0.00	14.95	1041	1041.0
A8-5R	47	7	909	909.00	909	0.00	0.89	11.72	909	901.0
A9-5R	53	7	1149	1149.00	1149	0.00	1.46	21.46	1149	1132.5
A10-5R	59	9	1258	1256.50	1256	0.04	0.69	27.29	1254	1247.9
A11-5R	61	8	1137	1137.00	1137	0.00	0.00	18.45	1137	1137.0
A12-5R	62	10	1207	1207.00	1207	0.00	0.57	26.43	1207	1200.2
A13-5R	64	9	1139	1139.00	1139	0.00	0.62	28.05	1139	1132.0
A14-5R	79	10	1628	1628.00	1628	0.00	1.07	46.04	1628	1610.8
B1-5R	30	5	616	616.00	616	0.00	1.48	9.80	616	607.0
B2-5R	34	5	804	804.00	804	0.00	3.13	10.49	804	779.6
B3-5R	38	5	508	508.00	508	0.00	4.63	12.72	508	485.5
B4-5R	42	6	638	638.00	638	0.00	0.00	10.41	638	638.0
B5-5R	44	5	681	681.00	681	0.00	0.35	13.93	681	678.6
B6-5R	49	7	664	664.00	664	0.00	0.00	13.44	664	664.0
B7-5R	50	7	911	911.00	911	0.00	0.53	12.89	911	906.2
B8-5R	55	7	625	625.00	625	0.00	2.56	18.41	625	609.4
B9-5R	56	9	1331	1327.70	1327	0.05	1.37	29.10	1327	1309.8
B10-5R	63	9	807	807.00	807	0.00	1.98	19.69	807	791.3
B11-5R	66	10	908	906.80	906	0.09	0.80	24.86	906	899.6
B12-5R	77	10	1130	1128.70	1128	0.06	3.09	36.58	1128	1094.9
p03-5R	100	3	557	557.00	557	0.00	0.00	20.60	557	557.0
p06-5R	50	2	421	421.00	421	0.00	0.00	11.40	421	421.0
p07-5R	75	3	549	549.00	549	0.00	0.00	20.27	549	549.0
p08-5R	100	3	532	532.00	532	0.00	0.00	21.05	532	532.0
p09-5R	150	5	704	704.00	704	0.00	1.78	45.24	704	691.7
p10-5R	199	6	806	805.10	805	0.01	1.39	96.83	805	794.1
p13-5R	120	3	652	650.60	650	0.09	8.76	47.11	650	598.2
p14-5R	100	4	587	587.00	587	0.00	4.15	28.11	587	563.6
p15-5R	100	4	588	588.00	588	0.00	4.37	28.05	588	563.4
p16-5R	199	6	800	800.00	800	0.00	1.06	95.57	800	791.6

Table 9: HGS results for the VRP-SL instances with 5 groups (random distribution)

Inst	n	m	Wor-10	Avg-10	Best-10	Gap _{BKS}	Gap _{LB}	T(s)	BKS	BKLB
A1-5C	31	5	724	724.00	724	0.00	0.00	9.59	724	724.0
A2-5C	32	6	708	708.00	708	0.00	1.51	10.35	708	697.5
A3-5C	35	5	739	739.00	739	0.00	0.00	10.56	739	739.0
A4-5C	36	6	818	818.00	818	0.00	0.00	9.82	818	818.0
A5-5C	38	5	713	713.00	713	0.00	0.00	10.47	713	713.0
A6-5C	43	6	897	897.00	897	0.00	0.17	13.30	897	895.5
A7-5C	44	7	1040	1040.00	1040	0.00	0.77	13.41	1040	1032.1
A8-5C	47	7	997	997.00	997	0.00	0.00	14.25	997	997.0
A9-5C	53	7	1013	1013.00	1013	0.00	0.00	17.75	1013	1013.0
A10-5C	59	9	1170	1170.00	1170	0.00	0.65	23.43	1170	1162.5
A11-5C	61	8	1188	1184.50	1184	0.04	2.87	33.89	1184	1151.5
A12-5C	62	10	1136	1136.00	1136	0.00	0.38	19.98	1136	1131.7
A13-5C	64	9	1089	1089.00	1089	0.00	0.83	19.95	1089	1080.0
A14-5C	79	10	1617	1612.50	1612	0.03	0.57	43.07	1612	1603.4
B1-5C	30	5	671	671.00	671	0.00	0.80	11.73	671	665.7
B2-5C	34	5	781	781.00	781	0.00	0.00	8.33	781	781.0
B3-5C	38	5	538	538.00	538	0.00	5.45	12.06	538	510.2
B4-5C	42	6	661	661.00	661	0.00	0.00	10.24	661	661.0
B5-5C	44	5	662	662.00	662	0.00	0.30	12.87	662	660.0
B6-5C	49	7	703	703.00	703	0.00	5.75	16.28	703	664.8
B7-5C	50	7	917	917.00	917	0.00	0.00	15.57	917	917.0
B8-5C	55	7	634	634.00	634	0.00	2.09	17.93	634	621.0
B9-5C	56	9	1352	1347.90	1346	0.14	2.47	25.17	1346	1315.4
B10-5C	63	9	817	817.00	817	0.00	2.99	21.12	817	793.3
B11-5C	66	10	979	979.00	979	0.00	2.93	28.43	979	951.1
B12-5C	77	10	1108	1108.00	1108	0.00	1.79	22.59	1108	1088.5
p03-5C	100	3	555	555.00	555	0.00	0.00	21.23	555	555.0
p06-5C	50	2	387	387.00	387	0.00	0.00	11.57	387	387.0
p07-5C	75	3	526	526.00	526	0.00	0.00	18.15	526	526.0
p08-5C	100	3	530	530.00	530	0.00	0.00	20.19	530	530.0
p09-5C	150	5	711	711.00	711	0.00	2.02	53.00	711	696.9
p10-5C	199	6	833	832.10	831	0.13	1.34	95.36	831	821.1
p13-5C	120	3	634	633.60	633	0.09	9.70	42.63	633	577.6
p14-5C	100	4	588	588.00	588	0.00	4.31	28.40	588	563.7
p15-5C	100	4	588	588.00	588	0.00	4.27	28.85	588	563.9
p16-5C	199	7	817	816.20	815	0.15	1.76	106.96	815	802.1

Table 10: HGS results for the VRP-SL instances with 5 groups (clustered)

Table 11: HGS results for the CPTP instances of [2]

Instance	n	UGHS – [3]			ILS – [3]			This paper				BKS
		Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	T*(s)	
p03-15-200	100	664.92	664.92	189.60	663.83	664.92	131.40	664.92	664.92	32.69	12.88	664.92
p03-2-50	100	57.75	57.75	128.40	57.75	57.75	111.00	57.75	57.75	5.95	0.05	57.75
p03-2-75	100	106.15	106.15	209.40	106.15	106.15	201.00	106.15	106.15	6.75	0.16	106.15
p03-2-100	100	158.21	158.21	295.20	158.21	158.21	253.20	158.21	158.21	7.52	0.25	158.21
p03-2-200	100	330.14	330.14	303.60	330.14	330.14	307.80	330.14	330.14	10.19	0.72	330.14
p03-3-50	100	80.82	80.82	96.60	80.82	80.82	88.80	80.82	80.82	6.60	0.10	80.82
p03-3-75	100	147.55	147.55	171.00	147.55	147.55	151.80	147.55	147.55	8.03	0.26	147.55
p03-3-100	100	218.63	218.63	225.60	218.63	218.63	208.80	218.63	218.63	8.49	0.40	218.63
p03-3-200	100	447.15	447.15	302.40	447.10	447.15	305.40	447.15	447.15	16.22	2.51	447.15
p03-4-50	100	100.36	100.36	100.20	100.36	100.36	84.00	100.36	100.36	7.55	0.15	100.36
p03-4-75	100	185.27	185.27	212.40	185.27	185.27	141.00	185.27	185.27	9.37	0.29	185.27
p03-4-100	100	268.34	268.34	268.80	268.34	268.34	219.60	268.34	268.34	11.80	1.14	268.34
p03-4-200	100	537.66	537.66	302.40	537.66	537.66	287.40	537.62	537.66	28.98	12.71	537.66
p06-10-160	50	259.24	259.24	22.20	258.97	258.97	18.60	259.24	259.24	12.91	0.12	259.24
p06-2-50	50	33.88	33.88	19.80	33.88	33.88	20.40	33.88	33.88	4.20	0.00	33.88
p06-2-75	50	72.28	72.28	35.40	72.28	72.28	30.60	72.28	72.28	4.91	0.06	72.28
p06-2-100	50	100.27	100.27	47.40	100.20	100.27	42.60	100.27	100.27	5.41	0.09	100.27
p06-2-160	50	168.60	168.60	73.20	168.60	168.60	42.00	168.60	168.60	6.69	0.15	168.60
p06-3-50	50	40.95	40.95	21.00	40.95	40.95	17.40	40.95	40.95	4.55	0.02	40.95
p06-3-75	50	92.32	92.32	22.20	92.32	92.32	19.80	92.32	92.32	5.69	0.06	92.32
p06-3-100	50	134.72	134.72	25.20	134.72	134.72	16.20	134.72	134.72	6.57	0.08	134.72
p06-3-160	50	219.36	219.36	50.40	219.36	219.36	30.60	219.36	219.36	9.73	0.55	219.36
p06-4-50	50	45.43	45.43	21.00	45.43	45.43	16.20	45.43	45.43	5.26	0.06	45.43
p06-4-75	50	99.37	99.37	16.20	99.37	99.37	17.40	99.37	99.37	6.74	0.11	99.37
p06-4-100	50	153.30	153.30	20.40	153.30	153.30	13.80	153.30	153.30	8.45	0.29	153.30
p06-4-160	50	258.97	258.97	25.80	258.78	258.97	18.00	258.97	258.97	12.26	0.50	258.97
p07-20-140	75	540.67	541.32	34.80	538.94	541.32	28.20	540.78	541.32	21.69	2.78	541.32
p07-2-50	75	49.18	49.18	63.60	49.18	49.18	52.80	49.18	49.18	5.00	0.00	49.18
p07-2-75	75	92.44	92.44	93.00	92.44	92.44	68.40	92.44	92.44	5.48	0.03	92.44
p07-2-100	75	132.70	132.70	132.60	132.70	132.70	114.60	132.70	132.70	6.03	0.08	132.70
p07-2-140	75	199.97	199.97	258.00	199.97	199.97	177.00	199.97	199.97	6.72	0.12	199.97
p07-3-50	75	69.94	69.94	61.80	69.94	69.94	60.60	69.94	69.94	5.29	0.02	69.94
p07-3-75	75	131.12	131.12	71.40	131.12	131.12	57.60	131.12	131.12	6.27	0.08	131.12
p07-3-100	75	185.25	185.25	106.80	185.25	185.25	93.00	185.25	185.25	7.35	0.20	185.25
p07-3-140	75	274.80	274.80	189.00	274.80	274.80	157.20	274.80	274.80	9.43	0.91	274.80
p07-4-50	75	90.65	90.65	50.40	90.65	90.65	45.60	90.65	90.65	6.06	0.02	90.65
p07-4-75	75	158.11	158.11	75.00	158.11	158.11	69.00	158.11	158.11	7.64	0.17	158.11
p07-4-100	75	233.40	233.40	88.80	233.40	233.40	87.60	233.40	233.40	8.53	0.30	233.40
p07-4-140	75	344.35	344.35	126.00	344.35	344.35	103.80	344.35	344.35	13.20	2.67	344.35
p08-15-200	100	664.92	664.92	190.80	663.83	664.92	130.80	662.74	664.92	32.20	9.60	664.92
p08-2-50	100	57.75	57.75	127.20	57.75	57.75	108.00	57.75	57.75	5.58	0.05	57.75
p08-2-75	100	106.15	106.15	210.60	106.15	106.15	202.20	106.15	106.15	6.64	0.15	106.15
p08-2-100	100	158.21	158.21	299.40	158.21	158.21	253.80	158.21	158.21	7.35	0.22	158.21
p08-2-200	100	330.14	330.14	303.60	330.14	330.14	307.80	330.14	330.14	9.94	0.78	330.14
p08-3-50	100	80.82	80.82	96.60	80.82	80.82	91.20	80.82	80.82	6.57	0.10	80.82
p08-3-75	100	147.55	147.55	171.00	147.55	147.55	147.60	147.55	147.55	8.01	0.27	147.55
p08-3-100	100	218.63	218.63	226.20	218.63	218.63	210.60	218.63	218.63	8.96	0.46	218.63
p08-3-200	100	447.15	447.15	302.40	446.44	447.15	304.20	447.15	447.15	15.42	2.02	447.15
p08-4-50	100	100.36	100.36	103.20	100.36	100.36	84.00	100.36	100.36	7.45	0.14	100.36
p08-4-75	100	185.27	185.27	212.40	185.27	185.27	139.80	185.27	185.27	9.18	0.23	185.27
p08-4-100	100	268.34	268.34	265.80	268.34	268.34	220.80	268.34	268.34	12.14	1.46	268.34

Continued on the next page

Table 11: HGS results for the CPTP instances of [2] (continued)

Instance	n	UGHS – [3]			ILS – [3]			This paper				BKS
		Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	T*(s)	
p08-4-200	100	537.66	537.66	302.40	537.66	537.66	288.60	537.58	537.66	28.77	12.43	537.66
p09-10-200	150	1214.99	1215.29	296.40	1213.40	1215.29	282.60	1215.29	1215.29	75.60	38.19	1215.29
p09-2-50	150	65.03	65.03	278.40	65.03	65.03	240.60	65.03	65.03	7.80	0.03	65.03
p09-2-75	150	117.66	117.66	303.00	117.66	117.66	301.20	117.66	117.66	7.89	0.15	117.66
p09-2-100	150	161.22	161.23	303.00	161.19	161.23	306.60	161.23	161.23	9.98	0.59	161.23
p09-2-200	150	347.90	347.90	306.60	346.76	347.90	321.00	347.90	347.90	13.56	1.38	347.90
p09-3-50	150	96.16	96.16	286.80	96.16	96.16	231.00	96.16	96.16	8.35	0.10	96.16
p09-3-75	150	160.96	160.96	302.40	160.96	160.96	304.20	160.96	160.96	10.51	0.64	160.96
p09-3-100	150	230.49	230.49	302.40	230.32	230.49	306.00	230.49	230.49	12.47	1.22	230.49
p09-3-200	150	502.34	502.34	304.80	501.58	502.34	309.60	502.34	502.34	19.16	4.94	502.34
p09-4-50	150	121.35	121.35	247.80	121.35	121.35	253.20	121.35	121.35	9.63	0.16	121.35
p09-4-75	150	204.25	204.25	302.40	204.25	204.25	296.40	204.25	204.25	12.44	0.91	204.25
p09-4-100	150	290.75	290.97	302.40	290.89	290.97	305.40	290.97	290.97	17.57	4.90	290.97
p09-4-200	150	641.78	642.72	303.00	640.42	642.72	309.00	642.10	642.72	34.82	17.30	642.72
p10-20-200	199	1788.60	1793.95	302.40	1776.49	1785.00	303.60	1798.79	1799.65	86.01	36.65	1793.95
p10-2-50	199	70.87	70.87	301.80	70.87	70.87	303.00	70.87	70.87	9.76	0.09	70.87
p10-2-75	199	124.85	124.85	303.60	124.85	124.85	306.60	124.85	124.85	10.35	0.24	124.85
p10-2-100	199	171.24	171.24	303.60	171.24	171.24	312.00	171.24	171.24	11.59	0.57	171.24
p10-2-200	199	382.00	382.41	307.20	382.03	382.41	325.80	382.41	382.41	14.77	1.24	382.41
p10-3-50	199	103.79	103.79	301.80	103.79	103.79	303.00	103.79	103.79	10.15	0.14	103.79
p10-3-75	199	177.90	177.90	302.40	177.90	177.90	306.00	177.90	177.90	11.67	0.36	177.90
p10-3-100	199	249.82	250.18	303.60	249.55	250.18	308.40	250.18	250.18	12.53	0.91	250.18
p10-3-200	199	559.66	560.12	306.00	556.76	560.12	319.20	560.12	560.12	19.71	3.15	560.12
p10-4-50	199	134.81	134.81	301.80	134.81	134.81	303.60	134.81	134.81	11.27	0.21	134.81
p10-4-75	199	229.27	229.27	302.40	229.27	229.27	306.00	229.27	229.27	12.98	0.47	229.27
p10-4-100	199	324.90	324.93	303.00	324.67	324.93	307.20	324.93	324.93	15.69	2.07	324.93
p10-4-200	199	724.65	725.06	306.60	724.46	725.06	315.60	725.06	725.06	32.19	13.16	725.06
p13-15-200	120	319.68	319.68	302.40	316.98	319.68	304.80	317.67	319.68	33.80	7.97	319.68
p13-2-50	120	64.12	64.12	106.80	64.12	64.12	131.40	64.12	64.12	6.93	0.13	64.12
p13-2-75	120	110.12	110.12	235.80	110.12	110.12	295.20	110.12	110.12	8.61	0.24	110.12
p13-2-100	120	145.75	145.75	301.80	145.75	145.75	304.20	145.75	145.75	9.66	0.18	145.75
p13-2-200	120	240.87	241.15	304.20	239.99	241.15	309.60	241.15	241.15	16.97	2.96	241.15
p13-3-50	120	87.25	87.25	89.40	87.25	87.25	106.80	87.25	87.25	8.27	0.28	87.25
p13-3-75	120	139.37	139.37	207.00	139.37	139.37	296.40	139.37	139.37	11.10	0.23	139.37
p13-3-100	120	181.63	181.63	303.00	163.25	181.63	274.20	181.63	181.63	13.25	0.55	181.63
p13-3-200	120	282.70	283.70	303.60	281.94	283.70	311.40	282.62	283.70	20.14	1.64	283.70
p13-4-50	120	104.18	104.18	115.20	104.18	104.18	111.00	104.18	104.18	10.54	0.77	104.18
p13-4-75	120	161.62	161.62	202.80	161.62	161.62	270.60	161.62	161.62	13.62	0.47	161.62
p13-4-100	120	202.30	202.36	294.00	202.12	202.36	305.40	202.36	202.36	16.20	1.13	202.36
p13-4-200	120	303.07	304.15	301.80	301.61	303.18	306.60	303.15	303.18	27.84	6.11	304.15
p14-10-200	100	890.44	890.44	37.20	890.44	890.44	35.40	890.44	890.44	26.10	0.44	890.44
p14-2-50	100	43.26	43.26	80.40	43.26	43.26	84.60	43.26	43.26	6.58	0.08	43.26
p14-2-75	100	77.09	77.09	108.00	77.09	77.09	111.60	77.09	77.09	6.81	0.12	77.09
p14-2-100	100	125.29	125.29	136.20	125.29	125.29	132.00	125.29	125.29	7.63	0.16	125.29
p14-2-200	100	303.37	303.37	282.00	303.37	303.37	279.00	303.37	303.37	9.39	0.89	303.37
p14-3-50	100	59.43	59.43	55.80	59.43	59.43	64.20	59.43	59.43	6.59	0.13	59.43
p14-3-75	100	112.56	112.56	69.00	112.56	112.56	78.60	112.56	112.56	7.75	0.17	112.56
p14-3-100	100	182.31	182.31	115.20	182.31	182.31	100.80	182.31	182.31	9.04	0.35	182.31
p14-3-200	100	423.36	423.36	291.60	423.27	423.36	227.40	423.36	423.36	16.59	4.53	423.36
p14-4-50	100	68.63	68.63	61.20	68.63	68.63	68.40	68.63	68.63	7.71	0.18	68.63
p14-4-75	100	139.88	139.88	54.00	139.88	139.88	62.40	139.88	139.88	8.45	0.25	139.88

Continued on the next page

Table 11: HGS results for the CPTP instances of [2] (continued)

Instance	n	UGHS – [3]			ILS – [3]			This paper				BKS
		Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	Avg-10	Best-10	T(s)	T*(s)	
p14-4-100	100	237.68	237.68	70.80	237.68	237.68	78.60	237.68	237.68	10.42	0.52	237.68
p14-4-200	100	537.80	537.80	150.00	537.80	537.80	130.80	537.80	537.80	17.07	2.10	537.80
p15-15-200	150	1179.39	1180.65	296.40	1170.14	1173.56	250.20	1179.96	1180.88	50.68	12.10	1180.65
p15-2-50	150	64.98	64.98	285.00	64.98	64.98	285.00	64.98	64.98	8.03	0.16	64.98
p15-2-75	150	120.93	120.93	303.00	120.93	120.93	304.20	120.93	120.93	8.78	0.15	120.93
p15-2-100	150	169.71	169.71	303.60	169.71	169.71	306.00	169.71	169.71	9.31	0.19	169.71
p15-2-200	150	377.58	378.09	304.80	376.96	378.09	316.20	378.09	378.09	11.97	0.73	378.09
p15-3-50	150	96.42	96.42	271.20	96.42	96.42	247.20	96.42	96.42	8.77	0.15	96.42
p15-3-75	150	174.58	174.58	302.40	174.58	174.58	304.80	174.58	174.58	10.16	0.29	174.58
p15-3-100	150	244.08	244.08	302.40	244.08	244.08	305.40	244.08	244.08	11.14	0.56	244.08
p15-3-200	150	521.43	522.81	304.80	519.56	522.81	313.80	522.57	522.81	17.86	3.15	522.81
p15-4-50	150	124.02	124.02	256.20	124.02	124.02	216.60	124.02	124.02	9.65	0.15	124.02
p15-4-75	150	219.22	219.22	300.60	219.22	219.22	303.00	219.22	219.22	11.77	0.59	219.22
p15-4-100	150	309.75	309.75	301.80	309.75	309.75	306.60	309.75	309.75	14.84	1.88	309.75
p15-4-200	150	661.80	663.40	303.00	659.45	663.40	309.00	663.40	663.40	25.11	7.17	663.40
p16-20-200	199	1804.00	1809.98	301.80	1797.76	1804.32	303.60	1813.69	1813.76	84.46	32.38	1809.98
p16-2-50	199	66.81	66.81	303.00	66.81	66.81	304.80	66.81	66.81	9.79	0.22	66.81
p16-2-75	199	123.38	123.38	302.40	123.38	123.38	306.00	123.38	123.38	10.52	0.27	123.38
p16-2-100	199	176.65	177.23	303.60	176.88	177.23	309.60	177.23	177.23	11.92	0.66	177.23
p16-2-200	199	394.05	394.05	309.60	393.23	394.05	324.60	394.05	394.05	15.28	1.52	394.05
p16-3-50	199	99.70	99.70	301.80	99.70	99.70	304.80	99.70	99.70	10.47	0.22	99.70
p16-3-75	199	179.55	179.55	302.40	179.55	179.55	306.60	179.55	179.55	12.21	0.50	179.55
p16-3-100	199	258.89	259.25	302.40	258.65	259.25	308.40	259.25	259.25	13.90	1.06	259.25
p16-3-200	199	567.84	568.13	309.00	566.32	568.13	329.40	568.13	568.13	28.12	11.77	568.13
p16-4-50	199	131.37	131.37	302.40	131.37	131.37	297.60	131.37	131.37	11.39	0.26	131.37
p16-4-75	199	235.03	235.03	302.40	234.78	235.03	303.60	235.03	235.03	13.73	0.93	235.03
p16-4-100	199	337.19	337.80	303.00	336.33	337.80	308.40	337.80	337.80	15.56	1.86	337.80
p16-4-200	199	735.69	736.52	306.60	733.92	736.52	319.20	736.37	736.52	37.73	18.63	736.52
Avg. Gap(%)		0.029	0.000		0.172	0.014		0.012	-0.002			
Avg. T(s)				211.68			205.86			14.07		
Avg. T*(s)											2.53	
CPU		Xe 3.07GHz			Xe 3.07GHz			Intel i7 3.4GHz				

Table 12: Branch-and-price results for the CPTP instances of [2]

Instance	n	B&P1 – [1]			B&P2 – [1]			This paper – B&P				BKUB
		UB	LB	T(s)	UB	LB	T(s)	UB ₀	UB	LB	T(s)	
p03-15-200	100	675.00	664.92	3600.00	674.68	–	3600.00	675.52	672.79	664.92	3600.00	674.68
p03-2-50	100	57.75	57.75	0.00	57.75	57.75	0.00	57.75	57.75	57.75	0.06	57.75
p03-2-75	100	106.15	106.15	3.00	106.15	106.15	3.00	106.24	106.15	106.15	0.50	106.15
p03-2-100	100	158.21	158.21	5.00	158.21	158.21	5.00	159.48	158.21	158.21	2.52	158.21
p03-2-200	100	337.59	330.14	3600.00	337.43	–	3600.00	339.14	331.74	330.14	3600.00	337.43
p03-3-50	100	80.82	80.82	0.00	80.82	80.82	0.00	81.43	80.82	80.82	0.15	80.82
p03-3-75	100	147.55	147.55	4.00	147.55	147.55	4.00	150.39	147.55	147.55	1.46	147.55
p03-3-100	100	218.63	218.63	11.00	218.63	218.63	11.00	221.71	218.63	218.63	8.84	218.63
p03-3-200	100	451.92	447.15	3600.00	451.68	–	3600.00	453.97	447.62	447.15	3600.00	451.68
p03-4-50	100	100.36	100.36	0.00	100.36	100.36	0.00	100.36	100.36	100.36	0.07	100.36
p03-4-75	100	185.27	185.27	3.00	185.27	185.27	3.00	188.26	185.27	185.27	2.56	185.27
p03-4-100	100	268.34	268.34	333.00	268.34	268.34	334.00	271.41	268.34	268.34	91.90	268.34
p03-4-200	100	552.05	537.66	3600.00	551.74	–	3600.00	550.59	544.46	537.66	3600.00	551.74
p06-10-160	50	265.99	259.24	3600.00	265.21	–	3600.00	270.33	262.66	259.24	3600.00	265.21
p06-2-50	50	33.88	33.88	0.00	33.88	33.88	0.00	35.60	33.88	33.88	0.11	33.88
p06-2-75	50	72.28	72.28	1.00	72.28	72.28	0.00	72.28	72.28	72.28	0.05	72.28
p06-2-100	50	100.27	100.27	2.00	100.27	100.27	2.00	100.27	100.27	100.27	0.09	100.27
p06-2-160	50	168.60	168.60	13.00	168.60	168.60	12.00	169.19	168.60	168.60	1.11	168.60
p06-3-50	50	40.95	40.95	0.00	40.95	40.95	0.00	41.99	40.95	40.95	0.08	40.95
p06-3-75	50	92.32	92.32	0.00	92.32	92.32	0.00	92.99	92.32	92.32	0.16	92.32
p06-3-100	50	134.72	134.72	2.00	134.72	134.72	1.00	134.72	134.72	134.72	0.11	134.72
p06-3-160	50	219.36	219.36	870.00	219.36	219.36	578.00	224.91	219.36	219.36	41.68	219.36
p06-4-50	50	45.43	45.43	0.00	45.43	45.43	0.00	45.43	45.43	45.43	0.02	45.43
p06-4-75	50	99.37	99.37	22.00	99.37	99.37	22.00	103.89	99.37	99.37	3.09	99.37
p06-4-100	50	153.30	153.30	16.00	153.30	153.30	16.00	156.13	153.30	153.30	2.26	153.30
p06-4-160	50	260.31	258.97	3600.00	259.67	256.55	3600.00	265.24	258.97 [†]	258.97	93.87	259.67
p07-20-140	75	548.65	541.32	3600.00	548.10	–	3600.00	552.48	544.79	541.32	3600.00	548.10
p07-2-50	75	49.18	49.18	0.00	49.18	49.18	0.00	49.18	49.18	49.18	0.03	49.18
p07-2-75	75	92.44	92.44	0.00	92.44	92.44	0.00	92.44	92.44	92.44	0.10	92.44
p07-2-100	75	132.70	132.70	1.00	132.70	132.70	1.00	132.70	132.70	132.70	0.13	132.70
p07-2-140	75	199.97	199.97	3.00	199.97	199.97	4.00	199.97	199.97	199.97	0.27	199.97
p07-3-50	75	69.94	69.94	0.00	69.94	69.94	0.00	69.94	69.94	69.94	0.03	69.94
p07-3-75	75	131.12	131.12	0.00	131.12	131.12	0.00	131.12	131.12	131.12	0.06	131.12
p07-3-100	75	185.25	185.25	3.00	185.25	185.25	2.00	185.25	185.25	185.25	0.16	185.25
p07-3-140	75	274.80	274.80	878.00	274.80	274.80	768.00	277.86	274.80	274.80	19.33	274.80
p07-4-50	75	90.65	90.65	0.00	90.65	90.65	0.00	90.65	90.65	90.65	0.03	90.65
p07-4-75	75	158.11	158.11	3.00	158.11	158.11	3.00	161.38	158.11	158.11	2.57	158.11
p07-4-100	75	233.40	233.40	3.00	233.40	233.40	3.00	233.40	233.40	233.40	0.13	233.40
p07-4-140	75	345.15	344.35	3600.00	344.82	344.35	3600.00	348.33	344.35 [†]	344.35	82.92	344.82
p08-15-200	100	674.99	664.92	3600.00	674.68	–	3600.00	675.52	672.77	664.92	3600.00	674.68
p08-2-50	100	57.75	57.75	0.00	57.75	57.75	0.00	57.75	57.75	57.75	0.07	57.75
p08-2-75	100	106.15	106.15	3.00	106.15	106.15	3.00	106.24	106.15	106.15	0.52	106.15
p08-2-100	100	158.21	158.21	5.00	158.21	158.21	5.00	159.48	158.21	158.21	2.78	158.21
p08-2-200	100	337.59	330.14	3600.00	337.42	–	3600.00	339.14	331.76	330.14	3600.00	337.42
p08-3-50	100	80.82	80.82	0.00	80.82	80.82	0.00	81.43	80.82	80.82	0.15	80.82
p08-3-75	100	147.55	147.55	4.00	147.55	147.55	4.00	150.39	147.55	147.55	1.31	147.55
p08-3-100	100	218.63	218.63	11.00	218.63	218.63	10.00	221.71	218.63	218.63	8.65	218.63
p08-3-200	100	451.90	447.15	3600.00	451.70	–	3600.00	453.97	447.54	447.15	3600.00	451.70
p08-4-50	100	100.36	100.36	0.00	100.36	100.36	0.00	100.36	100.36	100.36	0.07	100.36
p08-4-75	100	185.27	185.27	3.00	185.27	185.27	3.00	188.26	185.27	185.27	2.62	185.27
p08-4-100	100	268.34	268.34	320.00	268.34	268.34	330.00	271.41	268.34	268.34	92.44	268.34

Continued on the next page

Table 12: Branch-and-price results for the CPTP instances of [2] (continued)

Instance	n	B&P1 – [1]			B&P2 – [1]			This paper – B&P				BKUB
		UB	LB	T(s)	UB	LB	T(s)	UB ₀	UB	LB	T(s)	
p08-4-200	100	552.00	537.66	3600.00	551.74	–	3600.00	550.59	544.49	537.66	3600.00	551.74
p09-10-200	150	1233.57	1187.54	3600.00	1233.23	–	3600.00	1228.77	1225.95	1215.29	3600.00	1233.23
p09-2-50	150	65.03	65.03	0.00	65.03	65.03	0.00	65.03	65.03	65.03	0.16	65.03
p09-2-75	150	117.66	117.66	2.00	117.66	117.66	2.00	117.66	117.66	117.66	0.38	117.66
p09-2-100	150	161.23	161.23	41.00	161.23	161.23	41.00	163.71	161.23	161.23	5.34	161.23
p09-2-200	150	359.97	343.21	3600.00	359.92	–	3600.00	357.36	348.45	347.90	3600.00	359.92
p09-3-50	150	96.16	96.16	1.00	96.16	96.16	1.00	96.16	96.16	96.16	0.16	96.16
p09-3-75	150	160.96	160.96	8.00	160.96	160.96	8.00	161.82	160.96	160.96	4.52	160.96
p09-3-100	150	230.49	230.49	37.00	230.49	230.49	37.00	233.31	230.49	230.49	10.44	230.49
p09-3-200	150	516.45	500.87	3600.00	516.37	–	3600.00	509.74	502.61	502.34	3600.00	516.37
p09-4-50	150	121.35	121.35	1.00	121.35	121.35	1.00	121.35	121.35	121.35	0.17	121.35
p09-4-75	150	204.25	204.25	6.00	204.25	204.25	6.00	204.91	204.25	204.25	0.91	204.25
p09-4-100	150	290.97	290.97	632.00	290.97	290.97	634.00	297.19	290.97	290.97	185.31	290.97
p09-4-200	150	657.13	632.54	3600.00	656.93	–	3600.00	650.58	645.68	642.72	3600.00	656.93
p10-20-200	199	1829.59	1655.01	3600.00	1824.91	–	3600.00	1819.95	1817.52	1799.65	3600.00	1824.91
p10-2-50	199	70.87	70.87	1.00	70.87	70.87	1.00	70.87	70.87	70.87	0.24	70.87
p10-2-75	199	124.85	124.85	2.00	124.85	124.85	2.00	124.85	124.85	124.85	0.53	124.85
p10-2-100	199	171.24	171.24	18.00	171.24	171.24	19.00	171.59	171.24	171.24	2.08	171.24
p10-2-200	199	383.62	382.41	3600.00	383.58	–	3600.00	384.95	382.41 [†]	382.41	42.57	383.58
p10-3-50	199	103.79	103.79	1.00	103.79	103.79	1.00	103.79	103.79	103.79	0.25	103.79
p10-3-75	199	177.90	177.90	9.00	177.90	177.90	9.00	179.75	177.90	177.90	3.81	177.90
p10-3-100	199	250.18	250.18	32.00	250.18	250.18	33.00	251.19	250.18	250.18	15.43	250.18
p10-3-200	199	563.16	553.36	3600.00	563.08	–	3600.00	565.57	560.12 [†]	560.12	462.79	563.08
p10-4-50	199	134.81	134.81	2.00	134.81	134.81	1.00	134.81	134.81	134.81	0.25	134.81
p10-4-75	199	229.27	229.27	9.00	229.27	229.27	9.00	230.10	229.27	229.27	2.22	229.27
p10-4-100	199	324.93	324.93	57.00	324.93	324.93	56.00	326.85	324.93	324.93	22.63	324.93
p10-4-200	199	732.19	722.46	3600.00	732.19	–	3600.00	732.65	727.73	725.06	3600.00	732.19
p13-15-200	120	357.52	271.39	3600.00	357.30	–	3600.00	341.10	337.65	319.68	3600.00	357.30
p13-2-50	120	64.12	64.12	13.00	64.12	64.12	13.00	66.08	64.12	64.12	4.61	64.12
p13-2-75	120	110.12	110.12	10.00	110.12	110.12	10.00	110.12	110.12	110.12	0.50	110.12
p13-2-100	120	145.75	145.75	218.00	145.75	145.75	218.00	145.75	145.75	145.75	1.22	145.75
p13-2-200	120	257.62	219.29	3600.00	257.56	–	3600.00	247.99	243.75	241.15	3600.00	257.56
p13-3-50	120	87.25	87.25	37.00	87.25	87.25	38.00	88.60	87.25	87.25	4.46	87.25
p13-3-75	120	139.37	139.37	702.00	139.37	139.37	700.00	141.50	139.37	139.37	78.00	139.37
p13-3-100	120	196.21	177.07	3600.00	196.21	–	3600.00	183.27	181.63 [†]	181.63	43.37	196.21
p13-3-200	120	314.59	244.31	3600.00	314.44	–	3600.00	297.12	292.55	283.70	3600.00	314.44
p13-4-50	120	104.18	104.18	243.00	104.18	104.18	250.00	106.11	104.18	104.18	25.69	104.18
p13-4-75	120	161.62	161.62	2086.00	161.62	161.62	2089.00	163.99	161.62	161.62	320.20	161.62
p13-4-100	120	206.59	201.59	3600.00	206.59	–	3600.00	205.78	202.36 [†]	202.36	932.28	206.59
p13-4-200	120	347.68	255.11	3600.00	347.61	–	3600.00	329.91	320.75	303.18	3600.00	347.61
p14-10-200	100	925.01	878.35	3600.00	925.02	–	3600.00	903.67	898.53	890.44	3600.00	925.02
p14-2-50	100	43.26	43.26	0.00	43.26	43.26	0.00	43.26	43.26	43.26	0.01	43.26
p14-2-75	100	77.09	77.09	1.00	77.09	77.09	1.00	77.43	77.09	77.09	0.08	77.09
p14-2-100	100	125.29	125.29	7.00	125.29	125.29	7.00	125.78	125.29	125.29	0.22	125.29
p14-2-200	100	326.11	283.35	3600.00	325.95	–	3600.00	306.30	303.37 [†]	303.37	21.68	325.95
p14-3-50	100	59.43	59.43	0.00	59.43	59.43	0.00	59.43	59.43	59.43	0.01	59.43
p14-3-75	100	112.56	112.56	1.00	112.56	112.56	1.00	112.56	112.56	112.56	0.03	112.56
p14-3-100	100	182.31	182.31	8.00	182.31	182.31	8.00	184.26	182.31	182.31	0.89	182.31
p14-3-200	100	456.50	399.37	3600.00	456.27	–	3600.00	432.78	427.73	423.36	3600.00	456.27
p14-4-50	100	68.63	68.63	2.00	68.63	68.63	2.00	69.53	68.63	68.63	0.06	68.63
p14-4-75	100	139.88	139.88	2.00	139.88	139.88	2.00	139.88	139.88	139.88	0.03	139.88

Continued on the next page

Table 12: Branch-and-price results for the CPTP instances of [2] (continued)

Instance	n	B&P1 – [1]			B&P2 – [1]			This paper – B&P				BKUB
		UB	LB	T(s)	UB	LB	T(s)	UB ₀	UB	LB	T(s)	
p14-4-100	100	237.68	237.68	13.00	237.68	237.68	13.00	238.59	237.68	237.68	0.34	237.68
p14-4-200	100	567.31	474.06	3600.00	566.90	–	3600.00	539.23	537.80 [†]	537.80	7.49	566.90
p15-15-200	150	1210.22	1146.68	3600.00	1209.69	–	3600.00	1206.41	1203.74	1180.88	3600.00	1209.69
p15-2-50	150	64.98	64.98	1.00	64.98	64.98	1.00	65.00	64.98	64.98	0.33	64.98
p15-2-75	150	120.93	120.93	2.00	120.93	120.93	2.00	120.93	120.93	120.93	0.35	120.93
p15-2-100	150	169.71	169.71	13.00	169.71	169.71	13.00	171.60	169.71	169.71	24.64	169.71
p15-2-200	150	378.09	378.09	608.00	378.09	378.09	594.00	380.41	378.09	378.09	94.91	378.09
p15-3-50	150	96.42	96.42	1.00	96.42	96.42	1.00	96.42	96.42	96.42	0.16	96.42
p15-3-75	150	174.58	174.58	5.00	174.58	174.58	5.00	174.58	174.58	174.58	0.36	174.58
p15-3-100	150	244.08	244.08	31.00	244.08	244.08	31.00	245.75	244.08	244.08	13.70	244.08
p15-3-200	150	533.75	512.56	3600.00	533.70	–	3600.00	534.34	529.76	522.81	3600.00	533.70
p15-4-50	150	124.02	124.02	1.00	124.02	124.02	1.00	124.02	124.02	124.02	0.17	124.02
p15-4-75	150	219.22	219.22	9.00	219.22	219.22	9.00	219.22	219.22	219.22	0.38	219.22
p15-4-100	150	309.75	309.75	145.00	309.75	309.75	162.00	315.64	309.75	309.75	190.60	309.75
p15-4-200	150	669.93	653.40	3600.00	669.87	–	3600.00	666.30	663.40 [†]	663.40	320.10	669.87
p16-20-200	199	1848.57	1629.10	3600.00	1845.31	–	3600.00	1837.65	1835.70	1813.76	3600.00	1845.31
p16-2-50	199	66.81	66.81	1.00	66.81	66.81	1.00	66.81	66.81	66.81	0.25	66.81
p16-2-75	199	123.38	123.38	8.00	123.38	123.38	8.00	123.38	123.38	123.38	0.51	123.38
p16-2-100	199	177.23	177.23	23.00	177.23	177.23	22.00	178.70	177.23	177.23	11.95	177.23
p16-2-200	199	397.86	391.66	3600.00	397.83	–	3600.00	400.91	394.05 [†]	394.05	1403.30	397.83
p16-3-50	199	99.70	99.70	1.00	99.70	99.70	1.00	99.70	99.70	99.70	0.23	99.70
p16-3-75	199	179.55	179.55	20.00	179.55	179.55	20.00	179.75	179.55	179.55	1.71	179.55
p16-3-100	199	259.25	259.25	40.00	259.25	259.25	40.00	260.72	259.25	259.25	6.84	259.25
p16-3-200	199	579.70	562.62	3600.00	579.27	–	3600.00	581.71	574.43	568.13	3600.00	579.27
p16-4-50	199	131.37	131.37	2.00	131.37	131.37	2.00	131.37	131.37	131.37	0.25	131.37
p16-4-75	199	235.03	235.03	12.00	235.03	235.03	12.00	235.03	235.03	235.03	0.55	235.03
p16-4-100	199	337.80	337.80	33.00	337.80	337.80	33.00	339.97	337.80	337.80	18.22	337.80
p16-4-200	199	745.15	733.95	3600.00	745.03	–	3600.00	745.87	739.86	736.52	3600.00	745.03
Average CPU		273.07	263.12	1083.69	272.96	151.91	1080.75	272.44	268.74	268.73	784.28	272.96
		Xe 3.07GHz			Xe 3.07GHz			Intel i7 3.4GHz				

†: New optimal solution.

References

- [1] Archetti, C., N. Bianchessi, M.G. Speranza. 2013. Optimal solutions for routing problems with profits. *Discrete Applied Mathematics* **161**(4-5) 547–557.
- [2] Archetti, C., D. Feillet, A. Hertz, M.G. Speranza. 2009. The capacitated team orienteering and profitable tour problems. *Journal of the Operational Research Society* **60**(6) 831–842.
- [3] Vidal, T., N. Maculan, L.S. Ochi, P.H.V. Penna. 2016. Large neighborhoods with implicit customer selection for vehicle routing problems with profits. *Transportation Science* **50**(2) 720–734.