Workshop on Traffic Assignment with Equilibrium Methods

Presented by:
David Boyce and Michael Florian
Northwestern University and University of Montreal

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Workshop Objectives and Overview

• Introduce the concept of user equilibrium (UE) in a road network
• Describe and classify the alternative concepts and models of route choice with fixed and variable trip tables (demand)
• Describe the solution of the problem with respect to measures of convergence
• Present case studies of applications to illustrate the concepts
• Provide as much time for discussion as possible, following the two presentations
What we will NOT do in this Workshop

• Present mathematical formulations
• Describe specific solution algorithms
• Describe or compare travel forecasting software systems
• Discuss dynamic traffic assignment or other extensions to the basic static problem
What is Road Traffic Assignment?

• Traditionally, a procedure for “loading” an origin-destination (OD) trip table onto links of a road network
• A behavioral model of route choice over a road network, describing an equilibrium between users’ route choices and OD travel times
• A critical step in the sequential procedure, which determines link and OD travel times, thereby influencing OD choice and mode choice through a “feedback” mechanism, or through the solution of a route choice model integrated with a model of variable OD flows (demand)
• A mathematical model, which can be solved by an iterative solution procedure, or algorithm
Traffic Assignment Assumptions - 1

• A time period of substantial length, compared with the duration of trips, in which the level of congestion in the network is relatively constant (either high or low); for example, a period of one or two hours. Such models are static, in contrast to dynamic models.

• An input trip table giving the flow per hour from each origin zone to each destination zone.

• A road network description consisting of nodes, links and link travel time-flow functions, which increases indefinitely as flow increases without limit (example).
Link 1: Travel Time vs. Flow

Link 1 Flow (vehicles/hour) vs. Link 1 Travel Time (minutes) for Link 1.
Traffic Assignment Assumptions - 2

- In practice, the travel time function for each link is typically defined on its own flow, ignoring the flows of opposing or conflicting links; however, the link’s nominal capacity may reflect the effect of intersecting links. The link capacity is not a strict upper limit on flow.

- Drivers have perfect information about travel times (deterministic), or perfect information plus a perception error (limited stochastic case); models with truly stochastic travel times are much more difficult, and not considered here.
User-Equilibrium Principles

• Deterministic: For each origin-destination pair of zones, all used routes have equal travel times, and no unused route has a lower travel time. (Wardrop)
• Stochastic: For each origin-destination pair of zones, all used routes have equal perceived travel times, and no unused route has a lower perceived travel time.
The Basic Two Link Problem - A Graphical Analysis -

- Consider two one-way links with fixed inflow and outflow in vehicles per hour
- Determine the flows on each link by graphically equating the link times to solve for the user-equilibrium flows
- Then, we explore the relation of this graphical solution to a more general problem that can be formulated mathematically.
Two-Link Example

Link 1

Node A

$\bar{d}$ (vehicles/hour)

Node B

Link 2

Two-Link Example
Link Travel Times vs. Flows

User Equilibrium Flows = (1,522; 2,478)
Equilibrium Travel Time = 27.1

Equation: Link 2 Flow = 4000 - Link 1 Flow
Area under Travel Time Functions vs. Link Flows

Link Travel Times (minutes) vs. Link 1 Flow (vehicles/hour)

Area (1521, 2479) = 79,569
Area (1900, 2100) = 83,174

Link 2 Flow = 4000 - Link 1 Flow
Areas under Travel Time Functions vs. Link Flows

Link Travel Times (minutes)

Area under Travel Time Functions

Area of each solution

Area (1900, 2100) = 83,174

Link 1 Flow (vehicles/hour)

Link 2 Flow = 4000 - Link 1 Flow
Areas and Link Travel Times vs. Flows

Area under Functions/4000 = Minimum

User Equilibrium Flows = (1,522; 2,478)

Link 2 Flow = 4000 - Link 1 Flow
Measuring the Solution’s Effectiveness

• In summary, only two-link problems can be solved graphically or algebraically; all others must be solved by some iterative method.
• To monitor the improvement in the solution as it approaches the user equilibrium (optimum), we need a measure of the solution’s effectiveness.
• One measure is the value of the “objective function” being minimized, the sum of the area under the link travel time functions; however, this is not the best measure because it changes very little as the solution approaches the equilibrium point.
Measuring the Solution (con’d)

- A better measure is the Total Excess Cost (or “Gap”)
- For each OD pair, the OD Excess Cost is the difference between each route’s time and the time of the shortest route, weighted by the route’s flow, and summed over all routes.
- The Total Excess Cost is the OD Excess Cost summed over all OD pairs.
- The Average Excess Cost is the Total Excess Cost divided by the total regional flow, which is the sum of the flows in the trip table.
- We use the term cost, in the sense of generalized cost, to represent a weighted sum of travel time and other variables such as vehicle operating costs, tolls, etc.
• The above definition of Total Excess Cost suggests that the **cost and flow of every used route** is needed for its computation. In fact, the TEC can be found for any solution to the assignment problem by computing one all-or-nothing assignment using the link costs from that **given solution**.

• Using the all-or-nothing flows and the given link costs, compute the total cost and call it the “Minimum Cost”.

• **Total Excess Cost** = 
  
  Travel Cost of the solution less the Minimum Cost
Total Excess Cost vs. Link Flows

Total Excess Cost at (1900, 2100) = (44.3 - 23.6) x 1900 = 39,300 minutes

Link 2 Flow = 4000 - Link 1
Average Excess Costs and Link Travel Times vs. Flows

- Link Travel Times (minutes)
- Average Excess Time (minutes)
- Link 2 Flow = 4000 - Link 1 Flow
- UE Flows = (1,522; 2,478)
- Average Excess Cost

Graph showing the relationship between Link 1 Flow (vehicles/hour) and various travel times and costs. The graph includes a red box indicating the UE Flows, and a purple arrow labeled Average Excess Cost.
Average Excess Cost Areas and Link Travel Times vs. Flows

Area/4000 + Average Excess Cost = Minimum
Area/4000 = Minimum

UE Flows = (1,522; 2,478)

Link 2 Flow = 4000 - Link 1 Flow
Monitoring the Convergence

• A useful property of Total Excess Cost is that it tends to zero as the assignment solution improves.
• The process of the solution moving towards zero Total Excess Cost is called Convergence.
• Some practitioners use the term Closure, which may be misleading, since it implies that the true equilibrium solution has actually been reached. In fact, for any problem other than a highly simplified one, the true equilibrium solution is never actually reached.
• So, we need criteria for judging when the solution has adequately converged for a specific study.
Convergence Based on Excess Cost

• When to stop the solution procedure, and accept the current solution as having satisfactorily converged, depends on several factors:
  1. How precise an answer is desired? For a scenario analysis, one would like the observed differences to reflect the scenarios and not errors in the assignments.
  2. How much clock time and computing resources are available?
  3. At what point do the link flows become stable? That is, when do link flows stop changing?
• Assuming that sufficient time and computing resources are available, stability of the answer should be the main criterion, so that differences found among scenarios do reflect the differences in the scenarios. Clearly, stability is a matter of judgment of the analyst, and cannot be determined in advance of examining the solution of a new analysis situation.

• Once the Excess Cost required to achieve a stable solution is determined, each scenario should be solved to the same Excess Cost, rather than to a fixed number of iterations.
Delaware Valley Region Case Study

- Two ramps were proposed to be added to a major interchange in South Jersey.
- Build / No-build analyses showed changes in link flows throughout the Delaware Valley Region.
- These changes were thought to stem from the poor convergence of the assignment.
- Our studies showed stability of flows in the interchange do require better convergence than best current practice.
- Moreover, what about other links in the Region?
Link Flow Differences (build less no-build) vs. Average Excess Cost
New Jersey freeway links with positive flow differences

Average Excess Cost (minutes) (Iterations of User-Equilibrium Assignment)

- WB Ramp
- EB Ramp
- WB I-295 west of Ramps
- EB I-295 west of Ramps
- NB SR-42 south of Ramps
- SB SR-42 south of Ramps
Link Flow Differences (build less no-build) vs. Average Excess Cost
New Jersey freeway links with negative flow differences
Link Flow Differences (build less no-build) vs. Average Excess Cost
Delsea Drive crossing I-295 west of SR-42

Average Excess Cost (minutes) (Iterations of User-Equilibrium Assignment)

Link Flow Difference (vehicles per day)

- NB Delsea Dr. N of I-295
- SB Delsea Dr. at I-295
- SB Delsea Dr. N of I-295
- NB Delsea Dr. at I-295
Convergence of the No-Build Traffic Assignment

Average Excess Cost (minutes)

Convergence Indicators (min)

Average Travel Cost
Average Minimum Cost
Objective Function/T
Objective Function/T - AEC
Average Excess Cost (AEC)
Total trips/hour (T)

Average Excess Cost (minutes)
Maps of Link Flow Differences

- Where do the largest link flow differences occur in relation to the proposed ramps?
- How does convergence of the assignments affect those flow differences?
- How large are these differences, as compared with link capacities?
- How are the largest errors in link flow differences, as compared with a highly converged solution, related to link capacities?

(Maps prepared by Biljana Dekic, CATS, Chicago.)
Link Flow Difference (vehicles/day)
Average Excess Cost - 0.2 minutes
Link Flow Difference (vehicles/day)
Average Excess Cost - 0.002 minutes
An Average Excess Cost (AEC) of 2 minutes corresponds roughly to 5-10 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.2 minutes corresponds roughly to 20-40 iterations of User-Equilibrium Assignment.
(Link Flow Difference/Link Capacity) * 100
Average Excess Cost - 2. minutes

- >+20%
- +10% to +20%
- +5% to +10%
- -5% to -10%
- -10% to -20%
- < -20%

1.0 0 0.1 Miles
(Link Flow Difference/Link Capacity) \times 100
Average Excess Cost - 0.2 minutes
(Link Flow Difference/Link Capacity) * 100
Average Excess Cost - 0.02 minutes
Average Excess Cost - 0.002 minutes

(Link Flow Difference/Link Capacity) * 100

Legend:
- >+20%
- +10% to +20%
- +5% to +10%
- -5% to -10%
- -10% to -20%
- <-20%

Scale: 1.0 0 0.1 Miles
An Average Excess Cost (AEC) of 2. minutes corresponds roughly to 5-10 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.2 minutes corresponds roughly to 20-40 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.02 minutes corresponds roughly to 80-120 iterations of User-Equilibrium Assignment.
Conclusions: Deterministic Assignment

- Deterministic user-equilibrium route choice models provide a simple, but effective traffic assignment method, if applied with care and understanding.
- Measures based on Excess Cost can be used intuitively to monitor convergence in comparing scenarios.
- Stability of link flows is important for making comparisons among scenarios and for improving the credibility of the results.
Integrated Models of OD, Mode and Route Choice

• Computational experiments were performed for various integrated models of variable demand and auto route choice for a single, aggregated class of travelers plus trucks.

• The following slides illustrate the convergence properties achieved with an integrated approach, as contrasted with the four-step procedure, for a model implemented for the zone system and road network of the Chicago Region: 1790 zones; 12,982 nodes; 39,018 links.
Guide to Comparing the Solutions

• Link flows at a given level of convergence are compared to a highly converged solution in two ways:
  - link flow at a given Average Excess Cost less the converged link flow.
  - ratio of link flow at a given Average Excess Cost less the converged link flow.

• Both the flow differences and the flow ratios are plotted against link capacity.

• A similar comparison is offered for origin-destination flows aggregated to districts.
Solutions in 2001 with a Compaq Alpha Unix Server DS20E with CPU speed of 666 MHz and 256 Mb RAM using the integrated solution method described by Bar-Gera and Boyce, Transportation Research, Part B (2003).
An Average Excess Cost (AEC) of 1.3 minutes corresponds roughly to 5-10 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.2 minutes corresponds roughly to 20-40 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.02 minutes corresponds roughly to 80-120 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.002 minutes corresponds to more than 500 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 1.3 minutes corresponds roughly to 5-10 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.2 minutes corresponds roughly to 20-40 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.02 minutes corresponds roughly to 80-120 iterations of User-Equilibrium Assignment.
An Average Excess Cost (AEC) of 0.002 minutes corresponds to more than 500 iterations of User-Equilibrium Assignment.
Average Misplaced Flow

- As with Average Excess Cost for link flows, a measure is needed for monitoring the convergence of a trip table with respect to its equilibrium solution.
- A definition of equilibrium for a given trip table is:
  - perform a user-equilibrium assignment of the given trip table, find the OD costs, and use them with the trip distribution model to determine a new trip table;
  - compare the given trip table and the new one; if they are equal, the given trip table is in equilibrium.
- A measure of equality is Average Misplaced Flow: sum of the absolute differences of the cells of the two tables divided by total flow.
CATS Zone System
Average Misplaced Flow (AMF) is a convergence measure for a trip table, similar to Average Excess Cost.
Average Misplaced Flow (AMF) is a convergence measure for a trip table, similar to Average Excess Cost.
Conclusions

• Examination of integrated models of origin-destination, mode and route choice suggests that substantial errors in road link flows may occur at convergence levels found in current practice.

• As in fixed demand models, the convergence of user equilibrium link flows with variable demand may be problematic for scenario analyses involving small changes in the road or transit networks.

• These issues may increase in importance for solution procedures involving feedback mechanisms that do not achieve an adequate approximation of the equilibrium solution.