General solution approaches for multi-attribute vehicle routing and scheduling problems

Extended abstract of the Ph.D. thesis of:

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1 Introduction

The Vehicle Routing Problem (CVRP) involves designing least cost delivery routes to service a geographically-dispersed customer set while respecting vehicle-capacity constraints. This NP-hard combinatorial optimization problem is linked with multiple applications in logistics, telecommunications, robotics, manufacturing and assembly-line design, crisis management in military and humanitarian frameworks, among others. Practical routing applications are usually quite distant from the scholarly cases, encompassing additional sets of specific constraints, objectives and decisions which breed further new problem variants. The resulting "Multi-Attribute" Vehicle Routing Problems (MAVRP) are the support of a vast literature, focused for the most part on introducing new problem-specific strategies. However, despite this growing number of contributions, the literature still critically lacks unified methods able to address or to be extended to several VRP. This deficiency limits our ability to apply current state-of-the art optimization methods to industrial cases, and our overall understanding of successful resolution concepts for a broad range of problems. In addition, some *rich* VRPs, combining several attributes, may be particularly difficult to address because of the wide array of combined, and possibly antagonistic, decisions they involve.

This thesis contributes to address these challenges related to the *variety* and *combination* of VRP attributes through three related axes of research on 1) problem-structure analysis, 2) metaheuristic strategies, and 3) general-purpose resolution of MAVRPs. In the thesis, each axis of research is described in a dedicated part, containing each between two and four chapters submitted or published as international journal articles.

2 Analysis of VRP attributes and resolution methods

The first chapter of the thesis introduces a unifying survey of multi-attribute vehicle routing problems and related solution methods. Attributes are classified relatively to their impact on solution methods, discerning three categories: Assign attributes, which impact the assignment of customers and routes to resources such as depots, days or vehicle types; Seq attributes, which impact the nature and structure of the routes, and Eval attributes, which impact route feasibility and cost evaluations, and which potentially bring forth additional decisions and sub-problems, e.g., packing or scheduling considerations in presence of fixed routes. Major resolution concepts are scrutinized. To that extent we selected fifteen notable MAVRPs, which have been the object of a consistent body of well-acknowledged research, and selected for each problem the three to five top metaheuristics for a detailed transversal analysis. This review points to the fact that the most successful heuristics do not rely on a single "winning" idea but rather result from a subtle balance and complementarity of concepts, different search spaces, scales of neighborhood search, short and long-term memories to manage problem knowledge, among others. In fact, no metaheuristic strategy stands out as the one best strategy for a majority of problems. Neighborhood-based searches such as Tabu Search or Simulated Annealing have been more widely considered in earlier literature (Gendreau et al. 1994, Cordeau et al. 1997), whereas the success of population-based searches is more recent (Prins 2004, Reimann et al. 2004) and has opened the way to many promising research leads.

It should be noted that many Eval attributes, impacting route cost and feasibility evaluations, are related to constraints and objectives on service dates. An efficient optimization of the delivery dates, or assessment of schedule feasibility, is thus critical to resolve the related MAVRP. These sub-problems of delivery date optimization, which we call *timing* problems, are also recurrently found in other literature domains, transportation problems, shortest path, resource allocation, and even isotonic regression in statistics. Yet, few relationships between domains have been actually exploited, and thus closely-related solution methods have been independently developed within different formalisms, being rarely put in a more general context. Thus, the thesis contributes to the *timing field* in a dedicated chapter, by means of a multidisciplinary review and analysis of timing features, problems and algorithmic approaches. A large assortment of timing problems are identified and classified in relation to their structure, and successful solution methods from different research fields are analyzed. This unifying review provides the keys for a stand-alone resolution of a large variety of timing problems, and also analyzes efficient timing solving as a subroutine in the context of global searches such as neighborhood-based heuristics and branch-and-bound-based approaches. For these latter cases, managing global information when successively solving related timing instances can lead to dramatic reductions of the overall solving complexity. The body of knowledge developed in this paper is critical for both modeling work and algorithmic design since in many cases, the difficulty of rich vehicle routing and scheduling problems is inherent to route evaluations, and thus to timing sub-problem resolution.

3 Metaheuristic developments

The second research line aims at offering novel efficient metaheuristic strategies for vehicle routing problems. To that extent, we propose a novel Hybrid Genetic Search with Advanced Diversity Control (HGSADC) for the capacitated VRP, the multi-depot VRP, the periodic VRP, and the multi-depot periodic VRP. The metaheuristic combines the exploration breadth of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based metaheuristics, and advanced population-diversity management schemes. HGSADC revisits the traditional *survival-of-the-fittest* paradigm, considering a bi-criteria evaluation of individuals, based on solution cost and diversity (distance-to-the-others) measures. Thus, diversity appears as part of the objective, as opposed to classical diversity-management procedures that have traditionally imposed diversity constraints for the acceptance of solutions in the population. Extensive computational experiments demonstrate the remarkable method performance, both in terms of computational efficiency and solution quality. It identifies either the best known solutions, including the known optimal ones, or presents best new solutions for all current classic benchmark instances. In addition, the new diversity-management mechanisms not only avoid premature population convergence efficiently, but also lead to higher quality solutions in reduced computation time when compared to traditional approaches.

The proposed HGSADC is also extended to address VRP with time windows (VRPTW) and other related variants to investigate the performance of the proposed concepts on a wider variety of problems. New contributions are introduced, to exploit penalized infeasible solutions w.r.t. time-window constraints during the search, and efficiently evaluate neighbor solutions. The time-window relaxation framework of Nagata et al. (2010) is used and extended, allowing penalized "returns in time" in case of late arrivals. Within this relaxation framework, we introduce a simple and efficient method to evaluate in amortized constant time $O(1)$ the penalized infeasible solutions generated by classic local-search moves issued from a bounded number of arc exchanges or node relocations. State-of-the-art results are obtained on vehicle routing variants with compound time windows, multiple periods, multiple depots, and/or vehicle-customer compatibility constraints. In addition, since the contribution of infeasible solutions in heuristic searches for vehicle routing problems is not a subject of consensus in the metaheuristics community, we also conduct an extensive analysis of the impact of time-window relaxations within local-search heuristics for vehicle routing. The results, generated for two different objectives, distance minimization and fleet-size minimization, validate the significant contribution of penalized intermediate infeasible solutions. We compare different relaxation frameworks, penalized late arrivals, early arrivals, returns in time and speed-ups for travel and services, and highlight significant performance differences. Experimental results demonstrate that the "returns in time" leads to the best results from both a quality and CPU time standpoint.

Finally, to further integrate different optimization decisions within heuristic and neighborhood search, we propose a new efficient method based on dynamic programming to evaluate in amortized $O(1)$ the optimal depot and vehicle type choice, as well as the choice of first customer to be visited in the route, called *optimal rotation* during route evaluations. This leads to richer neighborhood structures in local-search procedures and enhanced capabilities of the Split algorithm. Furthermore, the assignment choices, which weigh heavily in the problem difficulty, are no longer explicitly addressed in the solution structure, but implicitly determined during each move evaluation. Two metaheuristics based on these concepts, an Iterated Local Search (ILS) and a HGSADC, are presented. Extensive computational experiments demonstrate the remarkable performance of these methods on classic benchmark instances for multi-depot vehicle routing problems with and without fleet mix, as well as the significant contribution of the implicit depot choice and positioning methods to the search performance.

4 Rich VRPs and general-purpose resolution

The last line of research builds upon the previously-mentioned contributions to solve rich vehicle routing problems and advance towards effective general-purpose MAVRP solution methods . We first address a difficult *rich* Vehicle Routing and Truck Driver Scheduling Problem (VRTDSP) considering explicitly truck drivers' working hours and statutory pauses, in the presence of regulations from the European Union, United States, Canada, and Australia. Even for a fixed sequence of visits (route), the break-scheduling problem is very intricate and its polynomiality is still today an open question for most regulations. This problem is here solved by means of tree-search techniques, directly integrated at the level of route evaluations in HGSADC. To attain acceptable run times and enhance the solution quality, we introduce various speedup techniques, neighborhood and label pruning procedures, memory structures, and time-constraint relaxations. The resulting method produces new state-of-the-art solutions on classical benchmark instances for the VRTDSP. Even more importantly, it stands out from a "one-problem = one-method" view, in that it allows to deal with several regulations in a single algorithm. We can thus analyze the schedules generated by the method, in presence of different regulations, and assess their cost and relative risk using the fatigue index of Health and Safety Executive (2006). Our experiments demonstrate that E.U. rules lead to the highest safety level, while Canadian regulations are the most competitive in terms of economic efficiency. Australian regulations appear to have unnecessarily high risk rates with respect to operating costs. Policy makers, unions, and transport companies may thus use this approach to assess the impact of regulations to find the best trade-off between road safety, working conditions of truck drivers, as well as speed and costs of transportation.

This thesis also contributes to address the huge variety of multi-attribute VRP by means of a new component-based heuristic resolution framework and a Unified Hybrid Genetic Search (UHGS) for vehicle routing problems. Any unified method must ultimately account for the specific attributes, objectives, and constraints of the particular problem setting at hand. Yet, to achieve a high level of generality, these problem attributes are confined to restricted adaptive components. Thus, UHGS relies on unified problem-independent procedures: unified local search, crossover, Split algorithm and diversity management. Problem-specific strategies are restricted to a few modular components which take charge of assignment changes (of customers to depots, to days, or to any ASSIGN resources), enumerations of sequencing alternatives, and route evaluations. These components are self-adapted in relation to the attributes of the problem at hand. Extensive computational experiments demonstrate the groundbreaking performance of the method which, with a single implementation, parameter setting and termination criterion, matches or outperforms all current problem-tailored methods, issued from more than 180 articles, on 26 vehicle routing variants, 39 benchmark sets and a total of 1008 problem instances. In addition, all known optimal solutions from the literature have been retrieved. Hence, it appears that generality does not necessarily impede efficiency for the considered problem classes.

Finally, we contribute to progress further towards solving *rich* MAVRP with multiple compound attributes, by introducing a new parallel cooperative solution framework called "Integrative Cooperative Search (ICS)". The rationale of ICS methodology is to combine decomposition principles, cooperative guided search and opportunistic resolution of simpler sub-problems with fewer attributes by known state-of-the-art algorithms such as UHGS. The resulting sub-problem solutions are integrated by tailored procedures to yield complete solutions of the initial problem. All these methods cooperate through an adaptive search-guidance mechanism using the central-memory cooperative search paradigm. These new decomposition and cooperation principles enable to reach even better solutions for some compound multi-attribute problems, such as the multi-depot periodic VRP (MDPVRP). Several research avenues are also raised, investigating how to reconstruct complete solutions from partial sub-problem solutions, and how to efficiently exploit memories and knowledge on past search to guide the method farther still. Various experimental analyses have been developed to compare integration and guidance strategies.

5 Conclusions

A rich variety of contributions have been introduced in this thesis to progress towards a better resolution of MAVRP. In particular, the structure of MAVRPs and related heuristics was analyzed. A broad family of timing problems was unified. The traditional survival-of-the-fittest paradigm in evolutionary search was revisited to solve more efficiently vehicle routing variants. New efficient neighborhood search procedures in presence of time relaxations have been introduced as well as new large neighborhoods. The impact of hours of service regulations around the world has been assessed by means of our optimization methods. Finally, a new component-based heuristic resolution framework and a Unified Hybrid Genetic Search (UHGS), with efficient adaptive route evaluation components, is proposed and integrated into a parallel cooperative search. UHGS redefines the state-of -the-art of metaheuristics for vehicle routing. It outperforms all previous problem-dedicated methods from more than 180 articles on 26 different problem settings. These experiments demonstrate that a higher generality does not necessarily impede performance for the considered classes of problems. Thus, we believe that many previous problem-specific heuristic contributions should be investigated again on a broader range of MAVRP to assess their real scope of application.

Also, from a practical application perspective, UHGS stands out as the first general-purpose solver to be state-of-the-art in the academic literature, validated on numerous problem benchmarks, and quickly applicable to new industrial settings. The open research avenues are numerous. In addition to many specific questions related to timing problems and relaxation schemes, and from a broad metaheuristic perspective, we believe that the next generation of VRP metaheuristics should be improved by introducing new compound neighborhoods for rich problems, by exploiting more intelligently the information on the search history and the particular VRP search landscapes characteristics, and by exploiting simple and efficient method hybridizations. Our library of unified methods, including a unified ILS, VNS, and UHGS, may prove an important tool to further progress on these research lines since it allows quickly inserting and testing new methodological strategies on a rich array of MAVRPs.

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