Metaheuristics for vehicle routing: general-purpose resolution, new challenges and winning strategies

Thibaut Vidal

université de technologie Troyes

Université de Montréal

CIRRELT
Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport

CONSEIL REGIONAL CHAMPAGNE ARDENNE

Chaire de recherche industrielle du **CRSNG** en management logistique
NSERC Industrial Research Chair in Logistics Management

Seminar, Public University of Navarra,
17th December 2012, Pamplona

Research context

- Joint works with

  Christian PRINS, *Université de Technologie de Troyes, France*

  Teodor Gabriel CRAINIC, *Université de Québec à Montréal, Canada*

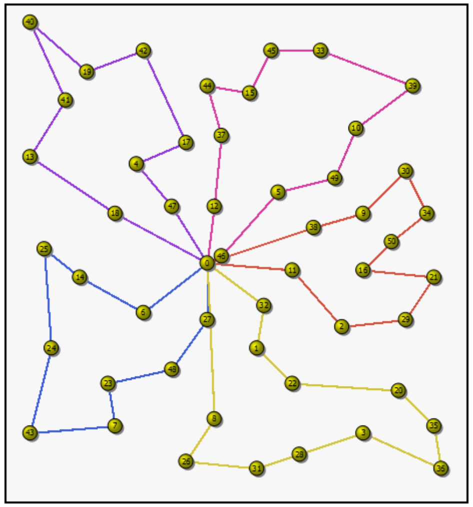
  and Michel GENDREAU, *Ecole Polytechnique, Montréal, Canada*

Presentation outline

- I) Vehicle Routing Problem, and attributes.
- II) Classic Heuristics and metaheuristics for vehicle routing
- III) An analysis of winning strategies
- IV) A new general-purpose solution approach
  - Attribute-based modular design
  - Unified Local Search
  - Unified Hybrid Genetic Search
  - Computational Experiments

Vehicle Routing Problem, and attributes

- **Capacitated vehicle routing problem (CVRP) :**
  - Designing a set of least cost delivery routes to service a geographically-dispersed set of n customers
  - For a set of identical capacitated vehicles
  - Respecting vehicle-capacity constraints



- NP-difficult problem.

- Exact methods for the CVRP can not consistently solve problem instances with more than 100-200 customers, thus emphasizing the research on heuristics and metaheuristics.

Vehicle Routing Problem, and attributes

- **Capacitated vehicle routing problem (CVRP) :**
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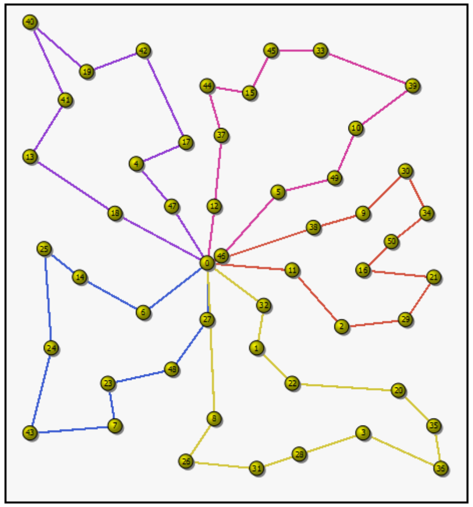
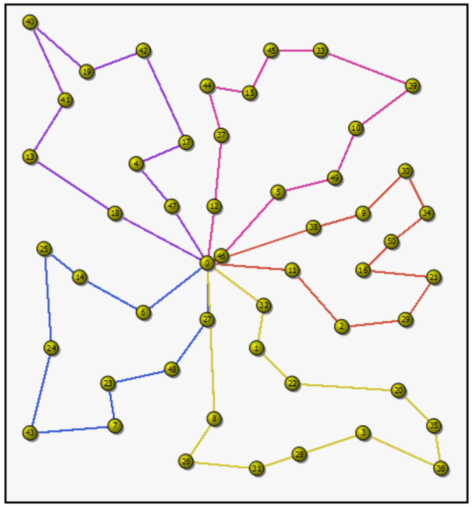


- “Scopus” : 2007-2011, 1258 articles with the key “vehicle routing”.
- Numerous applications, including transportation logistics, communications, manufacturing, military, relief systems…

Vehicle Routing Problem, and attributes

- Wide literature on CVRP…

- **…but, considerable challenges** related to the resolution of **VRP variants with additional *attributes*** (multi-attribute VRPs, MAVRPs)
  - modeling the specificities of application cases, customers requirements, network and vehicle specificities, operators abilities…
  - Combining **several attributes** together can lead to highly complex **rich VRPs.**
  - Dramatic increase in the literature dedicated to specific VRP variants.



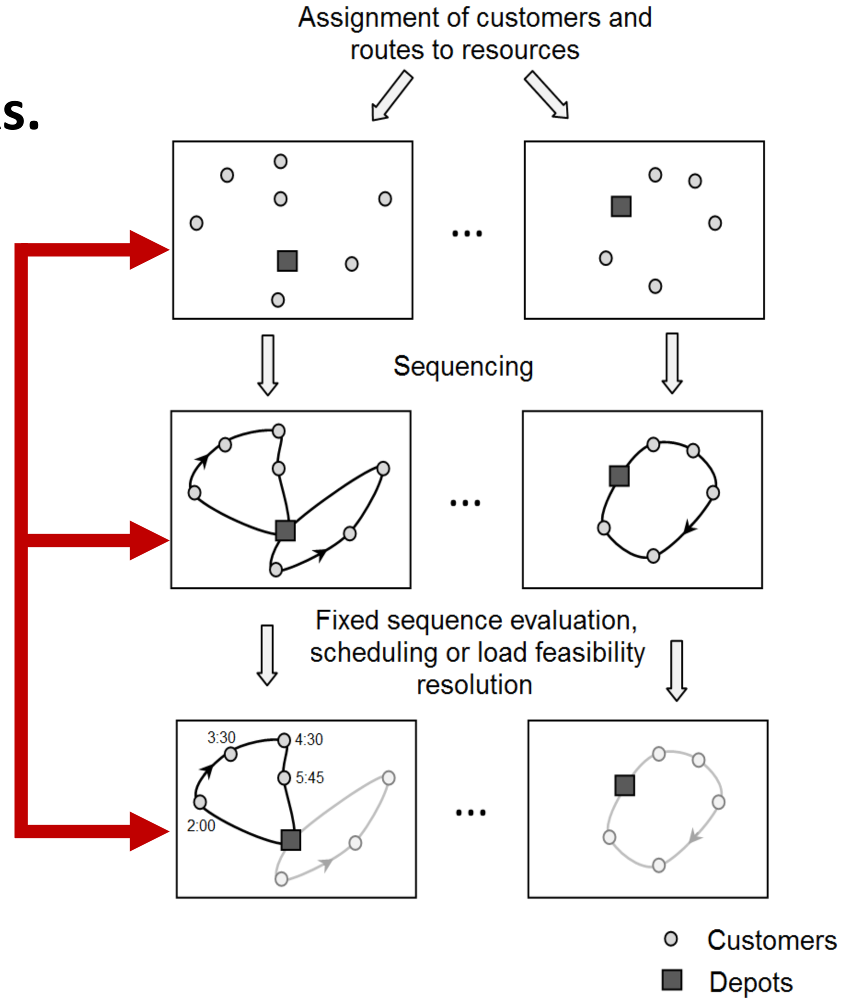
Vehicle Routing Problem, and attributes.

- **Generally, attributes can be classified into three types, relatively to the problem structure and the main resolution tasks.**

- **ASSIGN ATTRIBUTES:** impacting the assignment of customers and routes

- **SEQ ATTRIBUTES:** impacting the nature of the network and the sequences

- **EVAL ATTRIBUTES:** impacting the evaluation of fixed routes



Vehicle Routing Problem, and attributes

- **Some recurrent ASSIGN attributes :**
  - Multiple depots
  - Heterogeneous fleet
  - Multiple periods
  - Split deliveries
  - Prize Collection
  - Location Routing
  - Site-dependency
  - Inventory Routing
  - *Consistent service*

Vehicle Routing Problem, and attributes

- **Some recurrent SEQ attributes :**
  - Bakhauls
  - 1-to-1 pickup and deliveries
  - Multiple trips
  - Multi-Echelon
  - *Truck & Trailer*
  - *Generalized*
  - Other graph specifics : tree, shoreline…

Vehicle Routing Problem, and attributes

- **Some recurrent EVAL attributes :**
  - Open
  - Time windows
  - Time-dependent travel time and service costs
  - *Hours-of-service regulations*
  - 2D-3D loading
  - Soft and Multiple time windows
  - Duration constraints
  - Other time features
  - Cumulative costs
  - Simultaneous pickup & deliveries
  - *Pollution routing*
  - *Synchronization …*

Multi-attribute vehicle routing problems (MAVRPs)

- **A need for more flexible and general purpose solvers**
  - Solvers that can address a wide range of problems without need for extensive adaptation or user expertise.
  - **Necessary tools** for the timely application of current optimization methods to industrial settings.
  - Few such methods in the literature

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Classic Heuristics and Metaheuristics

- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

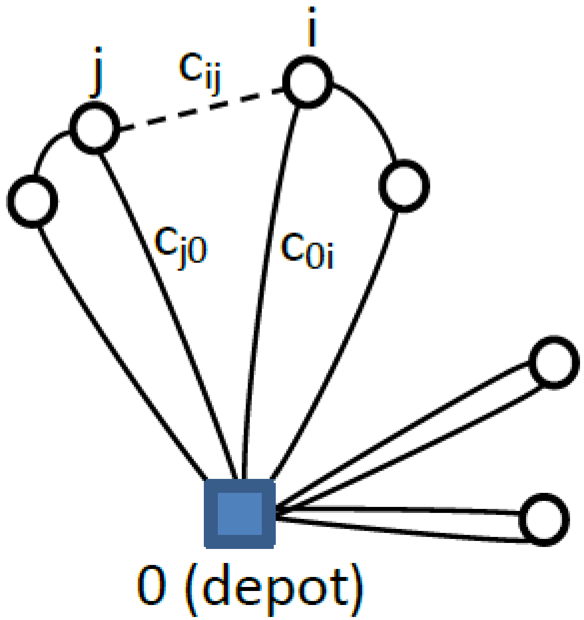
- Savings method (Clarke and Wright 1964)
  - Merge routes step by step based on a *savings* measure s_{ij}

$$s_{ij} = c_{i0} + c_{0j} - c_{ij}$$

  - Some refinements by Gaskell (1967) and Yellow (1970) :

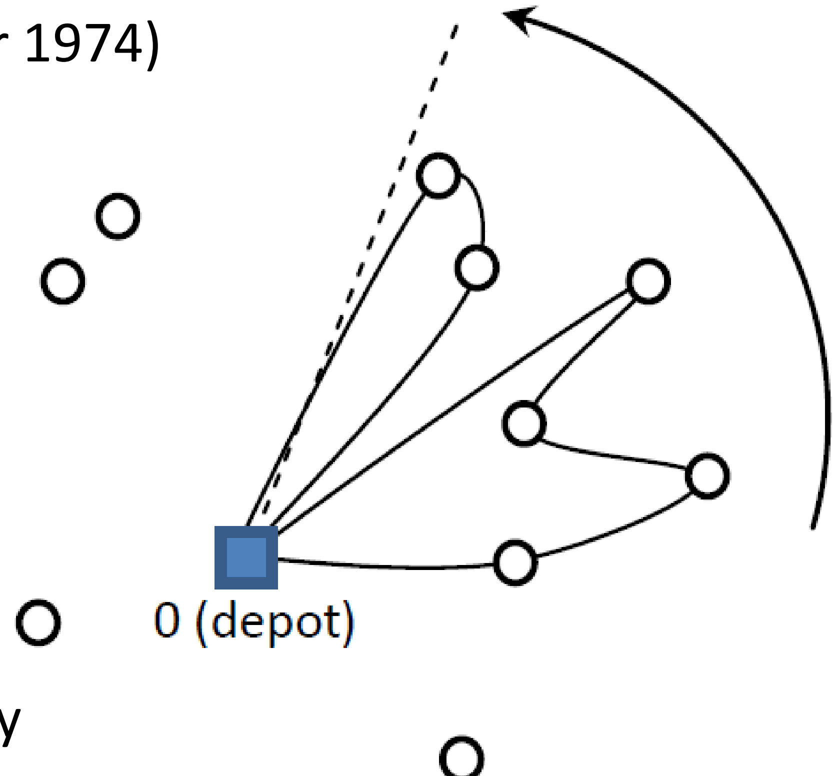
$$s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$$

  - Mole and Jameson (1976) and Solomon (1987) generalize the concepts and also consider insertions inside the routes.



- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

- Sweep algorithm (Gillett and Miller 1974)
  - Sweep the deliveries in circular order to create routes, a new route is initiated each time the capacity is exceeded.



- "Petal" methods : generate several alternative routes, called petals, and select a subset by solving a set-covering linear program.

Classic Heuristics and Metaheuristics

- **Constructive methods** : mostly between 1960s and 1980s.
  - Making step-by-step definitive decisions, which cannot be reversed afterwards

- Route first cluster second (Newton and Thomas 1974, Bodin and Berman 1979, Beasley 1979)
  - construct a giant circuit (TSP tour) that visits all customers.
  - Segmenting this tour into several routes. Optimal segmentation = assimilated to a shortest path problem in an auxiliary directed acyclic graph.

- Cluster-first Route-second (Fisher and Jaikumar 1981). First solve a generalized assignment problem (GAP) around *m* high-density locations to create clusters. Solve a TSP for each cluster.

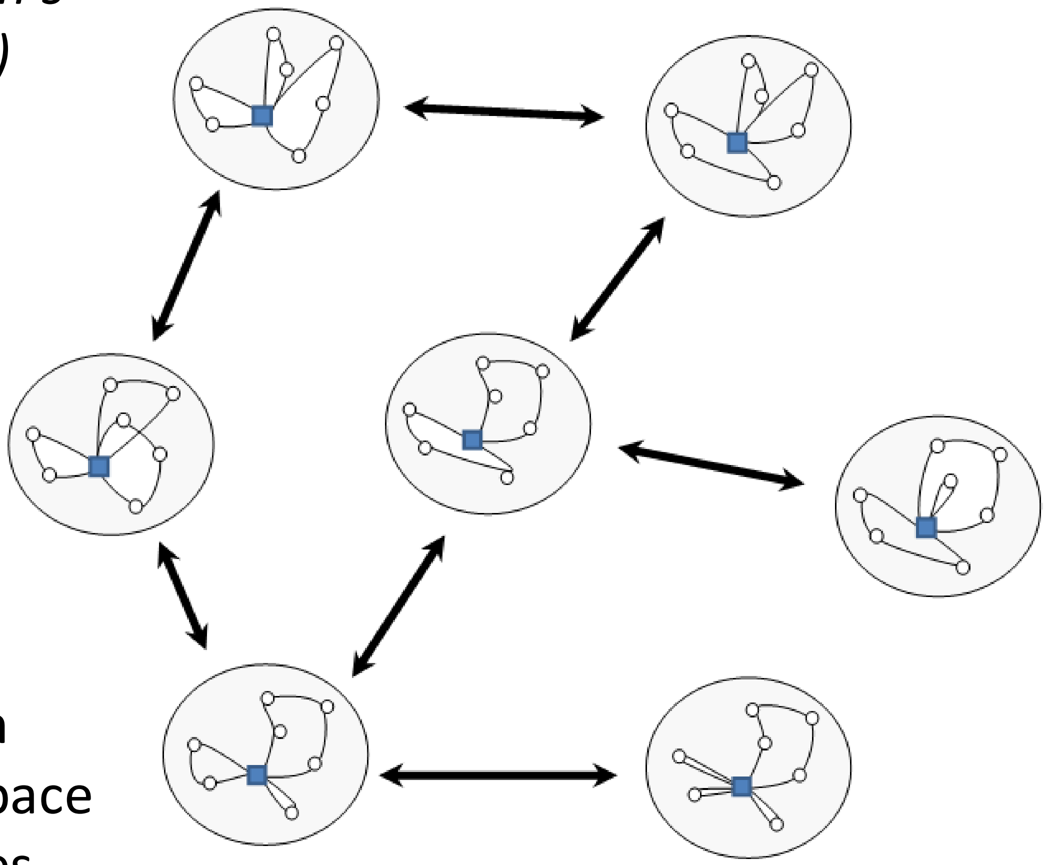
Classic Heuristics and Metaheuristics

- **Local-improvement procedures** :

  - From an *incumbent solution s* define a *neighborhood N(s)* of solutions obtained by applying some changes

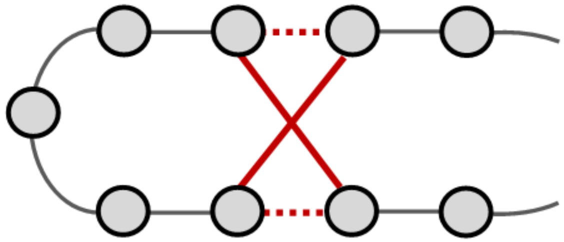
  - The set of solutions, linked by neighborhood relationships = *search space.*

  - LS-improvement method progress from one solution to another in this search space as long at the cost improves.

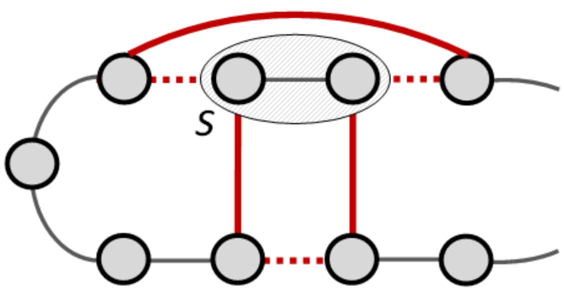


Classic Heuristics and Metaheuristics

- For optimizing a single route (TSP tour);
  - in the terminology of Lin (1965), *λ-opt* neighborhood = subset of moves obtained by deleting and reinserting λ arcs.
  - 2-opt and 3-opt are commonly used,
  - Or-opt which comes to relocate sequences of bounded size, and is a subset of 3-opt.



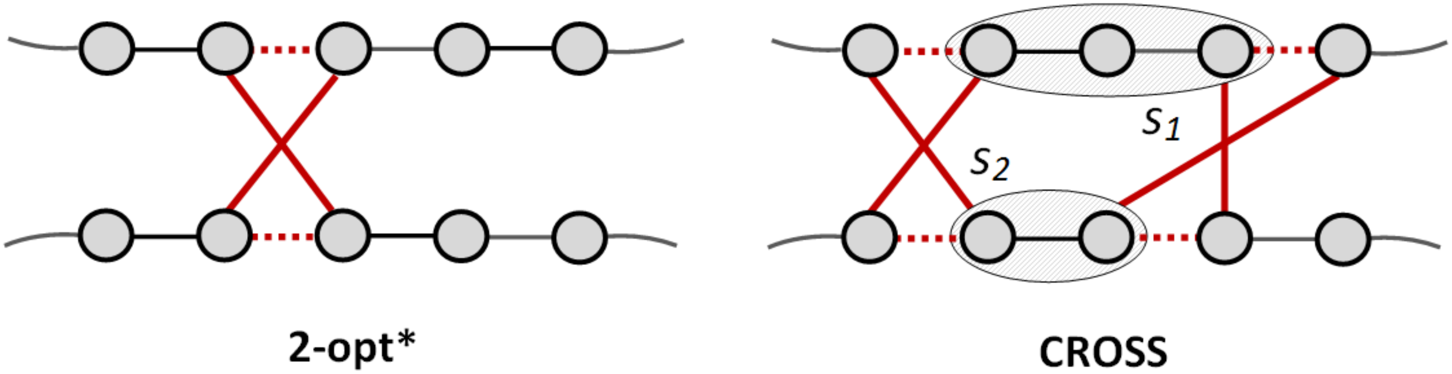
2-opt



Or-exchange

Classic Heuristics and Metaheuristics

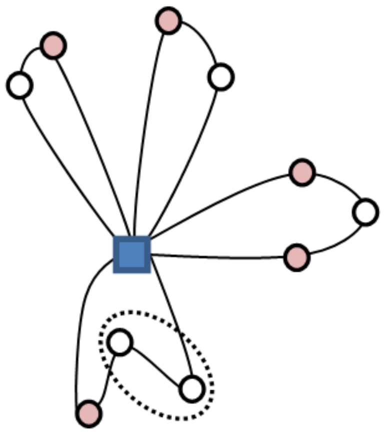
- For optimizing multiple routes together,
  - Insert neighborhood (relocate a delivery)
  - Swap neighborhoods (swap two deliveries from different routes)
  - CROSS-exchange (exchange two sequences of visits)
  - I-CROSS (exchange and reverse two sequences)
  - 2-opt* exchange two route tails (special case of CROSS)



Classic Heuristics and Metaheuristics

- These neighborhoods contain a polynomial number of moves.
  - For all moves except CROSS and I-CROSS, the number of neighbors is $O(n^2)$.
  - CROSS and I-CROSS are often limited of sequences of bounded size with less than k customers, in that case the number of neighbors is $O(k^2n^2)$.

- Other non-enumerative large-scale neighborhoods:
  - Lin and Kernighan 1973 for the TSP
  - Ruin-and-recreate (Schrimpf 2000, Shaw 1998)
  - Ejection chains (Glover 1992,1996)
  - Moving customers simultaneously between fixed points using IP (Sarvanov and Doroshko 1981, Toth 2008).



Classic Heuristics and Metaheuristics

- For enumerative neighborhoods, efficient move evaluations and pruning procedures are critical to address large-scale problem instances.

  - *Granular search* (Johnson and McGeoch 1997, Toth and Vigo 2003) : restrain the subset of moves to spatially related customers. (or related w.r.t. time constraints)

  - *Sequential search* (Christofides and Eilon 1972 for the TSP, Irnich et al 2006 for the CVRP) : any profitable move can be broken down into a list of arc exchanges $(a_1, \ldots, a_\lambda)$ with gains $(g_1, \ldots, g_\lambda)$ such that for any k, $g_1+\ldots+g_k \geq 0$. This condition enables to dynamically prune many non-promising moves.

Classic Heuristics and Metaheuristics

- Briefly discussing some classic, flexible and efficient metaheuristics.
- Discern between
  - **Neighborhood-centered** search, concerned with the iterative improvement of one single solution
    - Tabu, Simulated Annealing, Iterated LS, VNS…
  - **Population-based** search, managing and improving a population of solutions.
    - Genetic or Evolutionary Algorithm, ACO, Scatter Search, PR…
- Most successful early approaches (before 2000) were neighborhood-centered.

1. Adaptive memory programming -- Rochat and Taillard (1995)

Diversification

- Short term Tabu memories
- Intelligent randomization for diversification, driven by measures of attractiveness.

- Detection of good components that consistently appear in elite solutions → and creating new solutions from them to generate new search starting points
- Decomposition phases based on spatial proximity

Intensification

2. UTS – Cordeau et al. (1997)

- Tabu search centered on the choice of the best neighbor.
- Single family of moves (GENI insertions in the case of CVRP, simple insert for TW-constrained problems).
- Penalized infeasible solutions w.r.t. route constraints.
- Short term memory based on solution features to avoid cycling.
- Continuous diversification strategy : penalizing recurrent attributes in the solutions.

3. ALNS – Pisinger and Ropke (2007)

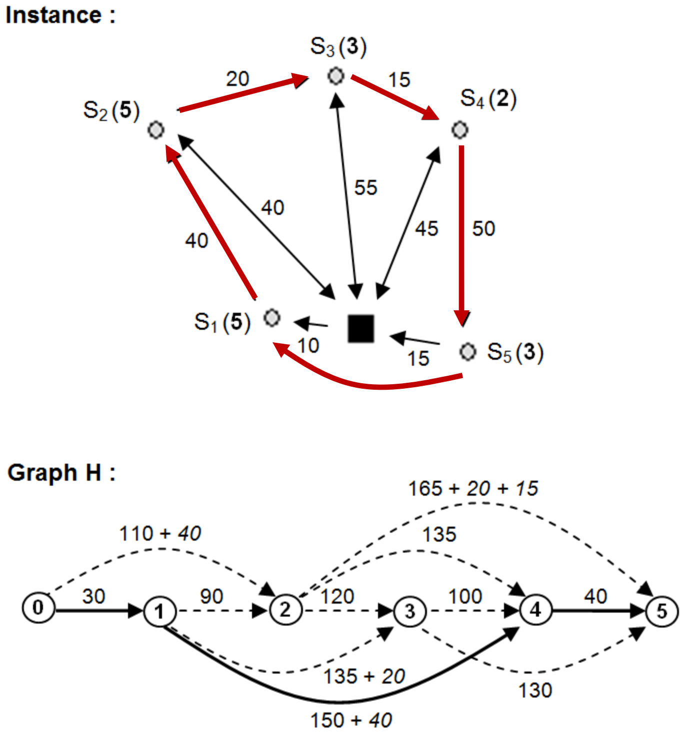
- Exploration of large neighborhoods based on the **Ruin-and-recreate** principle.
- Multiple operators (variety) for destroying the solution
  - Using randomness, quality measures, relatedness, or history
- SA criterion for acceptance of new solutions
- Adaptive probabilities for selecting the operators, driven by their success

4. ILS-RVND-SP – Subramanian et al. (2012)

- Rather simple iterated local search + Randomized Variable Neighborhood Descent (RVND)
- Rich neighborhoods :
  - Relocate1, Relocate2, Relocate3, Swap 1v1, Swap 1v2, Swap 2v2, 2Opt*, K-shift, Shift-Depot, Swap-Depot, 2Opt, Empty-Route…
- Multiple shaking operators
  - Multi-swap, Multi-shift, Double-bridge, Split
- Solving a set partitioning model on a pool of elite route, adaptation of the pool size and content relatively to the success of the IP. (another large neighborhood)

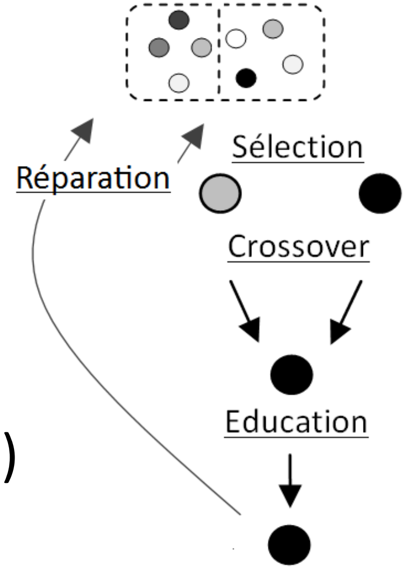
5. HGA – Prins (2004)

- First population-based method to achieve competitive results on VRP variants.
- Giant-tour solution representation
- Polynomial Split algorithm to obtain a complete solution
- Simple genetic operators : selection, crossover
- LS-improvement of the offspring
- Population management (spacing constraints)



6. HGSADC – Vidal et al. (2012)

- Giant-tour solution representation
  - Building on Prins (2004)
- Efficient granular local search
- Relaxations + two-populations management + "Repair operations"
- Generalized operators (in the unified version UHGS)
- **Promotion of diversity – biased fitness**



$$BF(I) = fit(I) + \left(1 - \frac{nbElit}{nbIndiv - 1}\right) \times dc(I)$$

Classic Heuristics and Metaheuristics

- **Range of problems addressed with some classic methods:**

- One drawback when dealing with a rich VRP model that includes several MAVRP as special cases

  → Still accounting for non-activated attributes

Type	Attribute	Acronym	UTS	ALNS	HYB	UHGS
	Multiple depots	MDVRP	X	X	X	X
	Multiple periods	PVRP	X			X
	Heterogeneous fleet	HVRP			X	X
	Site-dependent	SDVRP	X	X		X
	Pickup & deliveries	VRPPD	X	X		
	Backhauls	VRPB		X		X
	Open	OVRP		X	X	X
	Cumulative	CCVRP				X
	Load-dependent costs	LDVRP				X
	Simultaneous P.&D.	VRPSDP		X	X	X
	Vehicle Fleet Mix	VFMP			X	X
	Duration constraints	DurVRP	X			X
	Hard TW	VRPTW	X	X		X
	Soft TW	VRPSTW				X
	Multiple TW	VRPMTW				X
	General TW	VRPGTW				X
	Time-dep. travel time	TDVRP				X
	Lunch breaks	VRPLB				X
	Work hours reg	VRTDSP				X
	Service site choice	GVRP[2]				X

[2] Problem known as "Generalized Vehicle Routing Problem"

Classic Heuristics and Metaheuristics

- Quick glimpse on some other approaches, CVRP results on Golden et al (1998) instances – many types of successful methods

Acronym	Reference	Approach	Runs	Gap	T(min)	CPU	$T^{\#}$(min)
VCGLR11s	Vidal et al. (2012) slow	Hybrid GA	Avg 10	0.161%	113	Opt 2.4G	92.7
VCGLR11f	Vidal et al. (2012) fast	Hybrid GA	Avg 10	0.267%	34.8	Opt 2.4G	28.5
NB09	Nagata and Bräysy (2009)	Hybrid GA	Avg 10	0.273%	35.6	Opt 2.4G	29.2
GGW11	Groër et al. (2011)	Para. R-to-R	Best 5	0.296%	5.00	8×Xe 2.3G	129
MB07s	Mester and Bräysy (2007) slow	EA+ELS	Single	0.327%	24.4	P IV 2.8G	22.4
ZK10	Zachariadis and K. (2010a)	GLS+Tabu	Avg 10	0.430%	40.5	T5500 1.6G	26.7
JCL11	Jin et al. (2011)	Guided Tabu	Avg 10	0.448%	47.1	5×Xe 2.66G	180
MM11	Marinakis and Marinaki (2011)	Bees mating	Best 50	0.560%	3.96	P-M 1.86G	117
JCL12	Jin et al. (2012)	Coop Tabu	Avg 10	0.600%	41.9	8×Xe 3.0G	330
P09	Prins (2009a)	GRASP+ELS	Single	0.630%	7.27	P-IV 2.8G	6.09
RDH04	Reimann et al. (2004)	ACO	Avg 10	0.930%	49.3	P-III 900M	7.05
T05	Tarantilis (2005)	Ad.M.+Tabu	Single	0.931%	45.5	P-II 400M	2.02
CM11	Cordeau and M. (2012)	Iter. Tabu	Avg 10	0.939%	31.3	Xe 2.93G	30.8
MM10	Marinakis and Marinaki (2010)	GA+PSO	Avg 50	0.987%	4.20	P-M 1.86G	2.48
DK07	Derigs and Kaiser (2007)	ABHC	Single	1.017%	113	Cel 2.4G	106
GGW10	Groër et al. (2010)	R-to-R + EC	Single	1.186%	1.28	Xe 2.3G	0.82
MB07f	Mester and Bräysy (2007) fast	EA+ELS	Single	1.230%	0.22	P-IV 2.8G	0.20
PR07	Pisinger and Ropke (2007)	ALNS	Avg 10	1.347%	10.8	P-IV 3.0G	10.8
LGW05	Li et al. (2005)	R-to-R	Single	1.390%	1.13	Ath 1.0G	0.33
MMP06	Marinakis et al. (2006)	Hybrid GA	Single	1.559%	3.44	P-III 667G	0.23
P04	Prins (2004)	Hybrid GA	Single	1.662%	66.6	P-III 1.0G	10.6

Classic Heuristics and Metaheuristics

- **A plethora of metaheuristics specific to one or a few variants, often hybrids. Exponential literature growth.**
- Many existing concepts and methods, but… even more questions :
  - Why using a strategy of a given type
  - What is its scope of application, on which range of problems is it successful
  - Method quality = tradeoff between solution quality, speed, flexibility, robustness and simplicity

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An analysis of winning strategies

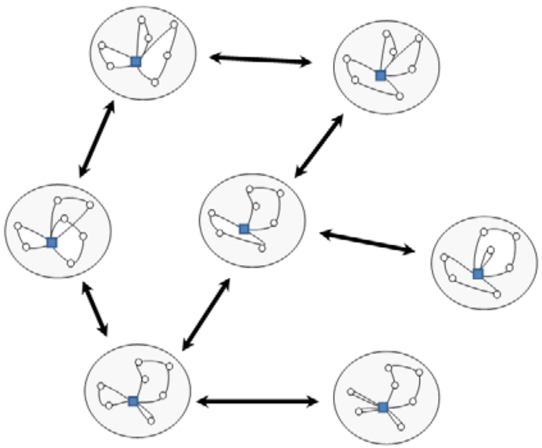
- Analyzing the method concepts, taking a broad perspective detached from problem attributes.

- **Methodology for this survey :**
  - Selecting 14 notable VRP variants. Criteria : classic benchmark instances available + large number of heuristics
  - Identifying the top 3 to 5 best metaheuristics w.r.t. solution quality
  - The resulting 64 methods are ``anatomized” to locate the recurrent successful elements of methodology.

An analysis of winning strategies

- 19 aspects of the methods have been scrutinized :

Search space	1) presence of infeasible solutions
	2) use of indirect representations of solutions
Neighbourhoods	3) presence of multiple neighbourhoods
	4) use of polynomially enumerable neighbourhoods
	5) use of pruning procedures
	6) use of large neighbourhoods
	7) use of solution recombinations
Trajectory	8) presence of random components
	9) continuous aspect of trajectories
	10) discontinuous aspect
	11) mixed aspect
Control and memories	12) use of populations
	13) diversity management
	14) parameter adaptation
	15) advanced guidance mechanisms
Hybrid strategies	16) use of hybridization
	17) matheuristics with integer programming
Parallelism	18) use of parallelism or cooperation concepts
Problem decompositions	19) use of problem decompositions



An analysis of winning strategies

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		SP.		NEIGHBOUR.					TRAJEC.				CONTROL							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		RELAXATION	SOL. REPRESENT.	MULT NEIGHB.	ENUMERATIVE	PRUNING	LARGE	RECOMBIN.	RANDOMNESS	CONTINUOUS	DISCONTINOUS	MIXED	POPULATIONS	DIV. MANAG.	PARAM. ADAPT.	GUIDANCE	HYBRIDIZATION	MATHEUR.	PARALLELISM	DECOMPOSITION
BACKHAULS																				
Brandão (2006)	Tabu	X		X	X				X	X					X	X				
Ropke and Pisinger (2006a)	ALNS	X		X			X		X	X					X	X	X			
Gajpal and Abad (2009)	ACO			X	X	X			X		X					X	X			
Zachariadis and K. (2012)	Attrib. driven LS			X	X	X				X						X				
PICK-UP AND DELIVERIES																				
Bent and V.H. (2006)	SA + LNS			X	X	X	X		X	X							X	X		
Ropke and Pisinger (2006b)	ALNS	X		X			X		X	X					X	X	X			
Cordeau and Laporte (2003)	Tabu	X			X	X				X					X	X				
Parragh et al. (2010)	VNS	X		X	X		X		X	X					X					
Nagata and Kobayashi (2011)	HGA			X	X	X	X	X	X		X		X			X	X			
MULTIPLE TRIPS																				
Taillard et al. (1996)	Adapt. M. + Tabu	X		X	X			X	X	X	X	X	X			X	X		X	X
Olivera and Viera (2007)	Adapt. M. + Tabu	X		X	X	X		X	X	X	X	X	X			X	X			
Alonso et al. (2008)	Tabu	X			X					X					X	X				
TIME WINDOWS																				
Hashimoto et Y. (2008)	Path Relinking	X		X	X	X	X	X	X		X		X		X	X				
Repoussis et al. (2009)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
P.-Gagnon et al. (2009)	LNS + Col. Gen.			X			X		X	X							X	X		
Nagata et al. (2010)	HGA	X		X	X	X		X	X		X		X				X			

…

An analysis of winning strategies

- Simple look at the metaheuristic frameworks in the champion methods :

Neighbourhood-centred	Freq.	Population-based	Freq.
Tabu Search	17	Genetic or Evolutionary Algorithm	16
Iterated Local Search	7	Ant Colony Optimization	4
Variable Neighbourhood Search	5	Scatter Search	2
Adaptive Large Neighbourhood Search	4	Path Relinking	2
Simulated Annealing & Record-to-record	3	Particle Swarm Optimization	1

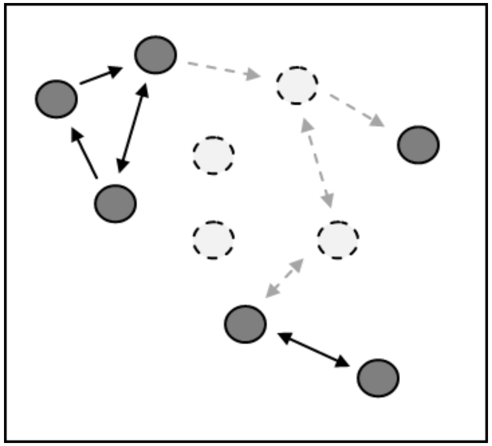
- Both classes of metaheuristics appear to be equally represented. Special emphasis on GA and Tabu.

- This observation goes against some popular claims for a “best” metaheuristic framework.

An analysis of winning strategies

- **Search Space : Relaxations (31/64 methods).**

- Most often, relaxations of route constraints. (capacity, duration, time windows…). Relaxing fleet size is usually not very convenient.



- Enables to transition more easily in the search space between feasible solutions.

- Simple procedure for fleet-size minimization.

- Strategic oscillation concept (Glover 1986), good solutions tend to be close to the borders of feasibility. Oscillating around these borders by adapting the penalty coefficients.

An analysis of winning strategies

- **Search Space : Relaxations (31/64 methods).**
- We conducted some experiments on this topic:
  - Solomon VRPTW instances, (several types of) relaxations of time windows, simple LS-improvement procedure.

Inst.	SolI1	NoUnf	Late	Twarp	Flex	NoUnf	Late	Twarp	Flex
		SolomonI1 Initial Solution				Random Initial Solution			
R1	1431.97	1225.25	1219.11	1220.13	**1219.41**	1268.97	1231.70	1226.08	1229.73
R2	1326.64	963.53	957.11	947.77	942.87	982.44	**940.38**	947.41	940.79
C1	936.48	844.00	835.17	835.67	**834.26**	860.82	835.14	840.73	835.32
C2	696.57	605.62	603.62	603.08	**600.59**	709.37	650.52	645.45	649.61
RC1	1578.28	1401.49	1399.11	**1389.83**	1396.54	1482.79	1406.57	1401.53	1404.57
RC2	1653.61	1139.12	1077.93	1093.02	1079.40	1130.21	1072.63	1075.60	**1070.59**
CTD	71633	58067	57319	57275	**57125**	60361	57679	57678	57621
T(sec)	0.03	3.41	20.06	6.59	7.90	6.25	17.55	8.03	10.00

  - Same observations on distance and load relaxations on CVRP, PVRP and MDVRP with advanced metaheuristics (Vidal et al 2012).

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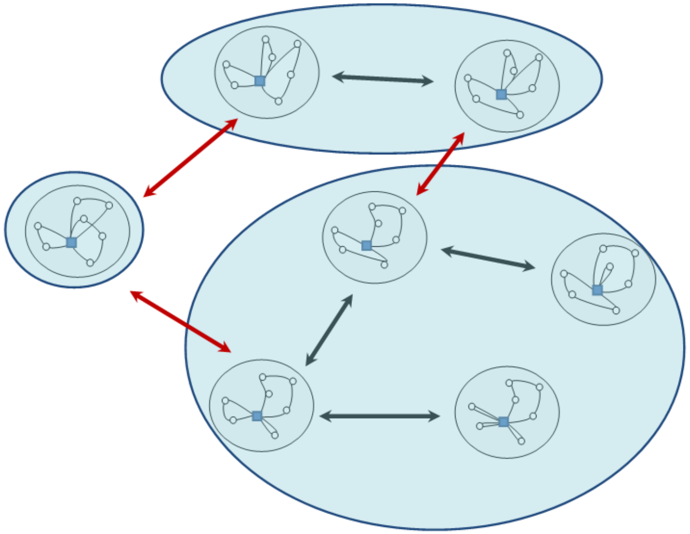
An analysis of winning strategies

- **Search Space : Indirect representations of solutions (12/64)**

- Indirect or incomplete representations
  - Giant-tour without trip delimiters (Prins 2004)
  - Only storing customer-to-visit-days choices (PVRP -- Alegre et al 2007)
  - Solution representation as a set of circular sectors (Salhi and Petch 2007)

- Using of a **decoding algorithm** to obtain the **best complete solution** from a **solution representation**
  - 1-to-many relationship
  - Shrinking the search space
  - Target : the *shrinking ratio* aims to be much larger than the additional effort related to decoding.



An analysis of winning strategies

- **Neighborhoods: Polynomially enumerable (almost all methods)**
  - All champion MAVRP metaheuristics use either LS or LNS.
  - LS neighborhoods are usually of $O(n^2)$ size for a given incumbent solution

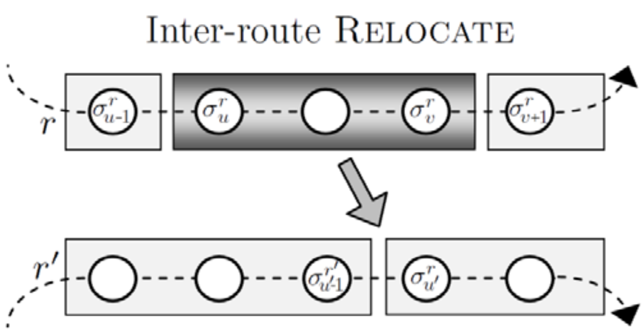
- **Neighborhoods: Multiple neighborhoods (60/64)**
  - The richness, variety, of the neighborhoods is determining to achieve high-quality solutions. Trade-off with search speed.
  - Addressing all attributes of the problems (sequencing, assignment to depots, vehicles, days) with purposeful, possibly compound, moves is often a key to success.

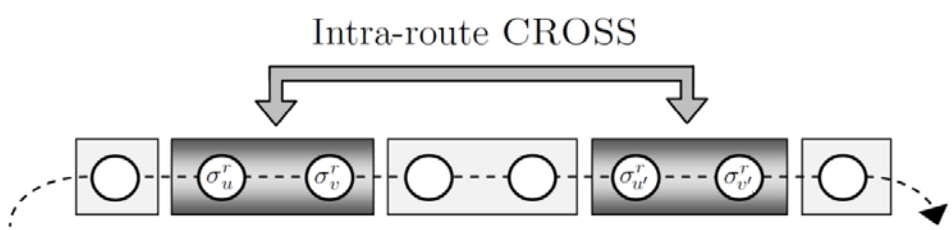
An analysis of winning strategies

- **Neighborhoods: Pruning and speed-up techniques (26/64)**
- Neighborhood search = bottleneck of most recent metaheuristics (also including population-based methods, which are usually hybrids)
- Speed-up techniques
  - **Neighborhood pruning** : either static (granular search) or dynamic (sequential search)
  - **Memory structures** : matrices for move evaluations, hashtables for route evaluations.

An analysis of winning strategies

- Speed-up techniques
  - **Information preprocessing** on subsequences to speed move evaluations in presence of complicating attributes :
    - Forward time slack for the VRPTW (Savelsbergh 1985,1992)
    - Generalized resources on segments (Irnich 2008), and timing re-optimization methods (Vidal et al 2012).
    - Using a simple property of classic VRP neighborhoods : any move resulting from a bounded number of arc exchanges or node relocations can be assimilated to a recombination of a bounded number of subsequences.





An analysis of winning strategies

- **Neighborhoods: Large neighborhoods (20/64)**
  - Ruin-and-recreate is commonly used
  - Also some cyclic improvement methods, e.g. in Ibaraki et al 2005
  - Sarvanov-Doroshko IP refinement heuristic, in Gulczynski et al. 2011

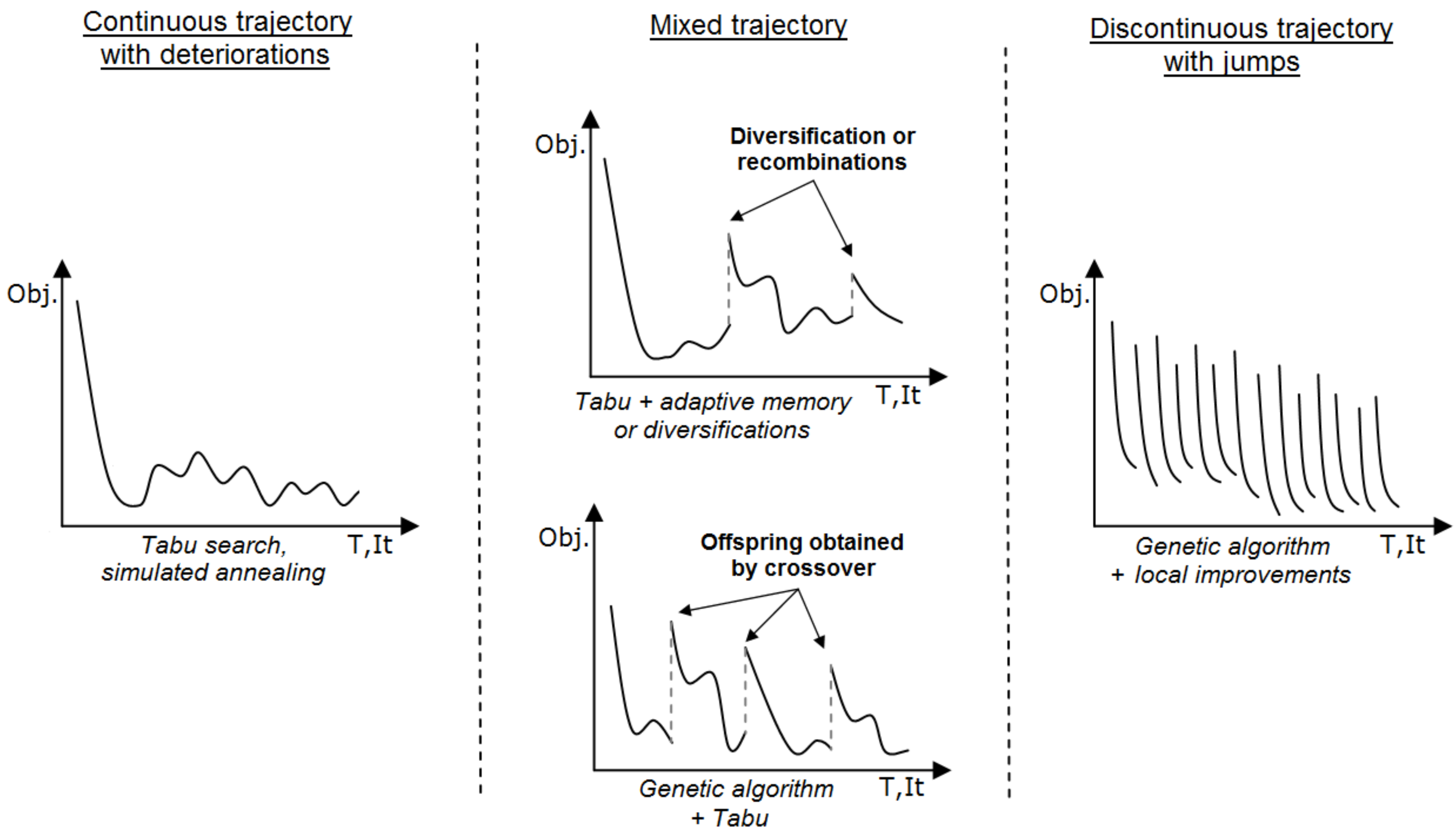
- **Neighborhoods: Solution recombinations (29/64)**
  - Combining fragments of good solutions leads to increased chances of finding new good solutions
    - Related to the building block hypothesis of Holland (1975)
    - MAVRP search landscapes often assimilated to ``big rugged valleys''
  - Not only GA or other population-based methods use recombination → c.f. adaptive memory Tabu search

An analysis of winning strategies

- **Trajectory: Randomization (56/64)**
  - Necessary for asymptotic convergence properties of SA and GA.
  - But, mostly used in recent metaheuristics as a simple way for avoiding cycling and bringing more diversity.
  - *[an intelligent use of randomization, which is not blindly uniform but embedded in probabilities that account for history and measures of attractiveness, offers a useful type of diversification that can substitute for more complex uses of memory]* (Rochat and Taillard 1995)

An analysis of winning strategies

- Trajectory : continuous (42/64), discontinuous (35/64), mixed aspect (12/64)



An analysis of winning strategies

- **Memories and control : populations (28/64)**

- Judicious acquisition, management, and exploitation of problem-knowledge → complex task that belongs to the core of metaheuristics.
- Glover (1975) discern several types of memories
  - Short term memories (e.g. tabu lists) – evade local optima
  - Medium and long-term memories – used to direct the overall exploration

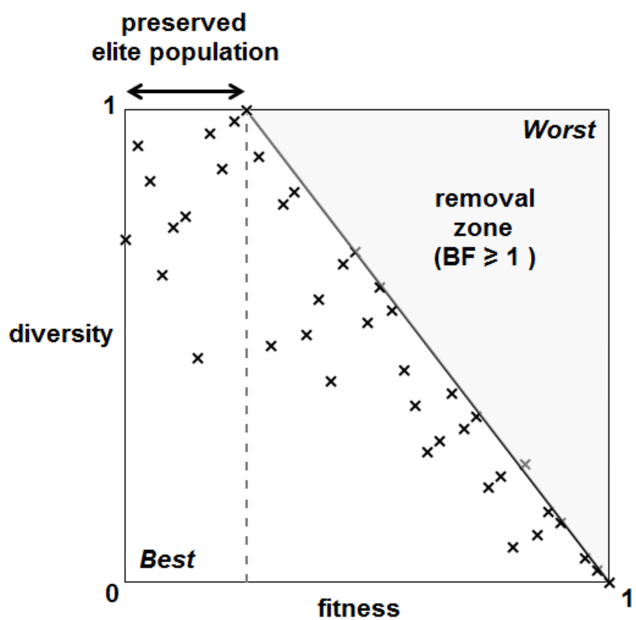
- Standard form of memory : populations (28/64) to store full solutions, solution representants, routes or other kind of fragments of solutions.

An analysis of winning strategies

- **Memories and control : population management (14/28)**

- Population-based methods : need diverse and high-quality solutions
  - PM is critical to avoid premature convergence, e.g. a state where the population information is poor and redundant. Needed to compensate the aggressive-improvement abilities of LS in hybrid population-based methods.

- Diversity management strategies (Prins 2004, Sörensen and Sevaux 2006)
- Promotion of diversity in the objective (Vidal et al 2012)
- Based on distance measures, in objective or solution space.



An analysis of winning strategies

- **Memories and control : population management (14/28)**

- Some experiments on this topic (Vidal 2012), solution-quality with HGSADC on standard PVRP, MDVRP, and MDPVRP instances.
  - **HGA** : No diversity management method
  - **HGA-DR** : Dispersal rule on objective space (Prins 2004)
  - **HGA-PM** : Dispersal rule on solution space (Sörensen and Sevaux 2006)
  - **HGSADC** : Promotion of diversity in the objective (Vidal et al 2012)

Benchmark		**HGA**	**HGA-DR**	**HGA-PM**	**HGSADC**
PVRP	T	6.86 min	7.01 min	7.66 min	8.17 min
	%	+0.64%	+0.49%	+0.39%	+0.13%
MDVRP	T	7.93 min	7.58 min	9.03 min	8.56 min
	%	+1.04%	+0.87%	+0.25%	-0.04%
MDPVRP	T	25.32 min	26.68 min	28.33 min	40.15 min
	%	+4.80%	+4.07%	+3.60%	+0.44%

An analysis of winning strategies

- **Memories and control : population management (14/28)**

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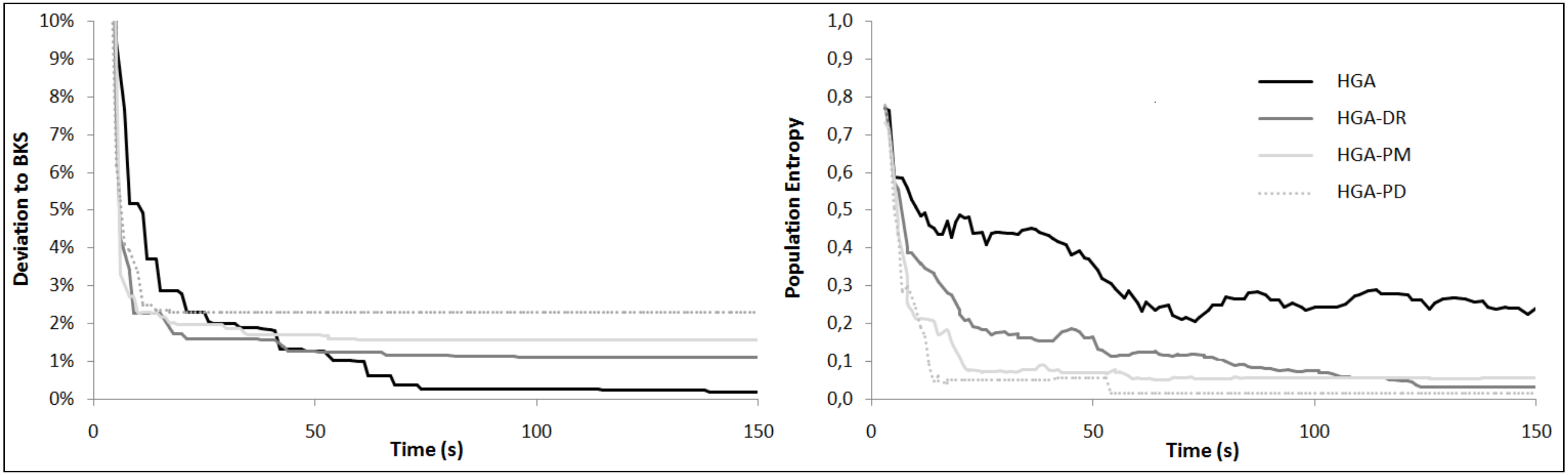


An analysis of winning strategies

- **Control and memories : guidance**

- A very simple form of guidance : parameters adaptation (30/64)
  - Driving infeasibility penalties, mutation and crossover rates, frequency of use of some operators or strategies. Method adaptation is a fundament of hyper-heuristics (Burke 2010)

An analysis of winning strategies

- **Control and memories : guidance**

- More advanced forms of guidance *:* explicitly collect, analyze, and exploit knowledge on the past search to orient the future trajectories.

- **Acquisition of guidance information** :
  - Historical statistics on solution features, arcs, sets of arcs, routes, or problem specic attributes.
  - Search context, value of incumbent and best solution
  - Possibly using data mining (Santos et al 2006)

Classic Heuristics and Metaheuristics

- **Control and memories : guidance**

- **Exploitation of guidance information** :
  - Guidance actions to
    - Either **intensify** the search around promising solution features
    - Or **diversify** the search around promising unexplored areas.
  - Applying penalties or incentives on solution features
  - Jumps toward elite solutions or restarts
  - Target solutions in path relinking
  - Neighborhood choice driven by pheromone matrices in ACO

- Continuous during all the search, or discreet through a purposeful move

An analysis of winning strategies

- **Hybridizations (39/64)**
  - Multiple methods combine different concepts
  - Among the most frequent in the heuristics surveyed : GA+LS, ACO+LS or ACO+LNS, Tabu + recombinations, ILS + VNS…
- On a general level, metaheuristics are inherently hybrids … described sometimes as "heuristics that guide other heuristics"
- **Matheuristics (9/64)**, blending metaheuristics with integer programming components. In the methods surveyed, IP used for
  - Handling problem-attributes (e.g. loading constraints or split deliveries)
  - Exploring large neighborhoods
  - Recombining solution elements.

An analysis of winning strategies

- **Parallelism and cooperation (6/64)**
  - Parallel Tabu searches that cooperating through an adaptive memory of solution elements (Ichoua et al 2003), or through a central memory of complete solutions.
  - Cooperation by pheromone exchanges (Balseiro et al 2011)
  - Other works : Le Bouthillier and Crainic (2005) → central memory with heterogeneous methods, or Crainic et al (2012) ``Integrative Cooperative Search''

- **Decompositions**
  - MAVRPs lend themselves well to
    - Structural or geometrical problem decompositions (assignment, sequencing, attribute subsets),
    - or based on attribute resources (e.g. time)

An analysis of winning strategies

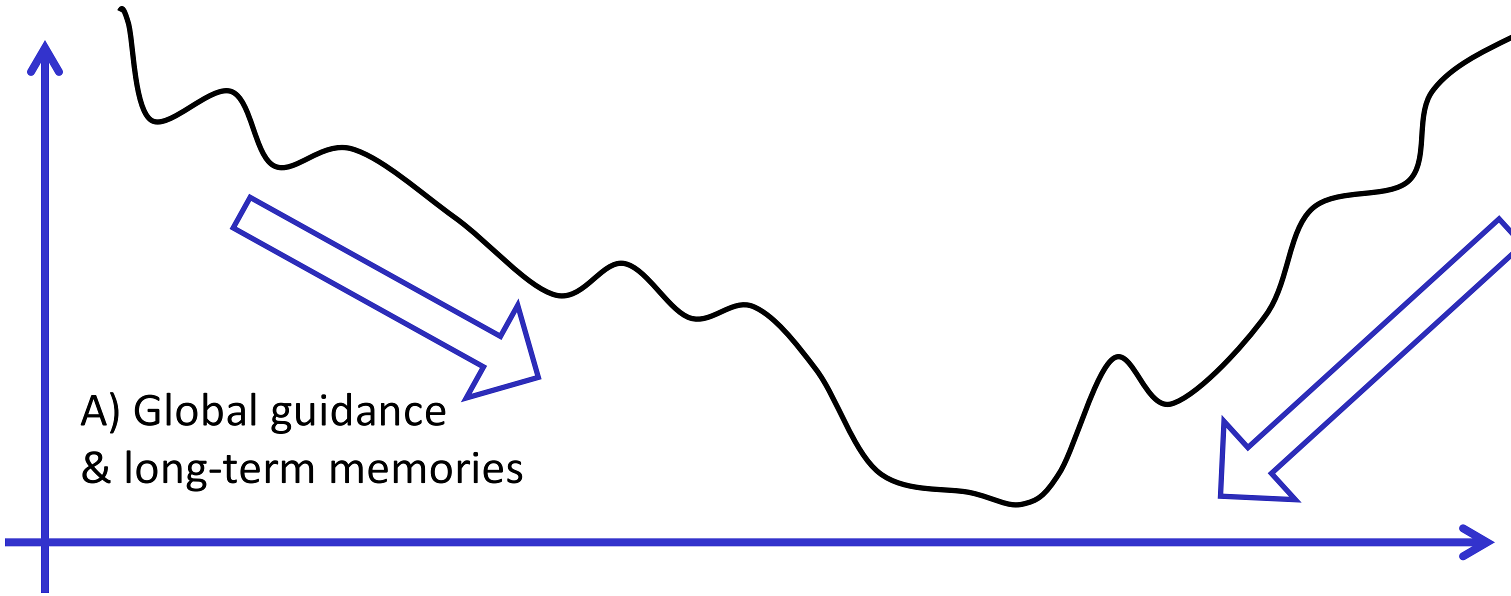
- **Some conclusions**:

  - **1.** Recurrent notions such as *mix* , *variability*, *hybridization*, *cooperation, diversity, multiplicity*, as well as *balance*, *equilibrium, trade-off* …
  - **Success is not related a single feature but rather to a combination of concepts.**

An analysis of winning strategies

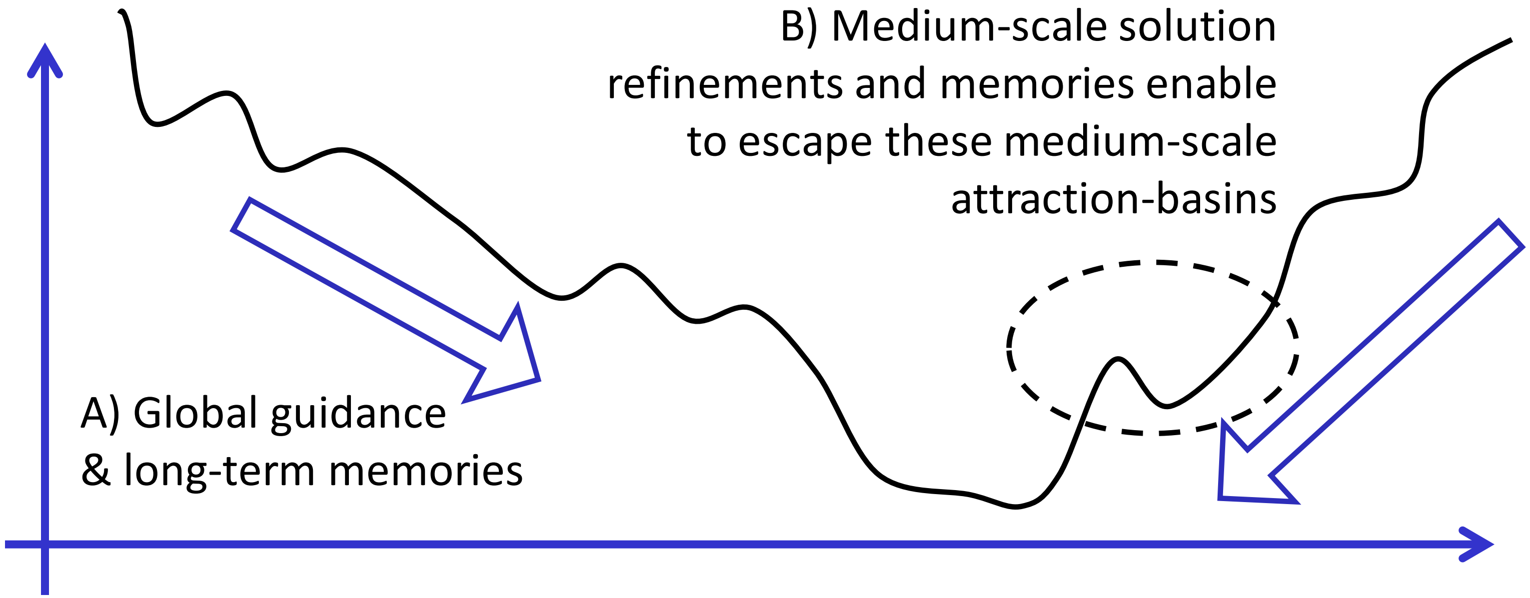
- **Some conclusions**:
  - **2. Interplay between different search levels**:
  - Long-term memories and guidance provide the necessary diversification to make the search progress in the general “big rugged valley”
  - LNS allow some medium-scale refinements
  - Short and medium-term memories and well-designed LS-improvement methods provide the aggressive search capabilities to refine the solutions.

- **Some conclusions**:
  - A personal mind picture :



An analysis of winning strategies

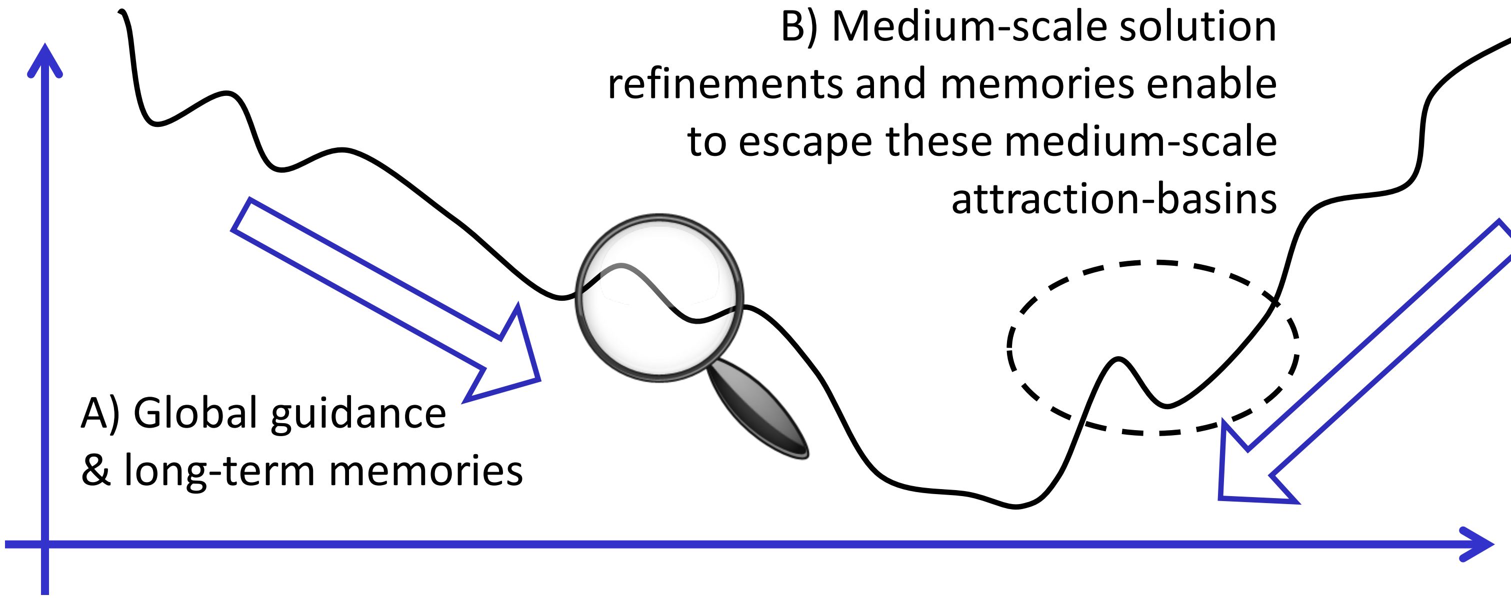
- **Some conclusions**:
  - A personal mind picture :



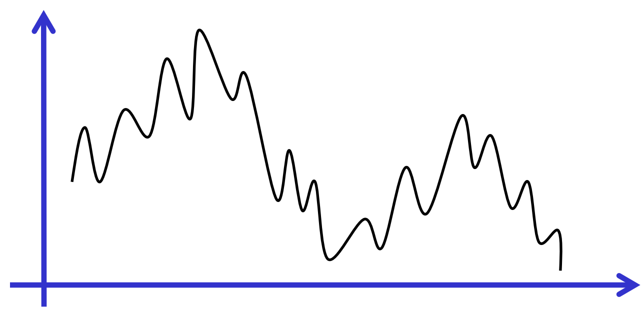
An analysis of winning strategies

- **Some conclusions**:

  - A personal mind picture :



C) Zooming-in → emphasizes small-scale ruggedness. Need LS to drive down the peaks



An analysis of winning strategies

- **Some conclusions**:

  - **3. Efficient neighborhood search and clever implementation of algorithms is a prerequisite for high performance** :
  - State-of-the-art move evaluations, reducing the complexity by keeping information on sequences,
  - neighborhood pruning and memories are critical.

  - **4. And good exploitation the search history to keep a suitable balance between intensification and diversification.**

- **Some conclusions**:

  - 5. Finally, **many other components can contribute to increase solution quality**, and there are often ways to improve a method by combining additional successful concept.
  - However, success should be more considered as a good tradeoff between performance and simplicity.
  - Only real critical components should be kept and presented. (it is critical to experimentally assess the impact of each separate component).

Presentation outline

- I) Vehicle Routing Problem, and attributes.
- II) Classic Heuristics and metaheuristics for vehicle routing
- III) An analysis of winning strategies
- IV) A new general-purpose solution approach
  - Attribute-based modular design
  - Unified Local Search
  - Unified Hybrid Genetic Search
  - Computational Experiments

A new general-purpose solution approach

- **Contributing with a new general-purpose method**, which exploits the successful concepts identified in this survey as well as the structure of the attributes.

- Additional challenge of this work, **designing a unified method : achieving generality & efficiency**
  - Drawback of current unified VRP methods: dealing with a rich VRP model that includes several MAVRP as special cases → Still accounting for non-activated attributes
  - Still need to address the problem → but relegating problem-specificities to small modular components
  - Each separate MAVRP shall be still addressed with state-of-the-art solution evaluation and search procedures

A new general-purpose solution approach

- **Contributing with a new general-purpose method**, which exploits the successful concepts identified in this survey as well as the structure of the attributes.

- Some elements of methodology that we opportunistically exploited :
  - **Modular design techniques based on attribute-structure**
  - **Successful heuristic strategies**
    - Relaxations
    - Solutions representants
    - Efficient LS with neighborhood pruning and memories
    - Population and Diversity Management
    - Diversification phases and guidance
    - Decomposition phases

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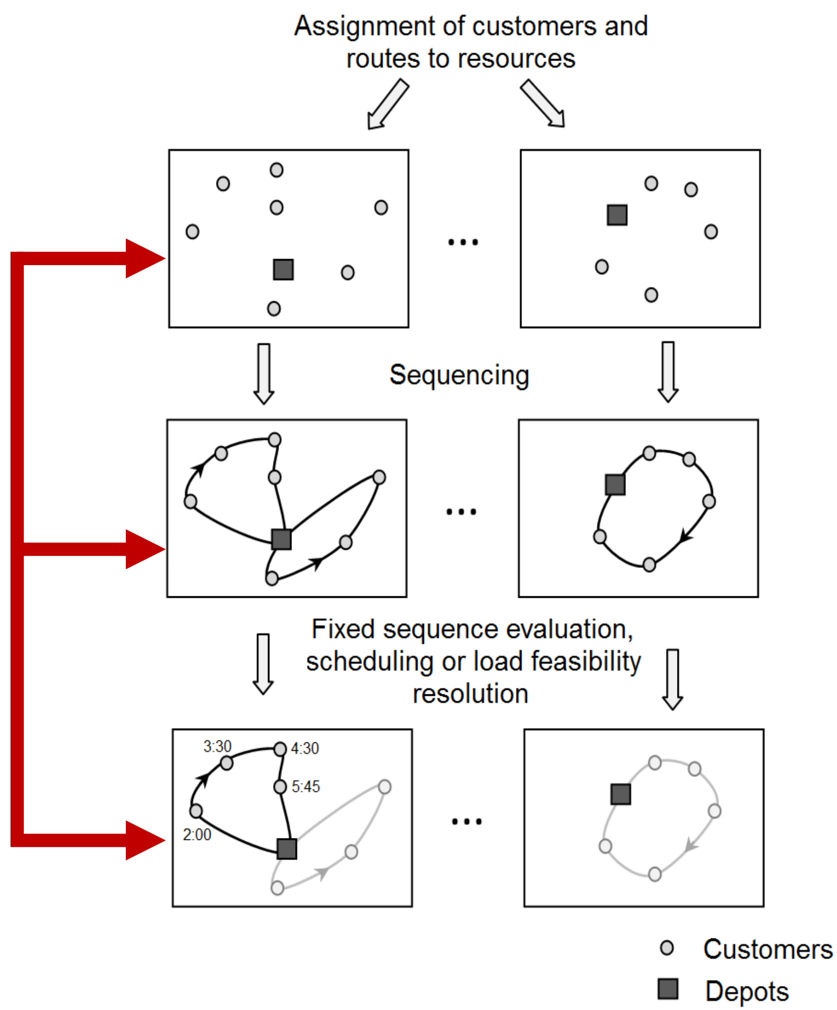
Attribute-based modular design

- **Back to the method-oriented attribute classification:**

- **ASSIGN ATTRIBUTES**: impacting the assignment of customers and routes

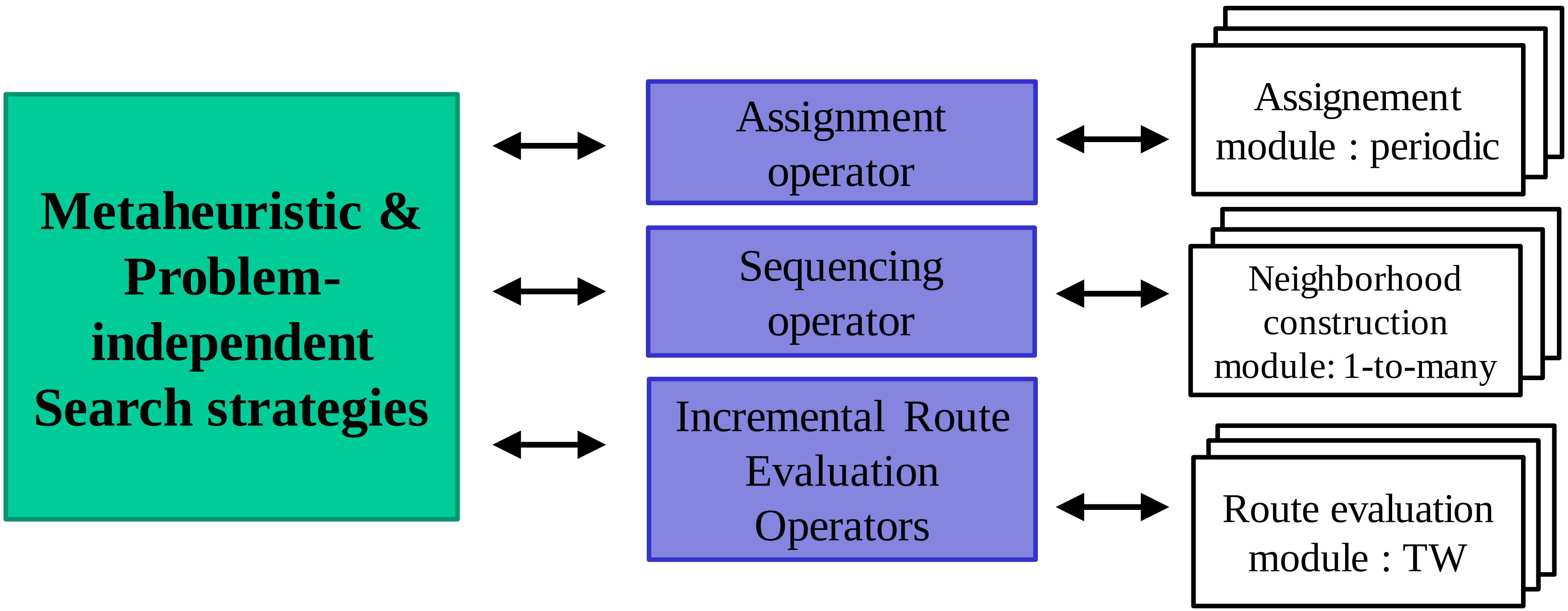
- **SEQ ATTRIBUTES:** impacting the nature of the network and the sequences

- **EVAL ATTRIBUTES:** impacting the evaluation of fixed routes



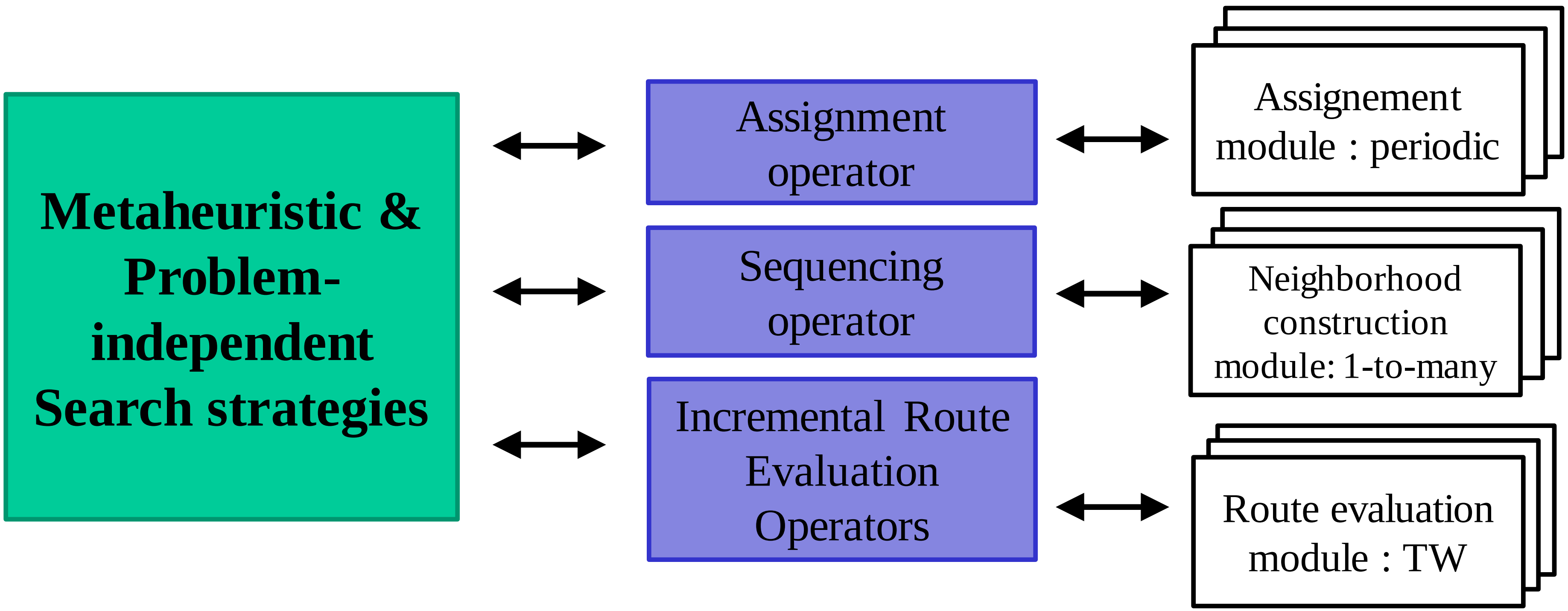
Attribute-based modular design

- Proposed unified framework:
  - Relying on assignment, sequencing & route evaluation operators → implemented in a generic way, based on a library of attribute-specific modules



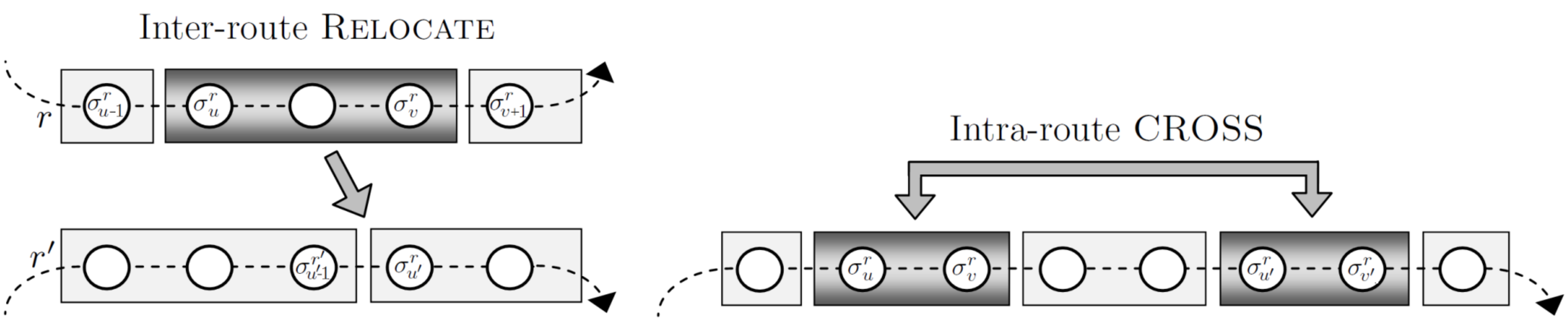
Attribute-based modular design

- Proposed unified framework:
  - **Attribute-dependent modules are automatically selected by the algorithm to serve as the basis for the assignment, sequencing, and route evaluation operators → Object-oriented programming, using inheritance and polymorphism.**



Route Evaluation Operators based on re-optimization

- Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
- The same subsequences appear many times during different moves



- Data preprocessing on sub-sequences to speed up the search (Savelsbergh 1985,1992 …)
- The route evaluation modules must allow for such preprocessing.

- Route Evaluation Operators based on re-optimization
  - Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
  - Hence, to manage and exploit information on subsequences, five families of route evaluation operators are used :

Operators for data construction:	
INIT(σ)	Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit.
FORW(σ)	Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence σ and vertex v_i.
BACK(σ)	Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex v_i and sub-sequence σ.
Operators for route evaluations:	
EVAL2(σ_1, σ_2)	Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \sigma_2$.
EVALN($\sigma_1, \dots, \sigma_n$)	Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \dots \oplus \sigma_n$.

Route evaluation operators examples

- **Example 1)** Route evaluation operators for **distance and capacity constraints**

What is managed ? → Partial loads $L(\sigma)$ and distance $D(\sigma)$

Init → For a sequence σ_0 with a single visit v_i , $L(\sigma_0) = q_i$ and $D(\sigma_0) = 0$

Forw and Back → increment $L(\sigma)$ and $D(\sigma)$

Eval → compute the data by induction on the concatenation operator

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2)$$
$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

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- **Example 2)** Route evaluation operators for **cumulated arrival time objectives**

What is managed ? → Travel time D(σ), Cumulated arrival time C(σ), Delay Cost W(σ) associated to one unit of delay in starting time

Init → For a sequence σ_0 with a single visit v_i , $D(\sigma_0) = 0$ and $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ if v_i is a customer, and $W(\sigma_0) = 0$ if v_i is a depot visit.

Forw & Back & Eval → induction on the concatenation operator:

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$
$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2)$$
$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2)$$

Route evaluation operators examples

- **Example 3)** Route evaluation operators for **time windows (and route duration constraints)**

What is managed ? → Travel time and service time T(σ), earliest feasible completion time E(σ), latest feasible starting date L(σ), statement of feasibility F(σ).

Init → For a sequence σ_0 with a single visit v_i , $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0)$ = *true*.

Forw & Back & Eval → induction on the concatenation operator:

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2)$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$

$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \;\wedge\; F(\sigma_2) \;\wedge\; (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$

Route evaluation operators examples

- **Example 4)** Route evaluation operators for **lunch break positioning in presence of time-window constraints**

What is managed ? → Same set of data ($T(\sigma)$, $E(\sigma)$, $L(\sigma)$, and $F(\sigma)$) as in the TW case, and it is duplicated to also provide $T'(\sigma)$, $E'(\sigma)$, $L'(\sigma)$, and $F'(\sigma)$ for the sequence where exactly one lunch break was inserted.

Init → As previously for $T(\sigma_0)$, $E(\sigma_0)$, $L(\sigma_0)$, and $F(\sigma_0)$. Furthermore, $T'(\sigma_0) = +\infty$, $E'(\sigma_0) = +\infty$, $L'(\sigma_0) = 0$, and $F'(\sigma_0) = \mathit{false}$.

Forw & Back & Eval → induction on the concatenation operator, see next page for the equations.

Route evaluation operators examples

- **Example 4)** Route evaluation operators for **lunch break positioning in presence of time-window constraints**

$$E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case i}} | F'_{\text{case i}} = true\} \cup +\infty)$$
$$L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case i}} | F'_{\text{case i}} = true\} \cup -\infty)$$
$$F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case 1}} \vee F'_{\text{case 2}} \vee F'_{\text{case 3}}$$

$$E'_{\text{case 1}} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$
$$E'_{\text{case 2}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\}$$
$$E'_{\text{case 3}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T'(\sigma_2), E'(\sigma_2)\}$$
$$L'_{\text{case 1}} = \min\{L'(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T'(\sigma_1)\}$$
$$L'_{\text{case 2}} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - s_{\text{LB}} - T(\sigma_1)\}$$
$$L'_{\text{case 3}} = \min\{L(\sigma_1), L'(\sigma_2) - p_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$
$$F'_{\text{case 1}} = F'(\sigma_1) \wedge F(\sigma_2) \wedge (E'(\sigma_1) + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$
$$F'_{\text{case 2}} = F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) \leq l_{\text{LB}}) \wedge (E(\sigma_1) + s_{\text{LB}} + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$
$$F'_{\text{case 3}} = F(\sigma_1) \wedge F'(\sigma_2) \wedge (E(\sigma_1) + p_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L'(\sigma_2))$$

- **Example 5)** Route evaluation operators for **soft and general time windows**

What is managed ? → Minimum cost F(σ)(t) to process the sequence σ while starting the last service before time t, minimum cost B(σ)(t) to process the sequence σ after time t.

Init → For a sequence σ_0 with a single visit v_i characterized by a service cost function $c_i(t)$, $F(\sigma_0)(t) = \min_{(x \leq t)} ci(x)$ and $B(\sigma_0)(t) = \min_{(x \geq t)} c_i(x)$.

Forw & Back →

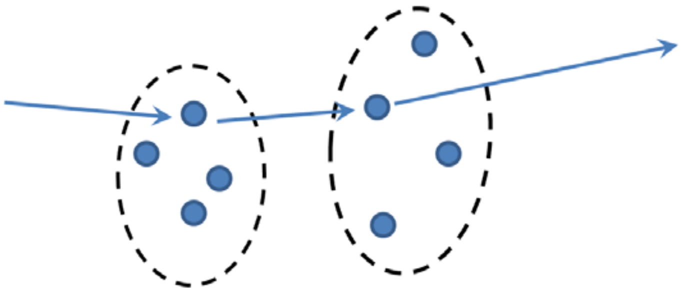
$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t} \{c_i(t) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\}$$

Eval 2 → $$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\}$$

Route evaluation operators examples

- **Example 6)** Route evaluation operators for **the generalized VRP :**



What is managed ? → The shortest path S(σ)[i,j] inside the sequence σ starting at the location *i* of the starting group and finishing at location *j* of the ending group.

Init → For a sequence σ_0 with a single visit v_i , S(σ)[i,j] = +∞ if i ≠ j, and S(σ)[i,i] = 0.

Forw & Back & Eval → induction on the concatenation operator:

$$S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)},\, 1 \leq y \leq \lambda_{\sigma_2(1)}} S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j]$$

$$\forall i \in \{1, \dots, \lambda_{\sigma_1(1)}\}, \forall j \in \{1, \dots, \lambda_{\sigma_2(|\sigma_2|)}\}$$

An efficient and unified local search for MAVRPs

- Generic local-search based on route evaluation operators

Algorithm 1 Unified local search based on route evaluation operators

Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
while some improving moves exist in the neighborhood $\mathcal{N}$ **do**
  for each move μ_i in $\mathcal{N}$ **do**
    for each route r_j^μ produced by the move **do**
      Determine the k sub-sequences $[\sigma_1, \dots, \sigma_k]$ that are concatenated to produce r_j^μ
      if $k = 2$, then NEWCOST(r) = EVAL2(σ_1,σ_2)
      else if $k > 2$, then NEWCOST(r) = EVALN$(\sigma_1,\dots,\sigma_k)$
    if ACCEPTCRITERIA(μ_i) **then** perform the move μ and update the re-optimization data on for each route r_j^μ using the INIT, FORW and BACK operators.

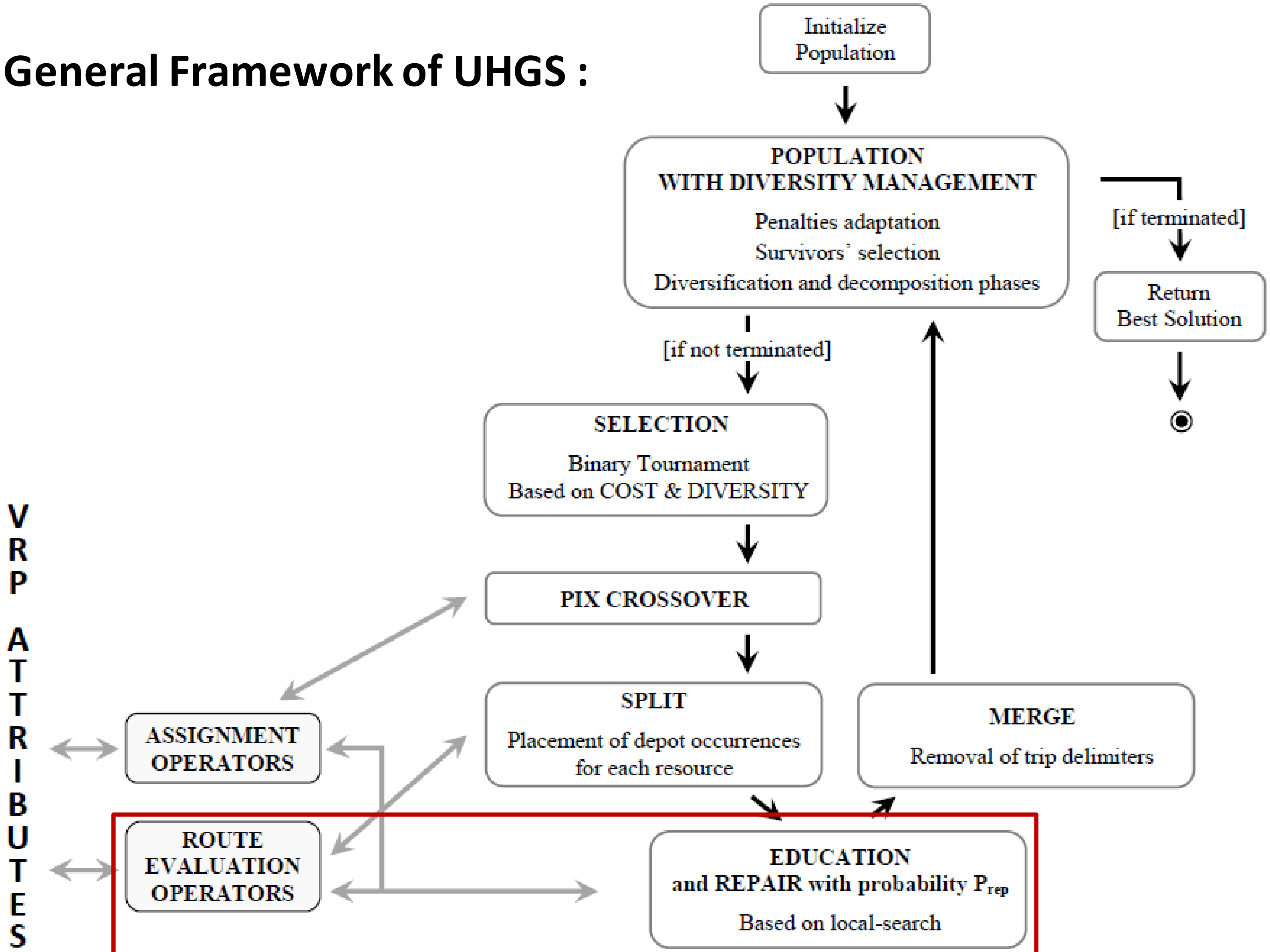
- Can serve as the basis to build any neighborhood-based unified solver based on VNS, Tabu, ILS for MAVRPs with EVAL attributes.
- Going one step further, designing a unified hybrid GA.

A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- UHGS = Classic GA framework + 4 main ingredients (Vidal et al. 2010)
  - Management of penalized infeasible solutions in two subpopulations
  - High-performance local search-based *Education* procedure
  - Solution Representation *without trip delimiters*
  - **Diversity & Cost objective for individuals evaluations**

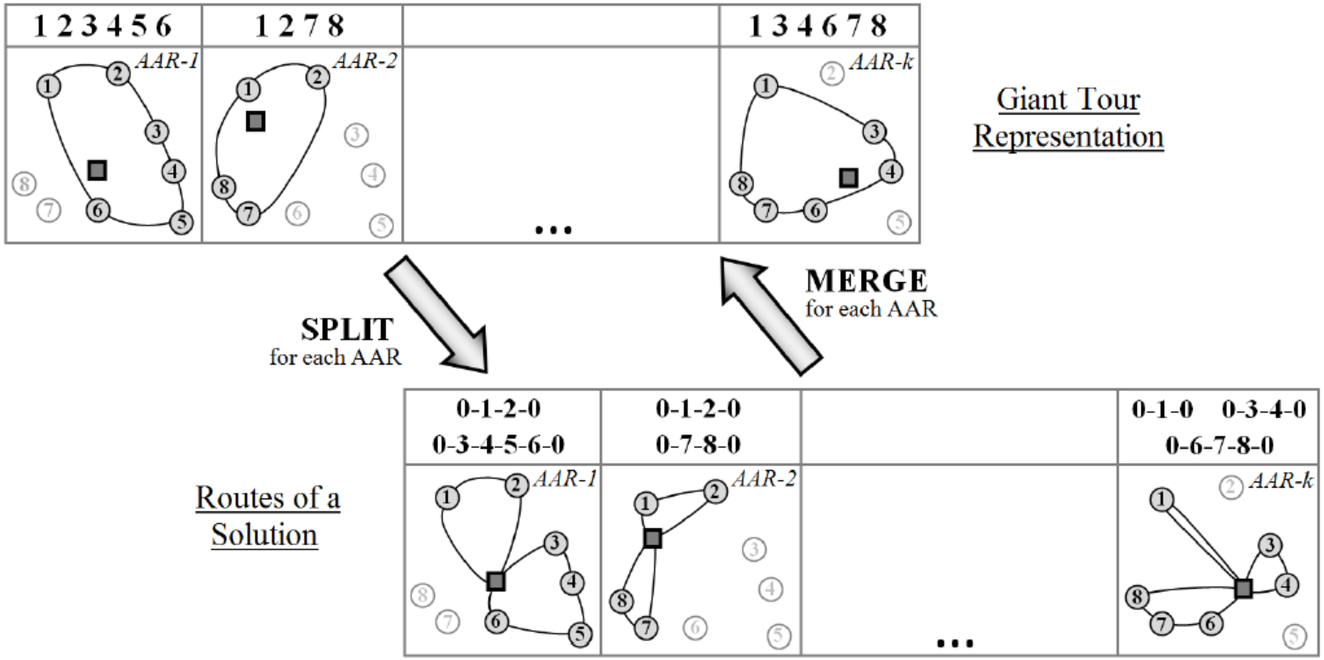
A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- **General Framework of UHGS :**



Unified Solution Representation and Split

- Now dealing with MAVRPs with both ASSIGN and EVAL attributes: Assignment of customer services to some ASSIGN attributes resources (AARs) + separate optimization of routes for each AARs.
  - Solution representation is designed accordingly.
  - Furthermore, representation without trip delimiters for each AAR.



Unified Solution Representation and Split

- Solution representation as a giant-tour per AAR → requires a Split algorithm (Prins 2004) for optimal segmentation into routes.
- We propose a **unified Split algorithm**.
  - As usual, the problem is solved as a m-shortest path
  - The route evaluation operators are used to build the auxiliary graph

Algorithm 2 Generic Split

1: **for** each node $i \in \{0, \dots, \nu\}$ **do**
2: $\quad SeqData(\sigma) = \textsc{Init}(\{v_0\})$ *//Initialize with depot vertex*
3: $\quad$ **for** each node $j \in \{i, \dots, \min(i + \bar{r}, \nu)\}$ **do**
4: $\quad\quad \phi(a_{ij}) = \textsc{Eval2}(\sigma,\{v_0\})$ *//Evaluate the route*
5: $\quad\quad SeqData(\sigma) = \textsc{Forw}(\sigma,\{\tau_j\})$ *//Append a new customer to the route end*
6: Solve the shortest path problem on $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc a_{ij}
7: Return the set of routes associated to the set of arcs of the shortest path

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Unified Crossover Operator

- 4 phases **Assignment and Insertion Crossover (AIX),** to produce a single offspring C from two parents P1 and P2.

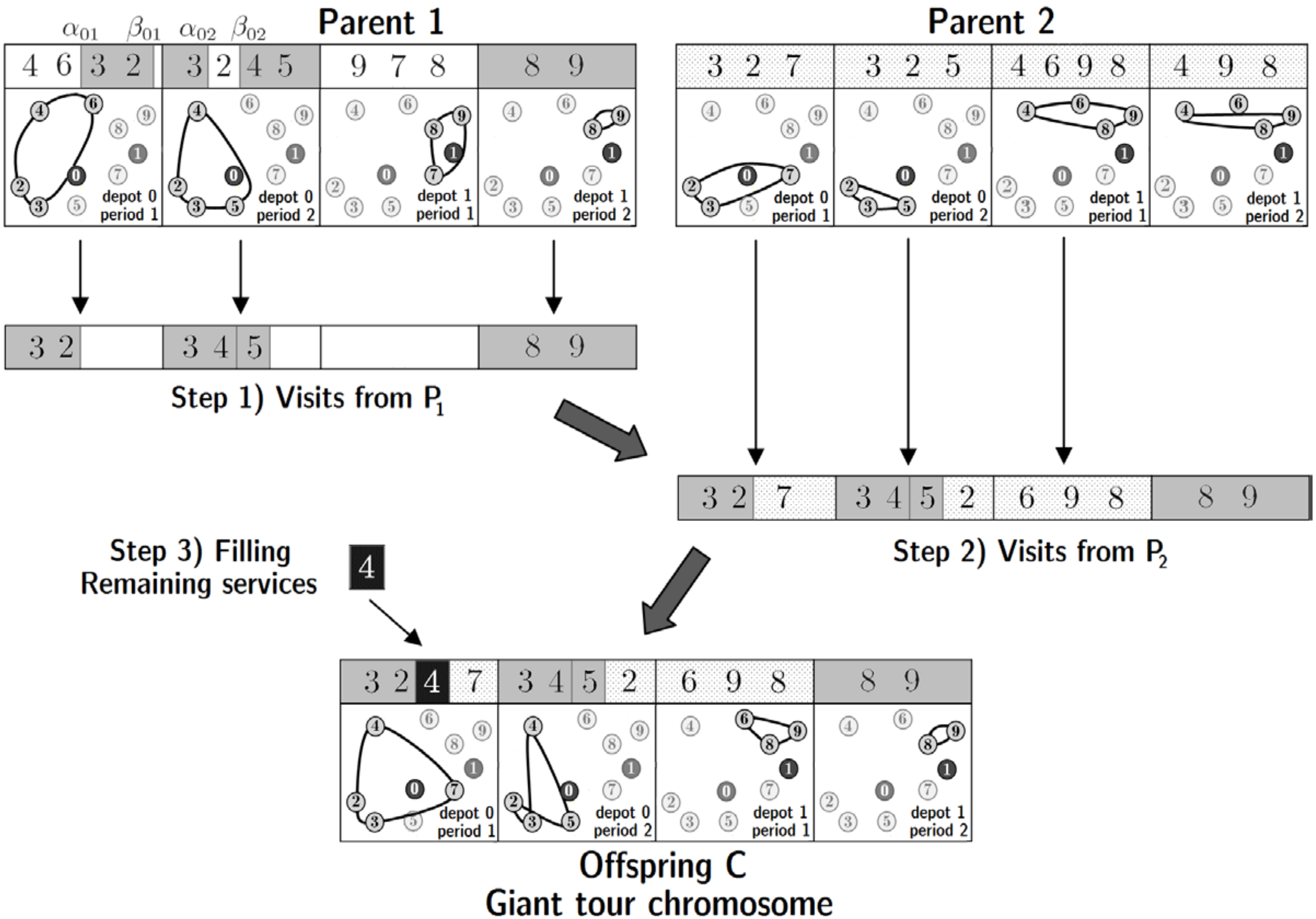
- Step 1) Choose for each AAR whether the genetic material of P1, P2, or both parents is inherited.

- Step 2) Fully transmit the selected material from P1

- Step 3) Complete with the selected material from P2, check at each step with an Assignment module whether the inheritance respects the ASSIGN attributes specifications.

- Step 4) Perform a best insertion of missing visits.

Unified Crossover Operator



Unified Education Procedure

- Based on the previously described Unified Local Search to perform route improvement (RI) on separate AAR.
  - Using CROSS, I-CROSS, Relocate, 2-Opt* and 2-Opt neighborhoods
  - Pruning procedures (granular search)
  - Hybrid acceptance strategy (intermediate between first improvement and best improvement)

- Combined with an assignment-improvement (AI) procedure to re-assign customer visits into different resources and routes.

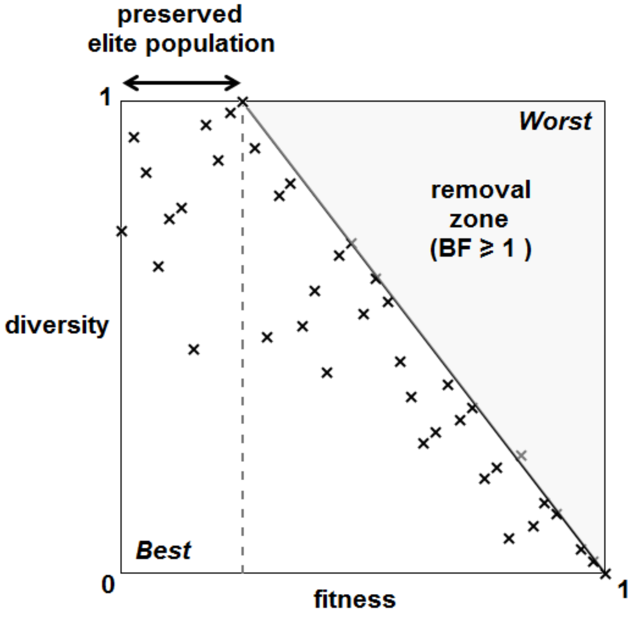
- These two procedures are called in the sequence RI-AI-RI.

Population management and search guidance

- **Biased Fitness** is a tradeoff between ranks in terms of **solution penalized cost** *cost(I),* and **contribution to the diversity** *dc(I),* measured as a distance to others individuals in the population.

$$BF(I) = fit(I) + (1 - \frac{nbElit}{nbIndiv - 1}) \times dc(I)$$

- Used during selection of the parents
  - Balancing strength with innovation during reproduction, and thus favoring exploration of the search space.
- and during selection of the survivors:
  - Removing the individual *I* with worst *BF(I)* also guarantees some elitism in terms of solution value.



Comparison with problem-tailored state-of-the-art methods

- Extensive computational experiments on 26 structurally different VRP variants and 39 sets of benchmark instances.

- Comparing UHGS with the best problem-tailored method for each benchmark.

- In the following, we indicate for each method
  - % Gap to the BKS of an average run (out of 10 for UHGS).
  - % Gap to the BKS of a best run (out of 10 for UHGS).
  - Computational effort (total work time) for an average run
  - Type of processor used.

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods: Author	Avg.%	Best%	T(min)	CPU
CVRP	CMT79	[50,199]	C	GG11:	—	+0.03%	8×2.38	8×Xe 2.3G
				MB07:	+0.03%	—	2.80	P-IV 2.8G
				UHGS*:	**+0.02%**	**+0.00%**	11.90	Opt 2.4G
CVRP	GWKC98	[200,483]	C	GG11:	—	+0.29%	8×5	8×Xe 2.3G
				NB09:	+0.27%	+0.16%	21.51	Opt 2.4G
				UHGS*:	**+0.15%**	**+0.02%**	71.41	Opt 2.4G
VRPB	GJ89	[25,200]	C	ZK12:	+0.38%	+0.00%	1.09	T5500 1.67G
				GA09:	+0.09%	+0.00%	1.13	Xe 2.4G
				UHGS:	**+0.01%**	**+0.00%**	0.99	Opt 2.4G
CCVRP	CMT79	[50,199]	C	NPW10:	+0.74%	+0.28%	5.20	Core2 2G
				RL12:	+0.37%	+0.07%	2.69	Core2 2G
				UHGS:	**+0.01%**	**-0.01%**	1.42	Opt 2.2G
CCVRP	GWKC98	[200,483]	C	NPW10:	+2.03%	+1.38%	94.13	Core2 2G
				RL12:	+0.34%	+0.07%	21.11	Core2 2G
				UHGS:	**-0.14%**	**-0.23%**	17.16	Opt 2.2G
VRPSDP	SN99	[50,199]	C	SDBOF10:	+0.16%	+0.00%	256×0.37	256×Xe 2.67G
				ZTK10:	—	+0.11%	—	T5500 1.66G
				UHGS:	**+0.01%**	**+0.00%**	2.79	Opt 2.4G
VRPSDP	MG06	[100,400]	C	SDBOF10:	+0.30%	+0.17%	256×3.11	256×Xe 2.67G
				UHGS:	+0.20%	+0.07%	12.00	Opt 2.4G
				S12 :	**+0.08%**	**+0.00%**	7.23	I7 2.93G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods Author	Avg.%	Best%	T(min)	CPU
VFMP-F	G84	[20,100]	C	ISW09:	—	+0.07%	8.34	P-M 1.7G
				SPUO12:	+0.12%	+0.01%	0.15	I7 2.93G
				UHGS:	**+0.04%**	**+0.01%**	1.13	Opt 2.4G
VFMP-V	G84	[20,100]	C	ISW09:	—	+0.02%	8.85	P-M 1.7G
				SPUO12:	+0.17%	+0.00%	0.06	I7 2.93G
				UHGS:	**+0.03%**	**+0.00%**	0.85	Opt 2.4G
VFMP-FV	G84	[20,100]	C	P09:	—	+0.02%	0.39	P4M 1.8G
				UHGS:	+0.01%	+0.00%	0.99	Opt 2.4G
				SPUO12:	**+0.01%**	**+0.00%**	0.13	I7 2.93G
LDVRP	CMT79	[50,199]	C	XZKX12:	+0.48%	+0.00%	1.3	NC 1.6G
				UHGS:	**-0.28%**	**-0.33%**	2.34	Opt 2.2G
LDVRP	GWKC98	[200,483]	C	XZKX12:	+0.66%	+0.00%	3.3	NC 1.6G
				UHGS:	**-1.38%**	**-1.52%**	23.81	Opt 2.2G
PVRP	CGL97	[50,417]	C	HDH09:	+1.69%	+0.28%	3.09	P-IV 3.2G
				UHGS*:	+0.43%	+0.02%	6.78	Opt 2.4G
				CM12:	**+0.24%**	**+0.06%**	64×3.55	64×Xe 3G
MDVRP	CGL97	[50,288]	C	CM12:	+0.09%	+0.03%	64×3.28	64×Xe 3G
				S12:	+0.07%	+0.02%	11.81	I7 2.93G
				UHGS*:	**+0.08%**	**+0.00%**	5.17	Opt 2.4G
GVRP	B11	[16,262]	C	BER11:	+0.06%	—	0.01	Opt 2.4G
				MCR12:	+0.11%	—	0.34	Duo 1.83G
				UHGS:	**+0.00%**	**-0.01%**	1.53	Opt 2.4G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
OVRP	CMT79 &F94	[50,199]	F/C	RTBI10:	0%/+0.32%	—	9.54	P-IV 2.8G
				S12:	—/+0.16%	0%/+0.00%	2.39	I7 2.93G
				UHGS:	**0%/+0.11%**	**0%/+0.00%**	1.97	Opt 2.4G
OVRP	GWKC98	[200,480]	F/C	ZK10:	0%/+0.39%	0%/+0.21%	14.79	T5500 1.66G
				S12:	0%/+0.13%	0%/+0.00%	64.07	I7 2.93G
				UHGS:	**0%/-0.11%**	**0%/-0.19%**	16.82	Opt 2.4G
VRPTW	SD88	100	F/C	RTI09:	0%/+0.11%	0%/+0.04%	17.9	Opt 2.3G
				UHGS*:	0%/+0.04%	0%/+0.01%	2.68	Xe 2.93G
				NBD10:	**0%/+0.02%**	**0%/+0.00%**	5.0	Opt 2.4G
VRPTW	HG99	[200,1000]	F/C	RTI09b:	—	+0.16%/+3.36%	270	Opt 2.3G
				NBD10:	+0.20%/+0.42%	+0.10%/+0.27%	21,7	Opt 2.4G
				UHGS*:	**+0.18%/+0.11%**	**+0.08%/-0.10%**	141	Xe 2.93G
OVRPTW	SD88	100	F/C	RTI09a:	+0.89%/+0.42%	0%/+0.24%	10.0	P-IV 3.0G
				KTDHS12:	0%/+0.79%	0%/+0.18%	10.0	Xe 2.67G
				UHGS:	**+0.09%/-0.10%**	**0%/-0.10%**	5.27	Opt 2.2G
TDVRPTW	SD88	100	F/C	KTDHS12:	+2.25%	0%	10.0	Xe 2.67G
				UHGS:	**-3.31%**	**-3.68%**	21.94	Opt 2.2G
VFMPTW	LS99	100	D	BDHMG08:	—	+0.59%	10.15	Ath 2.6G
				RT10:	+0.22%	—	16.67	P-IV 3.4G
				UHGS:	**-0.15%**	**-0.24%**	4.58	Opt 2.2G
VFMPTW	LS99	100	C	BDHMG08:	—	+0.25%	3.55	Ath 2.6G
				BPDRT09:	—	+0.17%	0.06	Duo 2.4G
				UHGS:	-0.38%	-0.49%	4.82	Opt 2.2G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
PVRPTW	CL01	[48,288]	C	PR08:	—	+1.75%	—	Opt 2.2G
				CM12:	+1.10%	+0.76%	64×11.3	64×Xe 3G
				UHGS*:	**+0.63%**	**+0.22%**	32.7	Xe 2.93G
MDVRPTW	CL01	[48,288]	C	PBDH08:	—	+1.37%	147	P-IV 3.6G
				CM12:	+0.36%	+0.15%	64×6.57	64×Xe 3G
				UHGS*:	**+0.19%**	**+0.03%**	6.49	Xe 2.93G
SDVRPTW	CL01	[48,288]	C	B10:	+2.23%	—	2.94	Qd 2.67G
				CM12:	+0.62%	+0.36%	64×5.60	64×Xe 3G
				UHGS*:	**+0.36%**	**+0.10%**	5.48	Xe 2.93G
VRPSTW (type 1, α=100)	SD88	100	F/TW/C	F10:	0%	—	9.69	P-M 1.6G
				UHGS:	**-3.05%**	**-4.42%**	18.62	Opt 2.2G
VRPSTW (type 1, α=1)	SD88	100	C+TW	KTDHS12:	+0.62%	+0.00%	10.0	Xe 2.67G
				UHGS:	**-0.13%**	**-0.18%**	5.82	Opt 2.2G
VRPSTW (type 2, α=100)	SD88	100	F/TW/C	FEL07:	0%	—	5.98	P-II 600M
				UHGS:	**-13.91%**	**-13.91%**	41.16	Opt 2.2G
VRPSTW (type 2, α=1)	SD88	100	C+TW	**UHGS:**	**+0.26%**	**0%**	29.96	Opt 2.2G
MDPVRPTW	New	[48,288]	C	**UHGS:**	**+0.77%**	**0%**	16.89	Opt 2.2G
VRTDSP (E.U. rules)	G09	100	F/C	PDDR10:	0%/0%	0%/0%	88	Opt 2.3G
				UHGS*:	**-0.56%/-0.54%**	**-0.85%/-0.70%**	228	Xe 2.93G

Comparison with problem-tailored state-of-the-art methods

List of acronyms for benchmarks					
B11	Bektas et al. (2011)	G84	Golden (1984)	LS99	Liu and Shen (1999)
CGL97	Cordeau et al. (1997)	G09	Goel (2009)	MG06	Montané and Galvão (2006)
CL01	Cordeau and Laporte (2001)	GH99	Gehring and Homberger (1999)	SD88	Solomon and Desrosiers (1988)
CMT79	Christofides et al. (1979)	GJ89	Goetschalckx and J.-B. (1989)	SN99	Salhi and Nagy (1999)
F94	Fisher (1994)	GWKC98	Golden et al. (1998)		
List of acronyms for state-of-the-art algorithms					
B10	Belhaiza (2010)	KTDHS12	Kritzinger et al. (2012)	RT10	Repoussis and Tarantilis (2010)
BDHMG08	Bräysy et al. (2008a)	MB07	Mester and Bräysy (2007)	RTBI10	Repoussis et al. (2010)
BER11	Bektas et al. (2011)	MCR12	Moccia et al. (2012)	RTI09a	Repoussis et al. (2009a)
BLR11	Balseiro et al. (2011)	NB09	Nagata and Bräysy (2009)	RTI09b	Repoussis et al. (2009b)
BPDRT09	Bräysy et al. (2009)	NBD10	Nagata et al. (2010)	S12	Subramanian (2012)
CM12	Cordeau and M. (2012)	NPW10	Ngueveu et al. (2010)	SDBOF10	Subramanian et al. (2010)
F10	Figliozzi (2010)	P09	Prins (2009)	SPUO12	Subramanian et al. (2012)
FEL07	Fu et al. (2007)	PBDH08	Polacek et al. (2008)	XZKX12	Xiao et al. (2012)
GA09	Gajpal and Abad (2009)	PDDR10	Prescott-Gagnon et al. (2010)	ZTK10	Zachariadis et al. (2010)
GG11	Groër and Golden (2011)	PR07	Pisinger and Ropke (2007)	ZK10	Zachariadis and Kiranoudis (2010)
HDH09	Hemmelmayr et al. (2009)	PR08	Pirkwieser and Raidl (2008)	ZK11	Zachariadis and Kiranoudis (2011)
ISW09	Imran et al. (2009)	RL12	Ribeiro and Laporte (2012)	ZK12	Zachariadis and Kiranoudis (2012)

General Conclusions and Research Perspectives

- **A unified hybrid genetic search**
  - Using a local-search framework which is generic and computationally efficient.
  - With generalized solution representation, Split procedure, genetic operators (Crossover) and population management methods.
  - State-of-the-art results when compared to each problem-tailored method for 26 VRP variants.

- **Generality does not necessarily alter performance for the considered classes of problems.**

General Conclusions and Research Perspectives

- **Some perspectives – on UHGS :**
  - Extend the range of problems (especially SEQ attributes, stochastic and multi-objective settings)
  - Use UHGS to conduct experiments on metaheuristic strategies on a wide range of VRPs
  - Further study of the combinatorial aspect of attributes relatively to UHGS operators.

- **Some perspectives – on VRP metaheuristics in general :**
  - Identify some “good” search spaces for broad MAVRP classes, and compound neighborhoods.
  - Diversity management and definition of better population-diversity metrics and distances
  - More intelligent pruning procedures.
  - Better exploiting the search history, and profiting for the very particular structure of MAVRP search spaces.
  - Finding good and simple hybridizations between classic methods.

Thank You

Thank you for your attention

- <u>For further reading , Survey on MAVRPs:</u>
  Vidal T., Crainic T.G., Gendreau M., Prins C. *Heuristics for Multi-Attribute Vehicle Routing Problems: A Survey and Synthesis* (2012), Tech. Rep. CIRRELT 2012-05.

- <u>Unified Hybrid Genetic Search (UHGS):</u>
  Vidal, T., Crainic, T. G., Gendreau, M., Lahrichi, N., & Rei, W. (2012*).* A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research*, 60(3), 611–624.

  Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2012). *A Unified Solution Framework for Multi-Attribute Vehicle Routing Problems*, Tech. Rep. CIRRELT 2012-23.