

Multi-Attribute Vehicle Routing Problems (Part II)

→ Unified methods and application cases

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Join Work With

- **A large part of this content derives from my Ph.D thesis with**
 - Teodor Gabriel Crainic – UQAM, Montréal
 - Michel Gendreau – Polytechnique, Montréal
 - Christian Prins – UTT, Troyes, France

Join Work With

□ Plus recent and current works on specific vehicle routing problems:

❖ **Prize-collecting VRP**

- Nelson Maculan – UFRJ
- Puca Huachi Vaz Penna – UFF
- Luis Satoru Ochi – UFF

❖ **Heterogeneous VRP**

- Puca Huachi Penna – UFF
- Luis Satoru Ochi – UFF

❖ **Clustered VRP**

- Maria Battarra – U. Southampton.
- Gunes Erdogan – U. Southampton.
- Anand Subramanian – UFPB

❖ **Online/Stochastic VRP**

- Patrick Jaillet -- MIT
- Richard Hartl – U. Vienna

❖ **Vehicle Routing and Truck Driver Scheduling Problem:**

- Asvin Goel – Jacobs University, Bremen

❖ **Workover Rig Routing Problem:**

- G. Ribeiro, B. Vieira – UFRJ
- G. Desaulniers, J. Desrosiers – U. Montréal

❖ **Pollution Routing Problem:**

- A. Subramanian – UFPB
- R. Kramer – UFPB

Presentation outline

- PART 1)**
 - ❑ **I) Vehicle Routing Problem, and attributes.**
 - ❑ **II) Classic Heuristics and metaheuristics for vehicle routing**
 - ❑ **III) An analysis of some winning strategies**
- PART 2)**
 - ❑ **IV) A new general-purpose solution approach**
 - **Attribute-based modular design**
 - **Unified Local Search**
 - **Unified Hybrid Genetic Search**
 - **Computational Experiments**
 - ❑ **V) Some application cases**

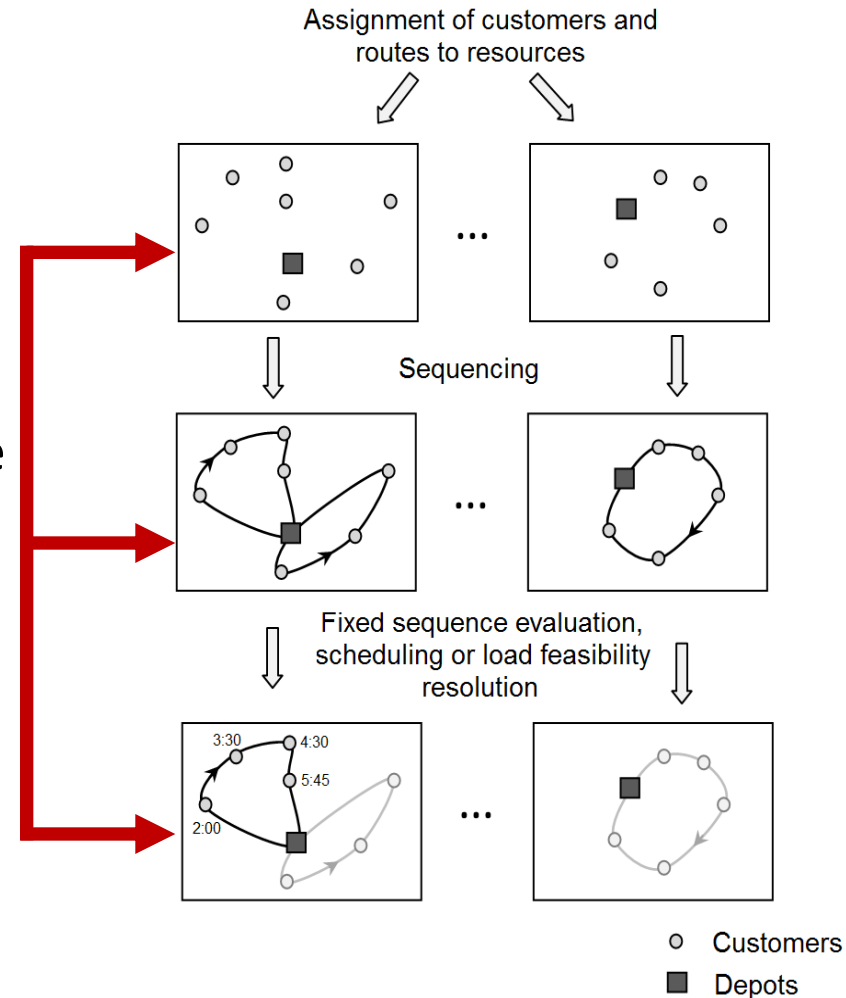
Attribute-based modular design

- ❑ **Contributing with a new general-purpose method**, which exploits the successful concepts identified in this survey as well as the structure of the attributes.

- ❑ Additional challenge of this work, **designing a unified method : achieving generality & efficiency**
 - Drawback of current unified VRP methods: dealing with a rich VRP model that includes several MAVRP as special cases → Still accounting for non-activated attributes
 - Still need to address the problem → but relegating problem-specificities to small modular components
 - Each separate MAVRP shall be still addressed with state-of-the-art solution evaluation and search procedures

Attribute-based modular design

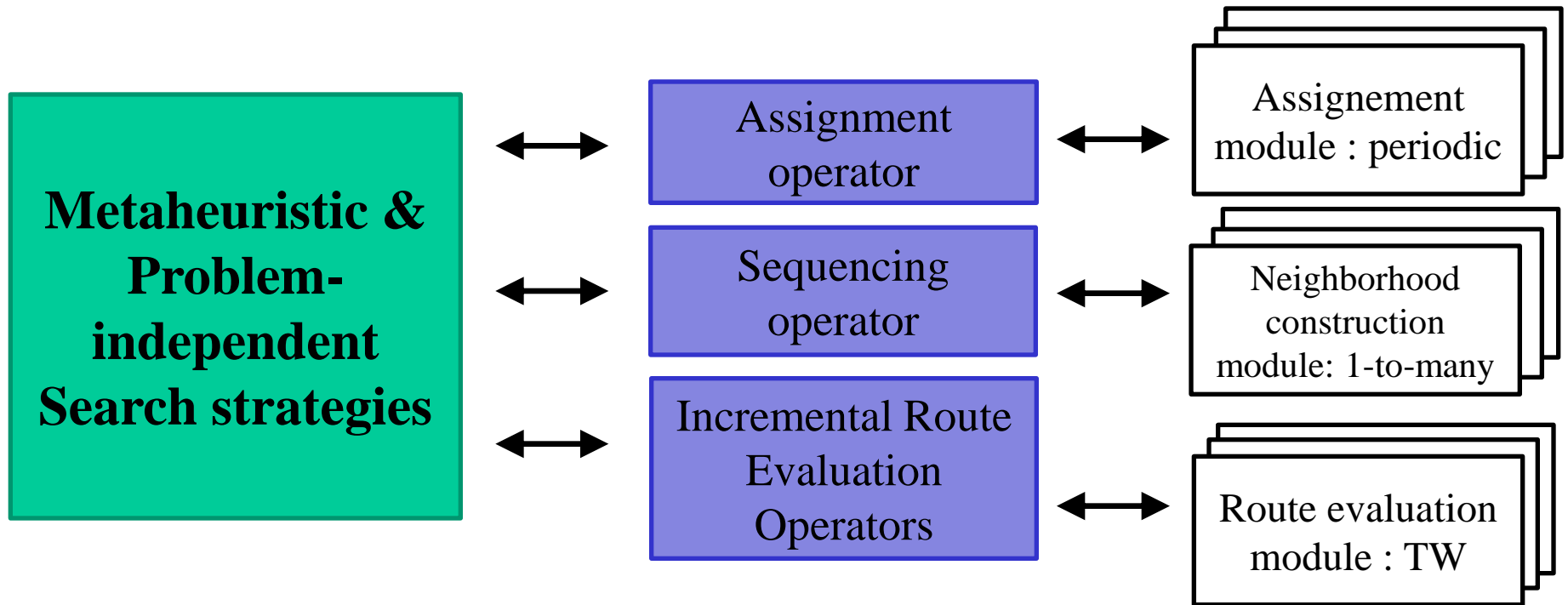
- ❑ Back to the method-oriented attribute classification:
- ❑ **ASSIGN ATTRIBUTES:** impacting the assignment of customers and routes
- ❑ **SEQ ATTRIBUTES:** impacting the nature of the network and the sequences
- ❑ **EVAL ATTRIBUTES:** impacting the evaluation of fixed routes



Attribute-based modular design

□ Proposed unified framework:

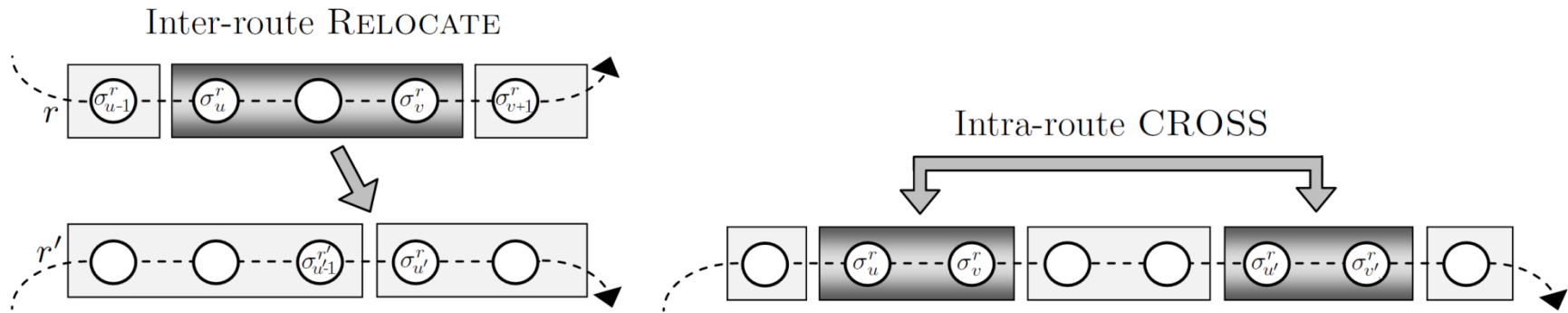
- Relying on assignment, sequencing & route evaluation operators → implemented in a generic way, based on a library of attribute-specific modules



An efficient and unified local search for MAVRPs

□ Route Evaluation Operators based on re-optimization

- Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
- The same subsequences appear many times during different moves



- Data preprocessing on sub-sequences to speed up the search (Savelsbergh 1985,1992 ...)
- The route evaluation modules must allow for such preprocessing.

An efficient and unified local search for MAVRPs

□ Route Evaluation Operators based on re-optimization

- Main Property : Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
- Hence, to manage and exploit information on subsequences, five families of route evaluation operators are used :

Operators for data construction:

INIT(σ)	Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit.
FORW(σ)	Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence σ and vertex v_i .
BACK(σ)	Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex v_i and sub-sequence σ .

Operators for route evaluations:

EVAL2(σ_1, σ_2)	Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \sigma_2$.
EVALN($\sigma_1, \dots, \sigma_n$)	Evaluate the cost and feasibility of the combined sequence $\sigma_1 \oplus \dots \oplus \sigma_n$.

Route evaluation operators examples

- **Example 1)** Route evaluation operators for **distance and capacity constraints**

What is managed ? → Partial loads $L(\sigma)$ and distance $D(\sigma)$

Init → For a sequence σ_0 with a single visit v_i , $L(\sigma_0) = q_i$ and $D(\sigma_0) = 0$

Forw and Back → increment $L(\sigma)$ and $D(\sigma)$

Eval → compute the data by induction on the concatenation operator

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2)$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

Route evaluation operators examples

- **Example 2)** Route evaluation operators for **cumulated arrival time objectives**

What is managed ? → Travel time $D(\sigma)$, Cumulated arrival time $C(\sigma)$, Delay Cost $W(\sigma)$ associated to one unit of delay in starting time

Init → For a sequence σ_0 with a single visit v_i , $D(\sigma_0) = 0$ and $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ if v_i is a customer, and $W(\sigma_0) = 0$ if v_i is a depot visit.

Forw & Back & Eval → induction on the concatenation operator:

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2)$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2)$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2)$$

Route evaluation operators examples

- **Example 3)** Route evaluation operators for **time windows (and route duration constraints)**

What is managed ? → Travel time and service time $T(\sigma)$, earliest feasible completion time $E(\sigma)$, latest feasible starting date $L(\sigma)$, statement of feasibility $F(\sigma)$.

Init → For a sequence σ_0 with a single visit v_i , $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = \text{true}$.

Forw & Back & Eval → induction on the concatenation operator:

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2)$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\}$$

$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2))$$

Route evaluation operators examples

- **Example 4)** Route evaluation operators for **soft and general time windows**

What is managed ? → Minimum cost $F(\sigma)(t)$ to process the sequence σ while starting the last service before time t , minimum cost $B(\sigma)(t)$ to process the sequence σ after time t .

Init → For a sequence σ_0 with a single visit v_i characterized by a service cost function $c_i(t)$, $F(\sigma_0)(t) = \min_{(x \leq t)} c_i(x)$ and $B(\sigma_0)(t) = \min_{(x \geq t)} c_i(x)$.

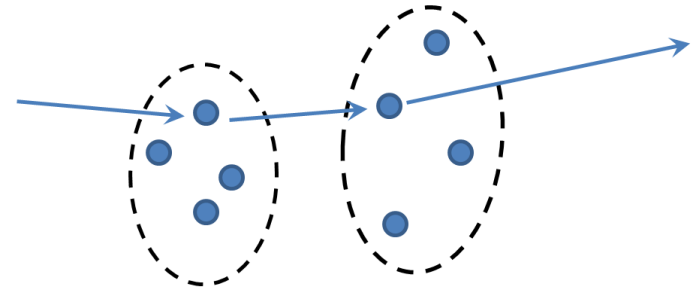
Forw & Back →

$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\}$$
$$B(v_i \oplus \sigma)(t) = \min_{x \geq t} \{c_i(t) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\}$$

Eval 2 → $Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\}$

Route evaluation operators examples

- **Example 5)** Route evaluation operators for the **generalized VRP** :



What is managed ? → The shortest path $S(\sigma)[i,j]$ inside the sequence σ starting at the location i of the starting group and finishing at location j of the ending group.

Init → For a sequence σ_0 with a single visit v_i , $S(\sigma)[i,j] = +\infty$ if $i \neq j$, and $S(\sigma)[i,i] = 0$.

Forw & Back & Eval → induction on the concatenation operator:

$$S(\sigma_1 \oplus \sigma_2)[i, j] = \min_{1 \leq x \leq \lambda_{\sigma_1}(|\sigma_1|), 1 \leq y \leq \lambda_{\sigma_2}(1)} S(\sigma_1)[i, x] + d_{xy} + S(\sigma_2)[y, j]$$
$$\forall i \in \{1, \dots, \lambda_{\sigma_1}(1)\}, \forall j \in \{1, \dots, \lambda_{\sigma_2}(|\sigma_2|)\}$$

An efficient and unified local search for MAVRPs

- Generic local-search based on route evaluation operators

Algorithm 1 Unified local search based on route evaluation operators

- 1: Detect the good combination of evaluation operators relatively to the problem attributes
- 2: Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
- 3: **while** some improving moves exist in the neighborhood \mathcal{N} **do**
- 4: **for** each move μ_i in \mathcal{N} **do**
- 5: **for** each route r_j^μ produced by the move **do**
- 6: Determine the k sub-sequences $[\sigma_1, \dots, \sigma_k]$ that are concatenated to produce r_j^μ
- 7: **if** $k = 2$, then $\text{NEWCOST}(r) = \text{EVAL2}(\sigma_1, \sigma_2)$
- 8: **else if** $k > 2$, then $\text{NEWCOST}(r) = \text{EVALN}(\sigma_1, \dots, \sigma_k)$
- 9: **if** $\text{ACCEPTCRITERIA}(\mu_i)$ **then** perform the move μ and update the re-optimization data on for each route r_j^μ using the INIT, FORW and BACK operators.

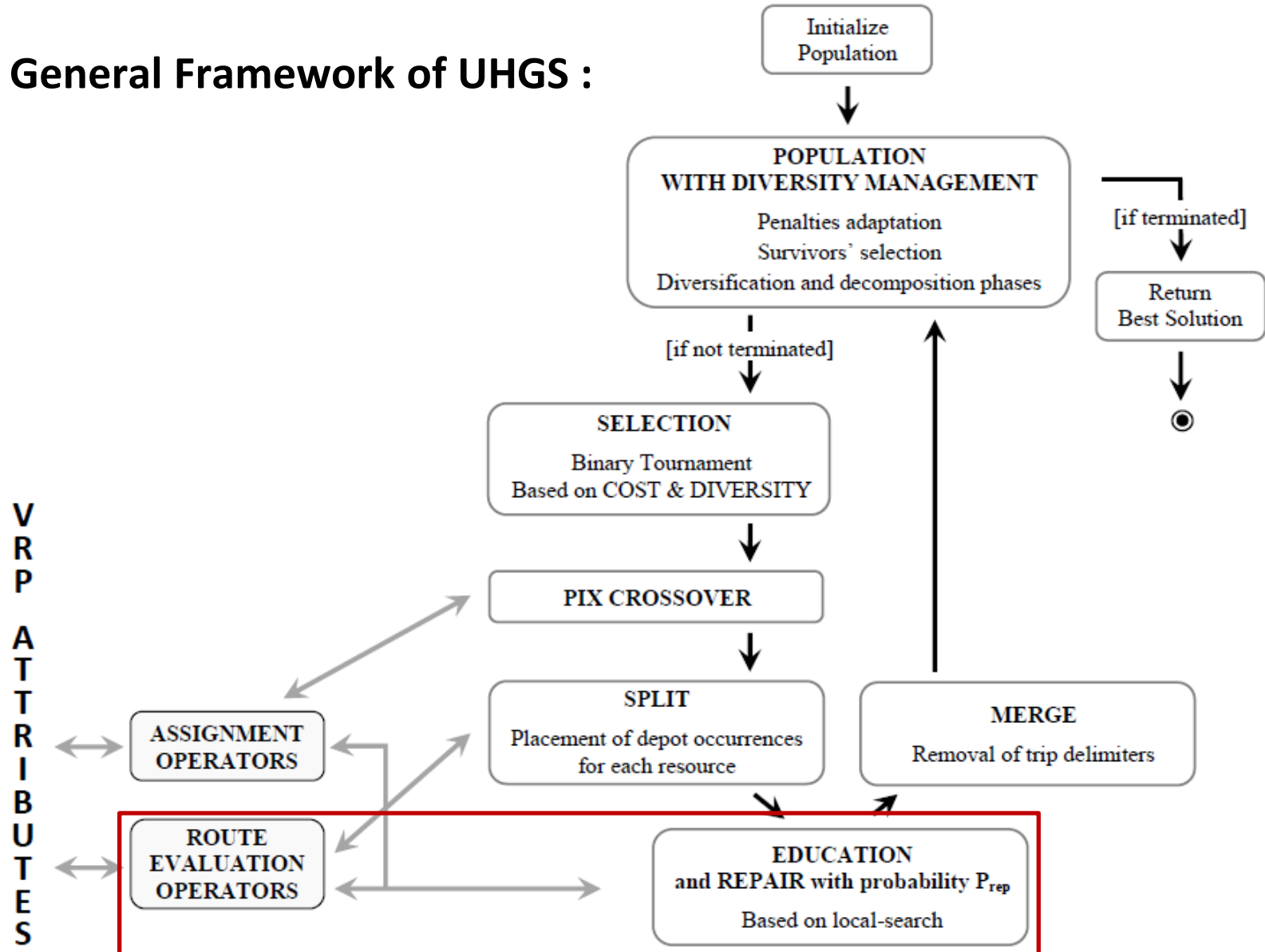
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- Can serve as the basis to build any neighborhood-based unified solver based on VNS, Tabu, ILS for MAVRPs with EVAL attributes.
 - Going one step further, designing a unified hybrid GA.

A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- UHGS = Classic GA framework + 4 main ingredients (Vidal et al. 2010)
 - Management of penalized infeasible solutions in two subpopulations
 - High-performance local search-based *Education* procedure
 - Solution Representation *without trip delimiters*
 - **Diversity & Cost objective for individuals evaluations**

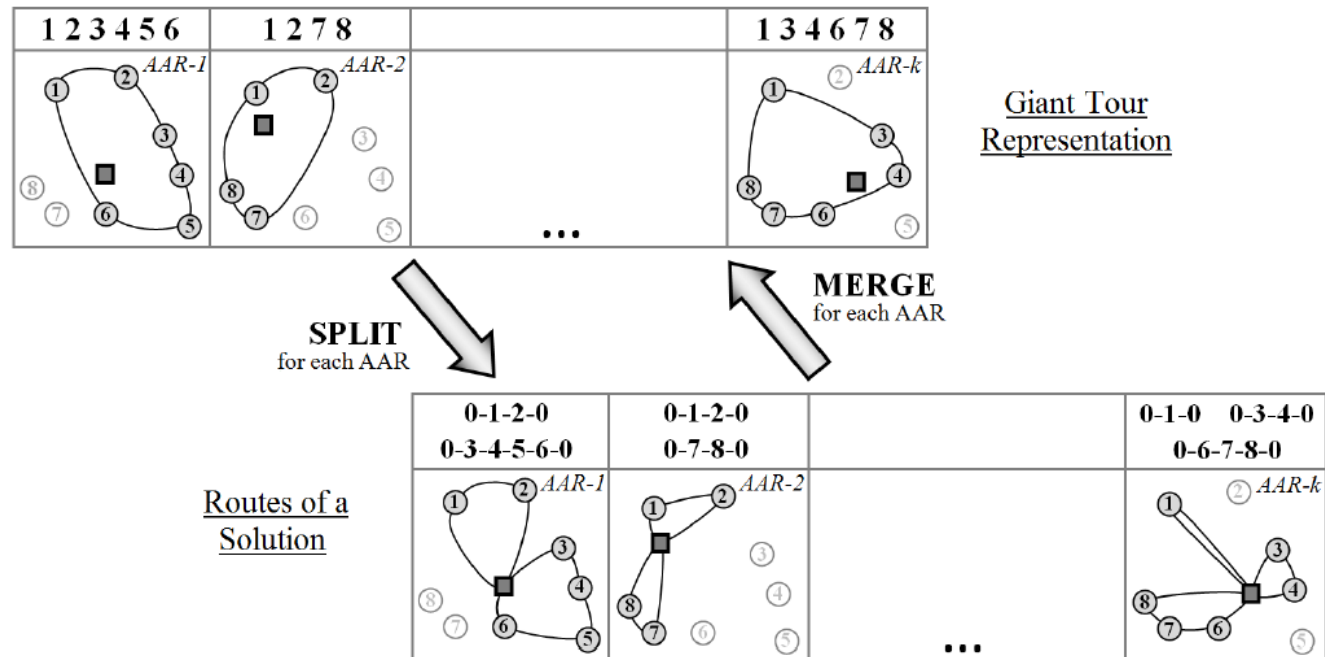
A Unified Hybrid Genetic Search (UHGS) for MAVRPs

□ General Framework of UHGS :



Unified Solution Representation and Split

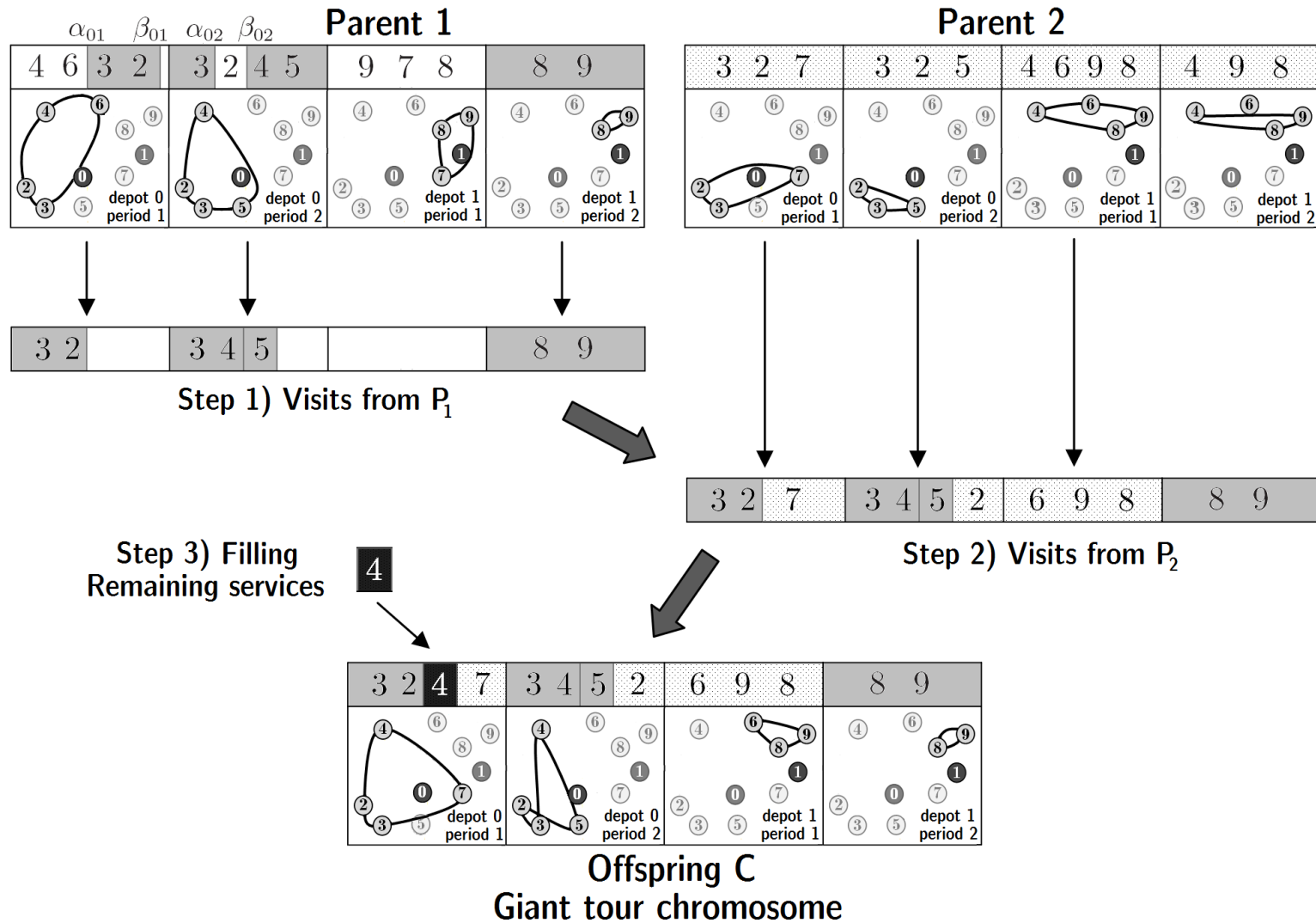
- ❑ Now dealing with MAVRPs with both ASSIGN and EVAL attributes: Assignment of customer services to some ASSIGN attributes resources (AARs) + separate optimization of routes for each AARs.
 - Solution representation is designed accordingly.
 - Furthermore, representation without trip delimiters for each AAR.



Unified Crossover Operator

- ❑ 4 phases **Assignment and Insertion Crossover (AIX)**, to produce a single offspring C from two parents P1 and P2.
- ❑ Step 1) Choose for each AAR whether the genetic material of P1, P2, or both parents is inherited.
- ❑ Step 2) Fully transmit the selected material from P1
- ❑ Step 3) Complete with the selected material from P2, check at each step with an Assignment module whether the inheritance respects the ASSIGN attributes specifications.
- ❑ Step 4) Perform a best insertion of missing visits.

Unified Crossover Operator



Unified Education Procedure

- ❑ Based on the previously described Unified Local Search to perform route improvement (RI) on separate AAR.
 - Using CROSS, I-CROSS, Relocate, 2-Opt* and 2-Opt neighborhoods
 - Pruning procedures (granular search)
 - Hybrid acceptance strategy (intermediate between first improvement and best improvement)

- ❑ Combined with an assignment-improvement (AI) procedure to re-assign customer visits into different resources and routes.

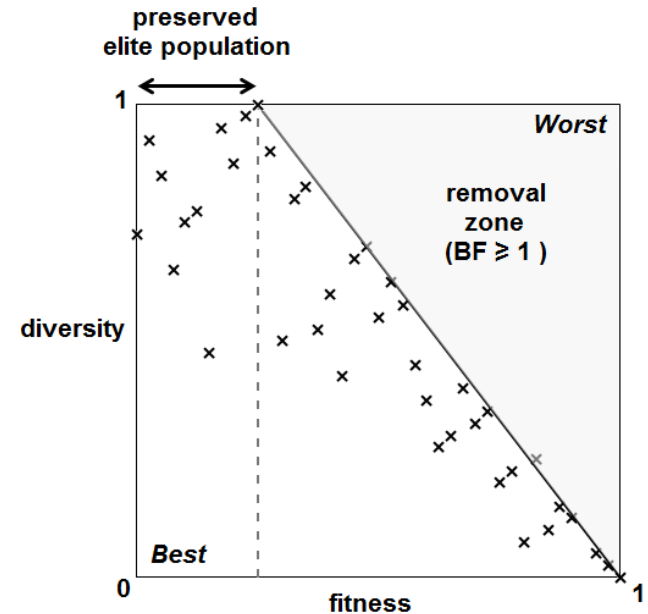
- ❑ These two procedures are called in the sequence RI-AI-RI.

Population management and search guidance

- **Biased Fitness** is a tradeoff between ranks in terms of **solution penalized cost** $cost(I)$, and **contribution to the diversity** $dc(I)$, measured as a distance to others individuals in the population.

$$BF(I) = fit(I) + \left(1 - \frac{nbElit}{nbIndiv - 1}\right) \times dc(I)$$

- Used during selection of the parents
 - Balancing strength with innovation during reproduction, and thus favoring exploration of the search space.
- and during selection of the survivors:
 - Removing the individual I with worst $BF(I)$ also guarantees some elitism in terms of solution value.



Comparison with problem-tailored state-of-the-art methods

- ❑ Extensive computational experiments on 26 structurally different VRP variants and 39 sets of benchmark instances.
- ❑ Comparing UHGS with the best problem-tailored method for each benchmark.
- ❑ In the following, we indicate for each method
 - % Gap to the BKS of an average run (out of 10 for UHGS).
 - % Gap to the BKS of a best run (out of 10 for UHGS).
 - Computational effort (total work time) for an average run
 - Type of processor used.

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
CVRP	CMT79	[50,199]	C	GG11:	—	+0.03%	8×2.38	8×Xe 2.3G
				MB07:	+0.03%	—	2.80	P-IV 2.8G
				UHGS*:	+0.02%	+0.00%	11.90	Opt 2.4G
CVRP	GWKC98	[200,483]	C	GG11:	—	+0.29%	8×5	8×Xe 2.3G
				NB09:	+0.27%	+0.16%	21.51	Opt 2.4G
				UHGS*:	+0.15%	+0.02%	71.41	Opt 2.4G
VRPB	GJ89	[25,200]	C	ZK12:	+0.38%	+0.00%	1.09	T5500 1.67G
				GA09:	+0.09%	+0.00%	1.13	Xe 2.4G
				UHGS:	+0.01%	+0.00%	0.99	Opt 2.4G
CCVRP	CMT79	[50,199]	C	NPW10:	+0.74%	+0.28%	5.20	Core2 2G
				RL12:	+0.37%	+0.07%	2.69	Core2 2G
				UHGS:	+0.01%	-0.01%	1.42	Opt 2.2G
CCVRP	GWKC98	[200,483]	C	NPW10:	+2.03%	+1.38%	94.13	Core2 2G
				RL12:	+0.34%	+0.07%	21.11	Core2 2G
				UHGS:	-0.14%	-0.23%	17.16	Opt 2.2G
VRPSDP	SN99	[50,199]	C	SDBOF10:	+0.16%	+0.00%	256×0.37	256×Xe 2.67G
				ZTK10:	—	+0.11%	—	T5500 1.66G
				UHGS:	+0.01%	+0.00%	2.79	Opt 2.4G
VRPSDP	MG06	[100,400]	C	SDBOF10:	+0.30%	+0.17%	256×3.11	256×Xe 2.67G
				UHGS:	+0.20%	+0.07%	12.00	Opt 2.4G
				S12 :	+0.08%	+0.00%	7.23	I7 2.93G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
VFMP-F	G84	[20,100]	C	ISW09:	—	+0.07%	8.34	P-M 1.7G
				SPUO12:	+0.12%	+0.01%	0.15	I7 2.93G
				UHGS:	+0.04%	+0.01%	1.13	Opt 2.4G
VFMP-V	G84	[20,100]	C	ISW09:	—	+0.02%	8.85	P-M 1.7G
				SPUO12:	+0.17%	+0.00%	0.06	I7 2.93G
				UHGS:	+0.03%	+0.00%	0.85	Opt 2.4G
VFMP-FV	G84	[20,100]	C	P09:	—	+0.02%	0.39	P4M 1.8G
				UHGS:	+0.01%	+0.00%	0.99	Opt 2.4G
				SPUO12:	+0.01%	+0.00%	0.13	I7 2.93G
LDVRP	CMT79	[50,199]	C	XZKX12:	+0.48%	+0.00%	1.3	NC 1.6G
				UHGS:	-0.28%	-0.33%	2.34	Opt 2.2G
LDVRP	GWKC98	[200,483]	C	XZKX12:	+0.66%	+0.00%	3.3	NC 1.6G
				UHGS:	-1.38%	-1.52%	23.81	Opt 2.2G
PVRP	CGL97	[50,417]	C	HDH09:	+1.69%	+0.28%	3.09	P-IV 3.2G
				UHGS*:	+0.43%	+0.02%	6.78	Opt 2.4G
				CM12:	+0.24%	+0.06%	64×3.55	64×Xe 3G
MDVRP	CGL97	[50,288]	C	CM12:	+0.09%	+0.03%	64×3.28	64×Xe 3G
				S12:	+0.07%	+0.02%	11.81	I7 2.93G
				UHGS*:	+0.08%	+0.00%	5.17	Opt 2.4G
GVRP	B11	[16,262]	C	BER11:	+0.06%	—	0.01	Opt 2.4G
				MCR12:	+0.11%	—	0.34	Duo 1.83G
				UHGS:	+0.00%	-0.01%	1.53	Opt 2.4G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
OVRP	CMT79 &F94	[50,199]	F/C	RTBI10:	0%/+0.32%	—	9.54	P-IV 2.8G
				S12:	—/+0.16%	0%/+0.00%	2.39	I7 2.93G
				UHGS:	0%/+0.11%	0%/+0.00%	1.97	Opt 2.4G
OVRP	GWKC98	[200,480]	F/C	ZK10:	0%/+0.39%	0%/+0.21%	14.79	T5500 1.66G
				S12:	0%/+0.13%	0%/+0.00%	64.07	I7 2.93G
				UHGS:	0%/-0.11%	0%/-0.19%	16.82	Opt 2.4G
VRPTW	SD88	100	F/C	RTI09:	0%/+0.11%	0%/+0.04%	17.9	Opt 2.3G
				UHGS*:	0%/+0.04%	0%/+0.01%	2.68	Xe 2.93G
				NBD10:	0%/+0.02%	0%/+0.00%	5.0	Opt 2.4G
VRPTW	HG99	[200,1000]	F/C	RTI09b:	—	+0.16%/+3.36%	270	Opt 2.3G
				NBD10:	+0.20%/+0.42%	+0.10%/+0.27%	21,7	Opt 2.4G
				UHGS*:	+0.18%/+0.11%	+0.08%/-0.10%	141	Xe 2.93G
OVRPTW	SD88	100	F/C	RTI09a:	+0.89%/+0.42%	0%/+0.24%	10.0	P-IV 3.0G
				KTDHS12:	0%/+0.79%	0%/+0.18%	10.0	Xe 2.67G
				UHGS:	+0.09%/-0.10%	0%/-0.10%	5.27	Opt 2.2G
TDVRPTW	SD88	100	F/C	KTDHS12:	+2.25%	0%	10.0	Xe 2.67G
				UHGS:	-3.31%	-3.68%	21.94	Opt 2.2G
VFMPTW	LS99	100	D	BDHMG08:	—	+0.59%	10.15	Ath 2.6G
				RT10:	+0.22%	—	16.67	P-IV 3.4G
				UHGS:	-0.15%	-0.24%	4.58	Opt 2.2G
VFMPTW	LS99	100	C	BDHMG08:	—	+0.25%	3.55	Ath 2.6G
				BPDRT09:	—	+0.17%	0.06	Duo 2.4G
				UHGS:	-0.38%	-0.49%	4.82	Opt 2.2G

Comparison with problem-tailored state-of-the-art methods

Variant	Bench.	n	Obj.	State-of-the-art methods				
				Author	Avg.%	Best%	T(min)	CPU
PVRPTW	CL01	[48,288]	C	PR08:	—	+1.75%	—	Opt 2.2G
				CM12:	+1.10%	+0.76%	64×11.3	64×Xe 3G
				UHGS*:	+0.63%	+0.22%	32.7	Xe 2.93G
MDVRPTW	CL01	[48,288]	C	PBDH08:	—	+1.37%	147	P-IV 3.6G
				CM12:	+0.36%	+0.15%	64×6.57	64×Xe 3G
				UHGS*:	+0.19%	+0.03%	6.49	Xe 2.93G
SDVRPTW	CL01	[48,288]	C	B10:	+2.23%	—	2.94	Qd 2.67G
				CM12:	+0.62%	+0.36%	64×5.60	64×Xe 3G
				UHGS*:	+0.36%	+0.10%	5.48	Xe 2.93G
VRPSTW (type 1, $\alpha=100$)	SD88	100	F/TW/C	F10:	0%	—	9.69	P-M 1.6G
				UHGS:	-3.05%	-4.42%	18.62	Opt 2.2G
VRPSTW (type 1, $\alpha=1$)	SD88	100	C+TW	KTDHS12:	+0.62%	+0.00%	10.0	Xe 2.67G
				UHGS:	-0.13%	-0.18%	5.82	Opt 2.2G
VRPSTW (type 2, $\alpha=100$)	SD88	100	F/TW/C	FEL07:	0%	—	5.98	P-II 600M
				UHGS:	-13.91%	-13.91%	41.16	Opt 2.2G
VRPSTW (type 2, $\alpha=1$)	SD88	100	C+TW	UHGS:	+0.26%	0%	29.96	Opt 2.2G
MDPVRPTW	New	[48,288]	C	UHGS:	+0.77%	0%	16.89	Opt 2.2G
VRTDSP (E.U. rules)	G09	100	F/C	PDDR10:	0%/0%	0%/0%	88	Opt 2.3G
				UHGS*:	-0.56%/-0.54%	-0.85%/-0.70%	228	Xe 2.93G

Comparison with problem-tailored state-of-the-art methods

List of acronyms for benchmarks

B11	Bektas et al. (2011)	G84	Golden (1984)	LS99	Liu and Shen (1999)
CGL97	Cordeau et al. (1997)	G09	Goel (2009)	MG06	Montané and Galvão (2006)
CL01	Cordeau and Laporte (2001)	GH99	Gehring and Homberger (1999)	SD88	Solomon and Desrosiers (1988)
CMT79	Christofides et al. (1979)	GJ89	Goetschalckx and J.-B. (1989)	SN99	Salhi and Nagy (1999)
F94	Fisher (1994)	GWKC98	Golden et al. (1998)		

List of acronyms for state-of-the-art algorithms

B10	Belhaiza (2010)	KTDHS12	Kritzing et al. (2012)	RT10	Repoussis and Tarantilis (2010)
BDHMG08	Bräysy et al. (2008a)	MB07	Mester and Bräysy (2007)	RTBI10	Repoussis et al. (2010)
BER11	Bektas et al. (2011)	MCR12	Moccia et al. (2012)	RTI09a	Repoussis et al. (2009a)
BLR11	Balseiro et al. (2011)	NB09	Nagata and Bräysy (2009)	RTI09b	Repoussis et al. (2009b)
BPDRT09	Bräysy et al. (2009)	NBD10	Nagata et al. (2010)	S12	Subramanian (2012)
CM12	Cordeau and M. (2012)	NPW10	Ngueveu et al. (2010)	SDBOF10	Subramanian et al. (2010)
F10	Figliozzi (2010)	P09	Prins (2009)	SPUO12	Subramanian et al. (2012)
FEL07	Fu et al. (2007)	PBDH08	Polacek et al. (2008)	XZKX12	Xiao et al. (2012)
GA09	Gajpal and Abad (2009)	PDDR10	Prescott-Gagnon et al. (2010)	ZTK10	Zachariadis et al. (2010)
GG11	Groër and Golden (2011)	PR07	Pisinger and Ropke (2007)	ZK10	Zachariadis and Kiranoudis (2010)
HDH09	Hemmelmayr et al. (2009)	PR08	Pirkwieser and Raidl (2008)	ZK11	Zachariadis and Kiranoudis (2011)
ISW09	Imran et al. (2009)	RL12	Ribeiro and Laporte (2012)	ZK12	Zachariadis and Kiranoudis (2012)

Conclusions on UHGS

- ❑ **A unified hybrid genetic search**
 - Using a local-search framework which is generic and computationally efficient.
 - With generalized solution representation, Split procedure, genetic operators (Crossover) and population management methods.
 - State-of-the-art results when compared to each problem-tailored method for 26 VRP variants.

- ❑ **Generality does not necessarily alter performance for the considered classes of problems.**

Conclusions on UHGS

□ Some perspectives – on UHGS :

- Extend the range of problems (especially SEQ attributes, stochastic and multi-objective settings)
- Use UHGS to conduct experiments on metaheuristic strategies on a wide range of VRPs
- Further study of the combinatorial aspect of attributes relatively to UHGS operators.

- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). A unified solution framework for multi-attribute vehicle routing problems. *European Journal of Operational Research, Forthcoming*.

Presentation outline

- PART 1)**
 - ❑ **I) Vehicle Routing Problem, and attributes.**
 - ❑ **II) Classic Heuristics and metaheuristics for vehicle routing**
 - ❑ **III) An analysis of some winning strategies**
- PART 2)**
 - ❑ **IV) A new general-purpose solution approach**
 - **Attribute-based modular design**
 - **Unified Local Search**
 - **Unified Hybrid Genetic Search**
 - **Computational Experiments**
 - ❑ **V) Some application cases**

Multi-attribute VRPs : some application cases

- ❑ Application cases originating from academic collaborations and personal projects.
- ❑ **Analysis of hours of service regulations around the world, considering optimized solutions**

<< SOME PREZI SLIDES >>

- ❑ Goel, A., & Vidal, T. (2013). Hours of service regulations in road freight transport: an optimization-based international assessment. *Transportation Science, Articles in Advance*.

□ Workover Rig Routing Problem

- One specialist (and author of the next 4 slides) is Glaydston Ribeiro
- We recently studied some advanced heuristics in
Ribeiro, G. M., Desaulniers, G., Desrosiers, J., Vidal, T., & Vieira, B. S.
(2013). Efficient Heuristics for the Workover Rig Routing Problem with
a Heterogeneous Fleet and a Finite Horizon. Submitted. Les Cahiers du
GERAD. Université de Montréal. G-2013-47

Multi-attribute VRPs : some application cases

❑ Workover Rig Routing Problem

- ❑ Onshore oil wells in Southeast and Northeast regions in Brazil use artificial lift methods
- ❑ Each oil well has a specialized and expensive equipment, which operates under hard conditions and for a long time
- ❑ Maintenance services are needed after some time : cleaning, reinstatement, stimulation
- ❑ In general, these maintenance services are performed by a short number of expensive workover rigs which are transported by trucks. Some rigs may have specific use, and not all rigs can provide all services.

Multi-attribute VRPs : some application cases

□ Workover Rig Routing Problem



Workover rig performing a maintenance service (Aloise et al., 2006)

Transportation of a workover rig (Aloise et al., 2006)



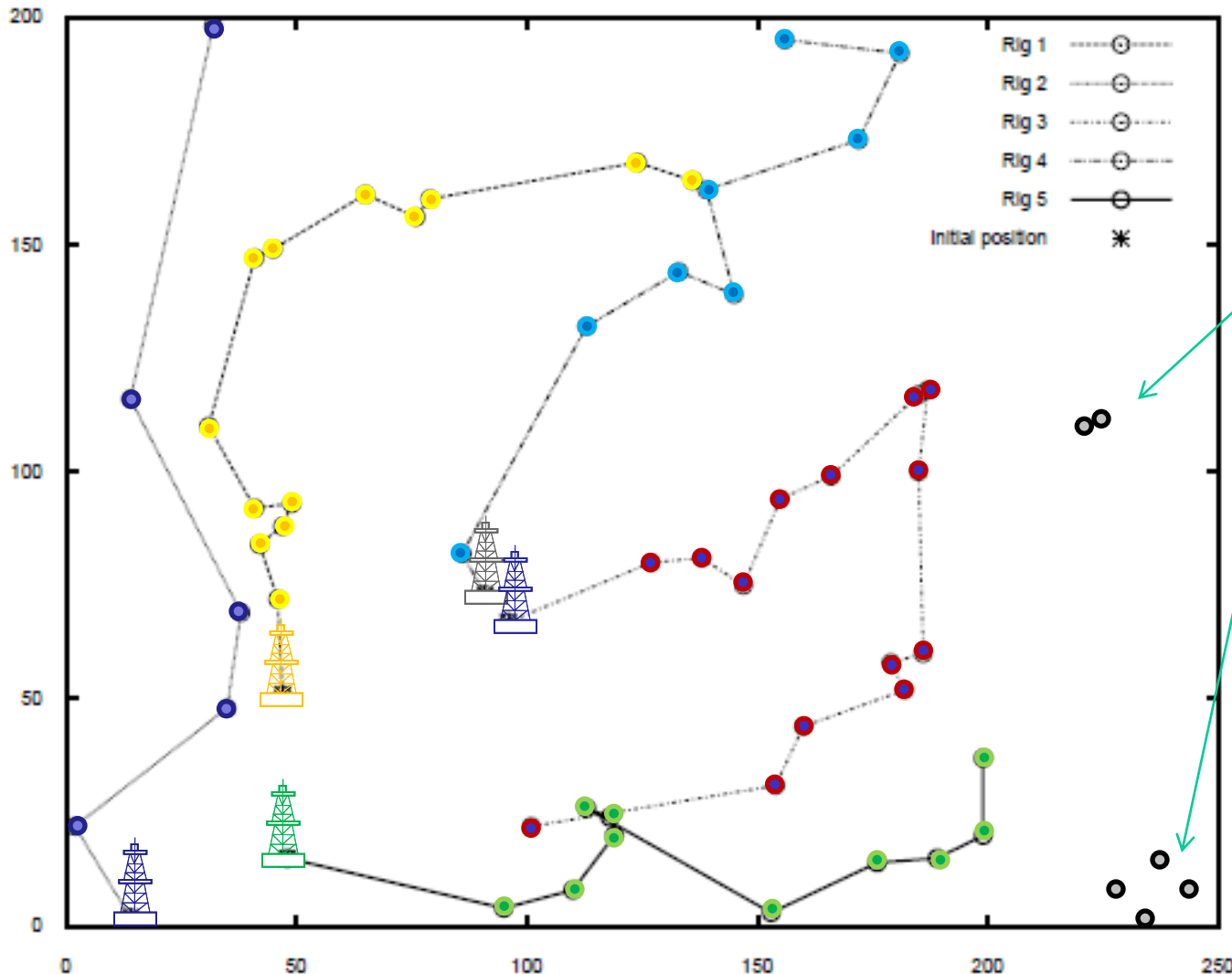
Multi-attribute VRPs : some application cases

❑ Workover Rig Routing Problem

- ❑ When a well requires a maintenance, its production is reduced or stopped for safety reasons.
 - A rig must be sent to perform the required service in order to reestablish its production.
 - The production loss of a well is obtained as its regular daily flow rate, multiplied by the number of days during which it does not operate.
- ❑ The rigs are located at different positions and may take considerable time to reach the wells.
- ❑ In our case:
 - all services must be performed within a planning horizon;
 - each well is serviced at most once;
 - total production loss must be minimized.

Multi-attribute VRPs : some application cases

□ Workover Rig Routing Problem



Example of a solution

Unserviced wells.

Multi-attribute VRPs : some application cases

- ❑ Workover Rig Routing Problem

- ❑ Developed four heuristics for this problem
 - Variable Neighborhood Search
 - Heuristic Branch-and-price-and-cut
 - Adaptive Large Neighborhood Search
 - Hybrid Genetic Algorithm → not using giant-tour solution representation in this case

- ❑ Experimentations on 80 instances from Ribeiro et al. (2012).
 - 100-200 wells,
 - 5-10 rigs
 - $H = 200-300$ time increments (equiv 14-21 days).

Multi-attribute VRPs : some application cases

□ Workover Rig Routing Problem

Combination	VNS			BPC			ALNS			HGA		
	Best (No.)	Time (s)	Dev (%)	Best (No.)	Time (s)	Dev (%)	Best (No.)	Time (s)	Dev (%)	Best (No.)	Time (s)	Dev (%)
100/ 5/200	0	6.5	6.96	10	0.5	0.00	10	6.8	0.01	10	5.7	0.00
100/10/200	0	9.2	6.39	9	2.3	0.00	9	9.3	0.04	10	4.8	0.01
100/ 5/300	0	9.4	7.88	4	30.7	0.08	9	9.5	0.04	10	5.5	0.00
100/10/300	0	7.8	6.87	5	23.9	0.02	10	7.5	0.02	10	4.3	0.00
200/ 5/200	0	25.7	8.92	9	14.1	0.01	7	26.6	0.23	10	26.5	0.01
200/10/200	0	51.9	9.89	8	27.9	0.03	7	52.4	0.22	10	26.7	0.02
200/ 5/300	0	44.5	12.56	8	296.7	0.00	2	45.4	0.52	10	26.7	0.06
200/10/300	0	67.6	12.87	2	815.6	0.28	3	66.2	0.46	10	21.1	0.06

- All heuristics provide results of high quality in very short CPU time.
 - HGA seems to produce all best known solutions, but BCP may be faster for some smaller-size problems.

Conclusions, Guidelines and Perspectives

□ Recipe -- Solving large and complex multi-attribute VRP :

- 1) Analyze the structure, and identify attributes that are just a matter of separate route evaluations
- 2) Possibly Relax linking constraints
- 3) Create a purposeful local search
- 4) Add some simple metaheuristic strategies
- 5) Adjust the balance between exploration and intensification

Conclusions, Guidelines and Perspectives

- **Research Perspectives – on VRP metaheuristics in general :**
 - Identify some “good” search spaces for broad MAVRP classes, and compound neighborhoods.
 - Diversity management and definition of better population-diversity metrics and distances
 - More intelligent pruning procedures.
 - Better exploiting the search history, and profiting for the very particular structure of MAVRP search spaces.
 - Finding good and simple hybridizations between classic methods.

Thank you for your attention !

□ For further reading, and follow-up works:

- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). A Unified Solution Framework for Multi-Attribute Vehicle Routing Problems. *European Journal of Operational Research, Forthcoming*
- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. *Computers & Operations Research, 40*(1), 475–489.
- Ribeiro, G. M., Desaulniers, G., Desrosiers, J., Vidal, T., & Vieira, B. S. (2013). Efficient Heuristics for the Workover Rig Routing Problem with a Heterogeneous Fleet and a Finite Horizon. *Cahiers du GERAD*.
- Vidal, T., Crainic, T. G., Gendreau, M., Lahrichi, N., & Rei, W. (2012). A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research, 60*(3), 611–624.
- Vidal T., Crainic T.G., Gendreau M., Prins C. Heuristics for Multi-Attribute Vehicle Routing Problems: A Survey and Synthesis (2013). *European Journal of Operational Research, Forthcoming*
- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2012). Implicit Depot Assignments and Rotations in Vehicle Routing Heuristics. Submitted to *European Journal of Operational Research. Revised*.
- Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2012). A Unifying View on Timing Problems and Algorithms. Submitted to *Networks. Tech Rep CIRRELT-2011-43*.
- Goel, A., & Vidal, T. (2012). Hours of service regulations in road freight transport : an optimization-based international assessment. *Transportation Science*
- **Links to other technical reports, papers and slides can be found at <http://w1.cirreлт.ca/~vidalt/>**