A General-Purpose Heuristic for Multi-Attribute Vehicle Routing Problems

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Presentation outline

1. General-purpose solvers

2. Multi-attribute vehicle routing problems (MAVRPs)

3. An efficient and unified local search for MAVRPs
   a) Route evaluation operators
   b) Implementation for several attributes

4. A Unified Hybrid Genetic Search (UHGS) for MAVRPs
   a) General framework
   b) Unified solution representation and Split
   c) Generic implementation of other genetic operators

5. Computational experiments
1. General-purpose solvers

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5. Computational experiments
General-purpose solvers for combinatorial optimization

- Solvers that can address a wide range of problems without need for extensive adaptation or user expertise.

Necessary tools for the timely application of current optimization methods to industrial settings.

Examples of such solvers:

- Integer & constraint programming solvers
- Local search-based methods: "LocalSolver" (Benoist et al. 2011).
- Methods designed to address a large compound problem model.

Libraries of metaheuristic components and classes libraries:
Open BEAGLE (Gagné and Parizeau 2002), ParadisEO (Cahon et al. 2004)...
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Multi-attribute vehicle routing problems (MAVRPs)

- Classical “vehicle routing problem (VRP)”
  → wide range of exact and heuristic methods

- **Challenges** related to the resolution of VRP variants with additional attributes (multi-attribute VRPs, MAVRPs)
  - modeling the specificities of application cases, customers requirements, network and vehicle specificities, operators abilities...
  - Combining **several attributes** together can lead to highly complex **rich VRPs**.
  - Dramatic increase in the literature dedicated to specific VRP variants.
Multi-attribute vehicle routing problems (MAVRPs)

- Some unified algorithms reporting high quality solutions on several MAVRPs:

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<th>Type</th>
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<th>Acronym</th>
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<th>HYB</th>
<th>VNS</th>
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² Problem known as "Generalized Vehicle Routing Problem"

- Issues: dealing with a rich VRP model that includes several MAVRP as special cases → Still accounting for non-activated attributes
We classified attributes into three categories related to their impact on VRP resolution methods:

- **Practical Setting**
  - Characteristics
    - NETWORK
    - PRODUCTS
    - VEHICLES
    - DEMANDS
    - CUSTOMERS
    - ROUTES
    - SCHEDULES
    - ...
  - SIZE
  - DATA AVAILABILITY

- **Academic Problem**
  - TYPE OF GRAPH
  - SIZE
  - PROBLEM ATTRIBUTES
    - DYNAMIC, STOCHASTIC setting?

- **Heuristic Resolution**
  - Metaheuristic strategies, decompositions, parallelism...
  - Assignment of routes and customers to resources
  - Sequences choices
  - Evaluation of fixed sequences, timing or loading sub-problems
Classification & Proposed Methodology

- **ASSIGN ATTRIBUTES**: impacting the assignment of customers and routes
  - Periodic, Multi-Depot, Heterogeneous Fleet, Location Routing...

- **SEQ ATTRIBUTES**: impacting the nature of the network and the sequences
  - P&D, Backhauls, Two Echelon, Multi Trips, Truck-and-Trailer...

- **EVAL ATTRIBUTES**: impacting the evaluation of fixed routes
  - Time windows, Time-dep. travel time, Loading constraints, HOS regulations, Lunch breaks, Load-Dependent costs...
Classification & Proposed Methodology

- **Challenge**: Achieving both genericity and efficiency
  - Still need to address the problem → but relegating problem-specificities to small modular components
  - Each separate MAVRP shall be still addressed with state-of-the-art solution evaluation and search procedures
  - Not dealing with “dummy” attributes
Proposed Methodology:

- Relying on assignment, sequencing & route evaluation operators → implemented in a generic way, based on a library of attribute-specific modules.

- Attribute-dependent modules are automatically selected by the algorithm to serve as the basis for the assignment, sequencing, and route evaluation operators → Object-oriented programming, using inheritance and polymorphism.
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An efficient and unified local search for MAVRPs

- Route Evaluation Operators based on re-optimization
  - Main Property: Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
  - The same subsequences appear many times during different moves.
  - Data preprocessing on sub-sequences to speed up the search (Savelsbergh 1985, 1992 ...)
  - The route evaluation modules must allow for such preprocessing.
Route Evaluation Operators based on re-optimization

- Main Property: Any local-search move involving a bounded number of node relocations or arc exchanges can be assimilated to a concatenation of a bounded number of sub-sequences.
- Hence, to manage and exploit information on subsequences, five families of route evaluation operators are used:

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**Operators for data construction:**

- **INIT(σ)**: Initialize the data $D(v_0)$ for a sub-sequence containing a single visit.
- **FORW(σ)**: Compute the data of $D(σ ⊕ v_i)$ from the data of sub-sequence $σ$ and vertex $v_i$.
- **BACK(σ)**: Compute the data of $D(v_i ⊕ σ)$ from the data of vertex $v_i$ and sub-sequence $σ$.

**Operators for route evaluations:**

- **EVAL2(σ₁, σ₂)**: Evaluate the cost and feasibility of the combined sequence $σ₁ ⊕ σ₂$.
- **EVALN(σ₁, ..., σₙ)**: Evaluate the cost and feasibility of the combined sequence $σ₁ ⊕ ... ⊕ σₙ$.
Example 1) Route evaluation operators for **distance and capacity** constraints

**What is managed?** → Partial loads \( L(\sigma) \) and distance \( D(\sigma) \)

**Init** → For a sequence \( \sigma_0 \) with a single visit \( v_i \), \( L(\sigma_0) = q_i \) and \( D(\sigma_0) = 0 \)

**Forw and Back** → increment \( L(\sigma) \) and \( D(\sigma) \)

**Eval** → compute the data by induction on the concatenation operator

\[
Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2)
\]

\[
D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)}\sigma_2(1) + D(\sigma_2)
\]
Example 2) Route evaluation operators for **cumulated arrival time objectives**

What is managed?  \( \rightarrow \) Travel time \( D(\sigma) \), Cumulated arrival time \( C(\sigma) \), Delay Cost \( W(\sigma) \) associated to one unit of delay in starting time

Init  \( \rightarrow \) For a sequence \( \sigma_0 \) with a single visit \( v_i \), \( D(\sigma_0) = 0 \) and \( C(\sigma_0) = 0 \), and \( W(\sigma_0) = 1 \) if \( v_i \) is a customer, and \( W(\sigma_0) = 0 \) if \( v_i \) is a depot visit.

Forw & Back & Eval  \( \rightarrow \) induction on the concatenation operator:

\[
\begin{align*}
D(\sigma_1 \oplus \sigma_2) &= D(\sigma_1) + d_{\sigma_1(|\sigma_1|)}(\sigma_2(1)) + D(\sigma_2) \\
C(\sigma_1 \oplus \sigma_2) &= C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)}(\sigma_2(1))) + C(\sigma_2) \\
W(\sigma_1 \oplus \sigma_2) &= W(\sigma_1) + W(\sigma_2)
\end{align*}
\]
Example 3) Route evaluation operators for time windows (and route duration constraints)

What is managed? → Travel time and service time \(T(\sigma)\), earliest feasible completion time \(E(\sigma)\), latest feasible starting date \(L(\sigma)\), statement of feasibility \(F(\sigma)\).

Init → For a sequence \(\sigma_0\) with a single visit \(v_i\), \(T(\sigma_0) = s_i\), \(E(\sigma_0) = e_i + s_i\), \(L(\sigma_0) = l_i\) and \(F(\sigma_0) = \text{true}\).

Forw & Back & Eval → induction on the concatenation operator:

\[
T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(\|\sigma_1\|)}\sigma_2(1) + T(\sigma_2) \\
E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(\|\sigma_1\|)}\sigma_2(1) + T(\sigma_2), E(\sigma_2)\} \\
L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(\|\sigma_1\|)}\sigma_2(1) - T(\sigma_1)\} \\
F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \land F(\sigma_2) \land (E(\sigma_1) + d_{\sigma_1(\|\sigma_1\|)}\sigma_2(1) \leq L(\sigma_2))
\]
Example 4) Route evaluation operators for lunch break positioning in presence of time-window constraints

What is managed? ➔ Same set of data \((T(\sigma), E(\sigma), L(\sigma), \text{and } F(\sigma))\) as in the TW case, and it is duplicated to also provide \(T'(\sigma), E'(\sigma), L'(\sigma), \text{and } F'(\sigma)\) for the sequence where exactly one lunch break was inserted.

Init ➔ As previously for \(T(\sigma_0), E(\sigma_0), L(\sigma_0), \text{and } F(\sigma_0)\). Furthermore, \(T'(\sigma_0) = +\infty, E'(\sigma_0) = +\infty, L'(\sigma_0) = 0, \text{and } F'(\sigma_0) = \text{false}\).

Forw & Back & Eval ➔ induction on the concatenation operator, see next page for the equations.
Example 4) Route evaluation operators for lunch break positioning in presence of time-window constraints

\[
E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case } i} | F'_{\text{case } i} = \text{true}\} \cup +\infty)
\]
\[
L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case } i} | F'_{\text{case } i} = \text{true}\} \cup -\infty)
\]
\[
F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case } 1} \lor F'_{\text{case } 2} \lor F'_{\text{case } 3}
\]

\[
E'_{\text{case } 1} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)}\sigma_2(1) + T(\sigma_2), E(\sigma_2)\}
\]
\[
E'_{\text{case } 2} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)}\sigma_2(1) + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\}
\]
\[
E'_{\text{case } 3} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)}\sigma_2(1) + T'(\sigma_2), E'(\sigma_2)\}
\]
\[
L'_{\text{case } 1} = \min\{L'(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)}\sigma_2(1) - T'(\sigma_1)\}
\]
\[
L'_{\text{case } 2} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - p_{\sigma_1(|\sigma_1|)}\sigma_2(1) - s_{\text{LB}} - T(\sigma_1)\}
\]
\[
L'_{\text{case } 3} = \min\{L(\sigma_1), L'(\sigma_2) - p_{\sigma_1(|\sigma_1|)}\sigma_2(1) - T(\sigma_1)\}
\]
\[
F'_{\text{case } 1} = F''(\sigma_1) \land F(\sigma_2) \land (E'(\sigma_1) + p_{\sigma_1(|\sigma_1|)}\sigma_2(1) \leq L(\sigma_2))
\]
\[
F'_{\text{case } 2} = F(\sigma_1) \land F(\sigma_2) \land (E(\sigma_1) \leq l_{\text{LB}}) \land (E(\sigma_1) + s_{\text{LB}} + p_{\sigma_1(|\sigma_1|)}\sigma_2(1) \leq L(\sigma_2))
\]
\[
F'_{\text{case } 3} = F(\sigma_1) \land F'(\sigma_2) \land (E(\sigma_1) + p_{\sigma_1(|\sigma_1|)}\sigma_2(1) \leq L'(\sigma_2))
\]
Example 5) Route evaluation operators for soft and general time windows

What is managed? → Minimum cost $F(\sigma)(t)$ to process the sequence $\sigma$ while starting the last service before time $t$, minimum cost $B(\sigma)(t)$ to process the sequence $\sigma$ after time $t$.

Init → For a sequence $\sigma_0$ with a single visit $v_i$ characterized by a service cost function $c_i(t)$, $F(\sigma_0)(t) = \min_{x \leq t} c_i(x)$ and $B(\sigma_0)(t) = \min_{x \geq t} c_i(x)$.

Forw & Back →

$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - s_\sigma(|\sigma|) - d_\sigma(|\sigma|, i))\}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t} \{c_i(t) + B(\sigma)(x + s_i + d_{i, \sigma(1)})\}$$

Eval 2 →

$$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\}$$
Example 6) Route evaluation operators for the generalized VRP:

What is managed? → The shortest path $S(\sigma)[i,j]$ inside the sequence $\sigma$ starting at the location $i$ of the starting group and finishing at location $j$ of the ending group.

Init → For a sequence $\sigma_0$ with a single visit $v_i$, $S(\sigma)[i,j] = +\infty$ if $i \neq j$, and $S(\sigma)[i,i] = 0$.

Forw & Back & Eval → induction on the concatenation operator:

$$S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j]$$

\[\forall i \in \{1, \ldots, \lambda_{\sigma_1(|\sigma_1|)}\}, \forall j \in \{1, \ldots, \lambda_{\sigma_2(|\sigma_2|)}\}\]
An efficient and unified local search for MAVRPs

- Generic local-search based on route evaluation operators

Algorithm 1 Unified local search based on route evaluation operators

1: Detect the good combination of evaluation operators relatively to the problem attributes
2: Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
3: while some improving moves exist in the neighborhood \( \mathcal{N} \) do
4:   for each move \( \mu_i \) in \( \mathcal{N} \) do
5:     for each route \( r_{ij}^\mu \) produced by the move do
6:       Determine the \( k \) sub-sequences \([\sigma_1, \ldots, \sigma_k]\) that are concatenated to produce \( r_{ij}^\mu \)
7:       if \( k = 2 \), then \( \text{NEWCOST}(r) = \text{EVAL}_2(\sigma_1, \sigma_2) \)
8:       else if \( k > 2 \), then \( \text{NEWCOST}(r) = \text{EVAL}_N(\sigma_1, \ldots, \sigma_k) \)
9:       if ACCEPTCRITERIA(\( \mu_i \)) then perform the move \( \mu \) and update the re-optimization data on for each route \( r_{ij}^\mu \) using the INIT, FORW and BACK operators.

- Can serve as the basis to build any neighborhood-based unified solver based on VNS, Tabu, ILS for MAVRPs with EVAL attributes.
- Going one step further, designing a unified hybrid GA.
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A Unified Hybrid Genetic Search (UHGS) for MAVRPs

- UHGS = Classic GA framework + 4 main ingredients (Vidal et al. 2010)
  - Management of penalized infeasible solutions in two subpopulations
  - High-performance local search-based *Education* procedure
  - Solution Representation *without trip delimiters*
  - Diversity & Cost objective for individuals evaluations
General Framework of UHGS:

- Initialize Population
- POPULATION WITH DIVERSITY MANAGEMENT
  - Penalties adaptation
  - Survivors' selection
  - Diversification and decomposition phases
- [if terminated]
  - Return Best Solution
- [if not terminated]
  - SELECTION
    - Binary Tournament
    - Based on COST & DIVERSITY
  - PIX CROSSOVER
  - SPLIT
    - Placement of depot occurrences for each resource
  - MERGE
    - Removal of trip delimiters
  - EDUCATION and REPAIR with probability $P_{\text{rep}}$
    - Based on local-search

VRP ATTRIBUTES

ASSIGNMENT OPERATORS

ROUTE EVALUATION OPERATORS
Now dealing with MAVRPs with both ASSIGN and EVAL attributes:
Assignment of customer services to some ASSIGN attributes resources (AARs) + separate optimization of routes for each AARs.

- Solution representation is designed accordingly.
- Furthermore, representation without trip delimiters for each AAR.
Unified Solution Representation and Split

- Solution representation as a giant-tour per AAR requires a Split algorithm (Prins 2004) for optimal segmentation into routes.

- We propose a unified Split algorithm.
  - As usual, the problem is solved as a $m$-shortest path
  - The route evaluation operators are used to build the auxiliary graph

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**Algorithm 2 Generic Split**

1. for each node $i \in \{0, \ldots, \nu\}$ do
2.   $SeqData(\sigma) = \text{INIT}(\{v_0\})$ //Initialize with depot vertex
3.   for each node $j \in \{i, \ldots, \min(i + \bar{r}, \nu)\}$ do
4.     $\phi(a_{ij}) = \text{EVAL2}(\sigma, \{v_0\})$ //Evaluate the route
5.     $SeqData(\sigma) = \text{FORW}(\sigma, \{\tau_j\})$ //Append a new customer to the route end
6.   Solve the shortest path problem on $\mathcal{G'} = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc $a_{ij}$
7.   Return the set of routes associated to the set of arcs of the shortest path
Unified Crossover Operator

- 4 phases Assignment and Insertion Crossover (AIX), to produce a single offspring C from two parents P1 and P2.

- Step 1) Choose for each AAR whether the genetic material of P1, P2, or both parents is inherited.

- Step 2) Fully transmit the selected material from P1

- Step 3) Complete with the selected material from P2, check at each step with an Assignment module whether the inheritance respects the ASSIGN attributes specifications.

- Step 4) Perform a best insertion of missing visits.
Unified Crossover Operator

Step 1) Visits from $P_1$

Step 2) Visits from $P_2$

Step 3) Filling Remaining services

Offspring C
Giant tour chromosome
Unified Education Procedure

- Based on the previously described Unified Local Search to perform route improvement (RI) on separate AAR.
  - Using CROSS, I-CROSS, Relocate, 2-Opt* and 2-Opt neighborhoods
  - Pruning procedures (granular search)
  - Hybrid acceptance strategy (intermediate between first improvement and best improvement)

- Combined with an assignment-improvement (AI) procedure to re-assign customer visits into different resources and routes.

- These two procedures are called in the sequence RI-AI-RI.
Population management and search guidance

- **Biased Fitness** is a tradeoff between ranks in terms of solution penalized cost $cost(I)$, and **contribution to the diversity** $dc(I)$, measured as a distance to others individuals in the population.

\[ BF(I) = fit(I) + (1 - \frac{nbElit}{nbIndiv - 1}) \times dc(I) \]

- Used during selection of the parents
  - Balancing strength with innovation during reproduction, and thus favoring exploration of the search space.

- and during selection of the survivors:
  - Removing the individual $I$ with worst $BF(I)$ also guarantees some elitism in terms of solution value.
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4. A Unified Hybrid Genetic Search (UHGS) for MAVRPs
   a) General framework
   b) Unified solution representation and Split
   c) Generic implementation of other genetic operators

5. Computational experiments
Comparison with problem-tailored state-of-the-art methods

- Extensive computational experiments on 26 structurally different VRP variants and 39 sets of benchmark instances.

- Comparing UHGS with the best problem-tailored method for each benchmark.

- In the following, we indicate for each method:
  - % Gap to the BKS of an average run (out of 10 for UHGS).
  - % Gap to the BKS of a best run (out of 10 for UHGS).
  - Computational effort (total work time) for an average run.
  - Type of processor used.
Comparison with problem-tailored state-of-the-art methods

<table>
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<tr>
<th>Variant</th>
<th>Bench.</th>
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Comparison with problem-tailored state-of-the-art methods

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### Comparison with problem-tailored state-of-the-art methods

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### Comparison with problem-tailed state-of-the-art methods

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Comparison with problem-tailed state-of-the-art methods

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List of acronyms for state-of-the-art algorithms

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<td>Repoussis et al. (2009a)</td>
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<td>Fu et al. (2007)</td>
<td>PBDH08</td>
<td>Polacek et al. (2008)</td>
<td>XZKX12</td>
<td>Xiao et al. (2012)</td>
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Conclusions and Research Perspectives

- A unified hybrid genetic search
  - Using a local-search framework which is generic and computationally efficient.
  - With generalized solution representation, Split procedure, genetic operators (Crossover) and population management methods.
  - State-of-the-art results when compared to each problem-tailored method for 26 VRP variants.

- It appears that **generality does not necessarily impede performance for a wide class of VRP variants.**
Perspectives:

- Extend the range of problems (especially SEQ attributes, stochastic and multi-objective settings)
- Use UHGS to conduct experiments on metaheuristic strategies on a wide range of VRPs
- Further study of the combinatorial aspect of attributes relatively to UHGS operators.
Thanks for your attention

THANK YOU

- For more details on this work:
  - The CIRRELT technical report on the unified algorithm will appear very soon.
Empirical studies on diversity management methods (1/2)

- Sensitivity analysis on diversity management methods:
  - **HGA**: No diversity management method
  - **HGA-DR**: Dispersal rule on objective space
  - **HGA-PM**: Dispersal rule on solution space
  - **HGSADC**: The proposed approach

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>HGA</th>
<th>HGA-DR</th>
<th>HGA-PM</th>
<th>HGSADC</th>
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</table>
Behavior of HGSADC during a random run:

- Higher entropy (average distance between two individuals)
- Better final solution
- Diversity can increase during run time

Empirical studies on diversity management methods (2/2)